$X$ and $Y$ are independent random variables having Poisson distributions with parameters $\alpha$ and $\beta$ respectively. By using probability generating functions, or otherwise, prove that $X + Y$ has a Poisson distribution and give its parameter.

Find the conditional distribution for $X$ given that $X + Y = n$, and give its mean and variance. Explain your result in words.