1993 Paper 3 Question 3

Foundations of Logic Programming

Consider the following set of clauses, where x is a variable and b is a Skolem constant:

$$\{\neg P(x), \neg Q(x), P(f(x))\} \tag{1}$$

$$\{\neg P(x), Q(x)\} \tag{2}$$

$$\{P(b)\} \tag{3}$$

$$\{\neg P(f(f(f(f(x)))))\} \tag{4}$$

(a) How many resolution steps are required to derive the empty clause from these clauses? Justify your answer clearly. [5 marks]

(b) Prolog uses resolution in a restricted form. How many Prolog-style resolution steps are required to derive the empty clause, taking (4) as the goal clause and (1)–(3) as program clauses? Justify your answer clearly. [5 marks]

(c) How do a resolution theorem-prover and a Prolog system differ in their implementation of resolution? [4 marks]

(d) The theorem $(\forall y)(\exists x)\neg(p(x,y) \leftrightarrow \neg((\exists z)(p(x,z) \land p(z,x))))$ is to be proved using the resolution method. Convert the problem to clause form, showing all steps. (Omit the resolution proof itself.) [6 marks]