1 Distributed Systems

A distributed software system follows the client-server model. The microkernel on which it is based supports multi-threaded processes. A remote procedure call (RPC) package is used for client-server interactions. The RPC system runs above an unreliable, datagram-based communications service.

(a) Explain how timers may be used in the RPC protocol to achieve client-server synchronisation. [10 marks]

(b) Discuss how the RPC system may support the location of remote procedures. [7 marks]

(c) Discuss the requirements on the RPC system that follow from the use of multi-threaded processes. [3 marks]

2 Distributed Systems

A distributed computation may involve related operations on a number of objects which reside at different nodes of a distributed system.

(a) Explain why the concept of transaction is suitable for modelling such a computation. [6 marks]

(b) Explain what is involved in committing a transaction in a distributed system. [14 marks]
3 Comparative Architectures

A revisionist view might be that a processor design is RISC if every transistor pays its way in terms of global system performance, i.e. the design is in some sense a local optimum. Explain how this view relates to the original definition of RISC. Can a CISC instruction set be RISC under such a definition? [4 marks]

Given a certain silicon budget (e.g. $10^6$ transistors) compare and contrast, under the above criterion, alternative ways of spending the excess over the (say) $10^4$ transistor cost of a simple load-store one-accumulator machine. You might find it helpful to consider the various features and instructions of common processors. [12 marks]

This budget is sufficient to build most, but not all, of a VAX. Which parts would you omit and how could you arrange to execute full VAX code? [4 marks]

4 Comparative Architectures

Compare and contrast possible implementations of instructions which

(a) load a single byte from memory [8 marks]

(b) store a single byte in memory [4 marks]

(c) move a sequence of $n$ bytes from one address to another [8 marks]

For each implementation, indicate briefly whether it conforms to the RISC philosophy, its effect on pipelining, and its likely efficiency compared with any alternatives.
5 Specification and Verification of Hardware

You are given components \textsc{mux}, \textsc{reg} and \textsc{comp} whose behaviour is defined by

\[
\textsc{mux}(\text{sw},\text{in1},\text{in2},\text{out}) = (\forall t. \text{out } t = (\text{sw } t \rightarrow \text{in1 } t \mid \text{in2 } t))
\]

\[
\text{reg } v (\text{in, out}) = (\text{out } 0 = v) \land (\forall t. \text{out}(t+1) = \text{in } t)
\]

\[
\text{comp}(\text{in1, in2, out}) = (\forall t. \text{out } t = (\text{in1 } t < \text{in2 } t))
\]

Use these to implement a device \textsc{max} that satisfies the specification

\[
\text{max}(\text{in, out}) \Rightarrow (\forall t. \text{out } t = \text{max } \text{in } t)
\]

where the function \text{max} is defined by

\[
(\text{max } \text{in } 0 = \text{in } 0) \land (\text{max } \text{in } (n+1) = (\text{max } \text{in } n < \text{in}(n+1) \rightarrow \text{in}(n+1) \mid \text{max } \text{in } n))
\]

[10 marks]

Prove that your implementation meets its specification. [10 marks]

6 Specification and Verification of Hardware

Describe the simple switch model of CMOS transistors. [5 marks]

Draw a circuit diagram of a CMOS inverter and give a proof that it is correct in the simple switch model. [2 + 3 marks]

Describe the unidirectional model of NMOS transistors. [5 marks]

Draw a circuit diagram of an NMOS inverter and give a proof that it is correct in the unidirectional model. [2 + 3 marks]
7 Numerical Analysis II

State a recurrence formula suitable for evaluating the sequence of Chebyshev polynomials \( \{T_n(x)\} \) for an argument \( x \). What are the starting values? [2 marks]

The error in Lagrange interpolation can be expressed in the form

\[
 f(x) - L_{n-1}(x) = \frac{f^{(n)}(\zeta)}{n!} \prod_{j=1}^{n} (x - x_j)
\]

for a suitable function \( f(x) \). Suggest a choice of the interpolation points \( \{x_j\} \) which tends to minimise this error over the interval \([-1, 1]\). [3 marks]

Hence justify and explain the method of economisation of a power series. [5 marks]

In what sense is an economised power series a best approximation? [2 marks]

Suppose \( P_n(x) \) is a polynomial formed by truncating a power series after the term in \( x^n \). Perform an economisation of the truncated power series

\[
 \cosh x \approx P_4(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!}
\]

[5 marks]

Given that the maximum error in \( P_4(x) \) over \([-1, 1]\) is approximately 0.0014, compare the error in your economised polynomial with the error in \( P_2(x) \). [3 marks]
8 Numerical Analysis II

If $B$ is a real symmetric $n \times n$ matrix such that $z^T B z \geq 0$ for any complex vector $z$, prove that any eigenvalue $\lambda$ of $B$ is such that $\lambda \geq 0$. Hence prove that the eigenvalues of $A^T A$, where $A$ is any real square matrix, are real and non-negative.

[3 marks]

Let $P, Q$ be real $n \times n$ matrices and let $\|P\|_2^2$ denote the maximum eigenvalue of $P^T P$. State Schwarz’s inequality for $\|PQ\|_2$. Explain how this is modified if $Q$ is replaced by a vector of $n$ elements.

[3 marks]

Derive the condition number $K$ for solution of the equations $Ax = b$. Hint: start by setting $e = x - \hat{x}$ where $\hat{x}$ is an approximate solution.

[5 marks]

Describe the singular value decomposition $A = UWV^T$

and explain how you would use it to solve the $n$ equations $Ax = b$ when $W$ has rank $n$.

[5 marks]

How may the singular value decomposition help in solving the equations $Ax = b$ when $A$ has rank $< n$? Use the case $n = 4$, $W = \text{diag}\{1,10^{-3},10^{-20},0\}$ to illustrate your answer. (You may assume that machine epsilon $\approx 10^{-16}$.)

[4 marks]

9 Graphics II

When scan-converting items for display, a Z-buffer is sometimes used to avoid some sorting. Outline its operation and limitations.

[12 marks]

The use of an A-buffer will improve matters. Explain why.

[8 marks]
An imperative language has boolean expressions $be$, integer expressions $ie$, and commands $C$, whose abstract syntax is specified by:

$$
ie ::= n | X | ie + ie | ie - ie$$

$$be ::= b | ie = ie$$

$$C ::= \text{skip} | X := ie | C ; C | \text{if } be \text{ then } C \text{ else } C | \text{while } be \text{ do } C$$

where $b$ is $\text{true}$ or $\text{false}$, $n$ is any integer, and $X$ ranges over a fixed set of variables. Describe the operational semantics of the language in terms of inductively defined evaluation relations

$$be, S \Rightarrow b \quad ie, S \Rightarrow n \quad \text{and} \quad C, S \Rightarrow S'$$

where $S$ and $S'$ are integer-valued functions on the set of variables.

In what sense are these evaluation relations deterministic? What is meant by the assertion that two commands are semantically equivalent?

For any choice of $be$, $C$ and $C'$, which of the following pairs of commands are semantically equivalent and which are not? Justify your answer in each case.

(a) $((\text{while } be \text{ do } C) ; C)$ and $(\text{if } be \text{ then } ((\text{while } be \text{ do } C) ; C) \text{ else } C)$

(b) $(C ; (\text{while } be \text{ do } C))$ and $(\text{if } be \text{ then } (\text{while } be \text{ do } C) \text{ else } C)$

(c) $(\text{while } be \text{ do } (\text{if } be \text{ then } C \text{ else } C'))$ and $(\text{while } be \text{ do } C)$
11 Additional Topics

Describe, for a competent programmer, the decoding of a Lempel-Ziv data compression code. Assume that the input is a stream of positive integers.

[10 marks]

Construct a Huffman Code for a seven-letter alphabet, given the following approximate frequencies:

\[
\begin{array}{cccccccc}
\text{a} & \text{b} & \text{c} & \text{d} & \text{e} & \text{f} & \text{g} \\
192 & 58 & 105 & 71 & 315 & 131 & 0
\end{array}
\]

[4 marks]

Give an error-correcting code and explain how it works, excluding the simple repetition code.

[3+3 marks]

12 Additional Topics

Explain the distinction between Public- and Secret-Key cryptosystems. [4 marks]

Name an example of each type. [1+1 marks]

Is a One Time Pad a Public- or Secret-Key system? [1 mark]

Discuss the advantages and disadvantages of One Time Pads as compared with alternative systems. When might One Time Pads be used and when should they be avoided? [13 marks]