Semantics

An imperative language has boolean expressions $be$, integer expressions $ie$, and commands $C$, whose abstract syntax is specified by:

\[
\begin{align*}
    ie & ::= n \mid X \mid ie + ie \mid ie - ie \\
    be & ::= b \mid ie = ie \\
    C & ::= \text{skip} \mid X := ie \mid C \mid \text{if } be \text{ then } C \text{ else } C \mid \text{while } be \text{ do } C
\end{align*}
\]

where $b$ is $true$ or $false$, $n$ is any integer, and $X$ ranges over a fixed set of variables.

Describe the operational semantics of the language in terms of inductively defined evaluation relations

\[
be, S \Rightarrow b \quad ie, S \Rightarrow n \quad \text{and} \quad C, S \Rightarrow S'
\]

where $S$ and $S'$ are integer-valued functions on the set of variables. \[5 \text{ marks}\]

In what sense are these evaluation relations determinstic? What is meant by the assertion that two commands are semantically equivalent? \[3 \text{ marks}\]

For any choice of $be$, $C$ and $C'$, which of the following pairs of commands are semantically equivalent and which are not? Justify your answer in each case.

(a) $((\text{while } be \text{ do } C) ; C)$ and $(\text{if } be \text{ then } ((\text{while } be \text{ do } C) ; C) \text{ else } C)$

(b) $(C ; (\text{while } be \text{ do } C))$ and $(\text{if } be \text{ then } (\text{while } be \text{ do } C) \text{ else } C)$

(c) $(\text{while } be \text{ do } (\text{if } be \text{ then } C \text{ else } C'))$ and $(\text{while } be \text{ do } C)$

\[12 \text{ marks}\]