Data Structures and Algorithms

A directed graph of \( n \) nodes numbered 1, 2, \ldots, \( n \) can be represented by an \( n \times n \) adjacency matrix \( G_1 \), where \( G_1[i, j] \) is true if there is an edge connecting node \( i \) to node \( j \), and \( G_1[i, j] \) is false otherwise.

By extension, define \( G_k \) to be that matrix such that \( G_k[i, j] \) is true if there is a path of length \( \leq k \) connecting node \( i \) to node \( j \), and \( G_k[i, j] \) is false otherwise.

Describe an algorithm to generate \( G_2 \) from \( G_1 \). \[12 \text{ marks}\]

How could this algorithm be used to generate the transitive closure of a graph given its adjacency matrix? \[5 \text{ marks}\]

What is the cost of this transitive closure algorithm in terms of \( n \) and \( m \), where \( m \) is the maximum path length in the transitive closure? \[3 \text{ marks}\]