1993 Paper 2 Question 1

Catalan numbers may be characterised through the set β of well-formed bracketings. The following rules define membership of β:

(a) the null string $\lambda \in \beta$;

(b) $S \in \beta \Rightarrow (S) \in \beta$;   (NESTING)

(c) $S_1 \in \beta, S_2 \in \beta \Rightarrow S_1 S_2 \in \beta$.   (CONCATENATION)

Show that the number of different well-formed bracketings that can be made with $2n$ brackets is

$$\frac{1}{n+1}\binom{2n}{n}.$$

Suppose that an extra rule

(b') $S \in \beta \Rightarrow\ < S >\ \in \beta$;   (ANGLE-NESTING)

is introduced in addition to (a)–(c). How many bracketings of length $2n$ will there now be?