Mobile Health
Basics of Signal Processing

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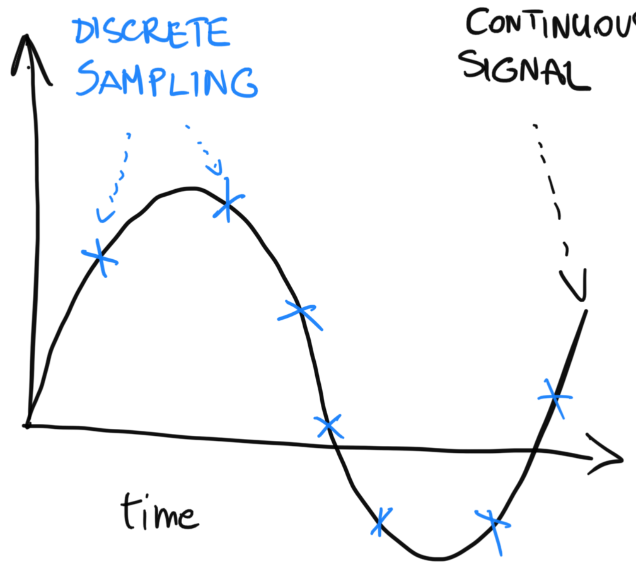
The sensor data

- Most of the sensor data collected by wearable devices' sensors is "time series data".
- A sensor is sampled at specific time intervals; data is discrete but could be seen as relatively "continuous".
- Time domain representation of signal:



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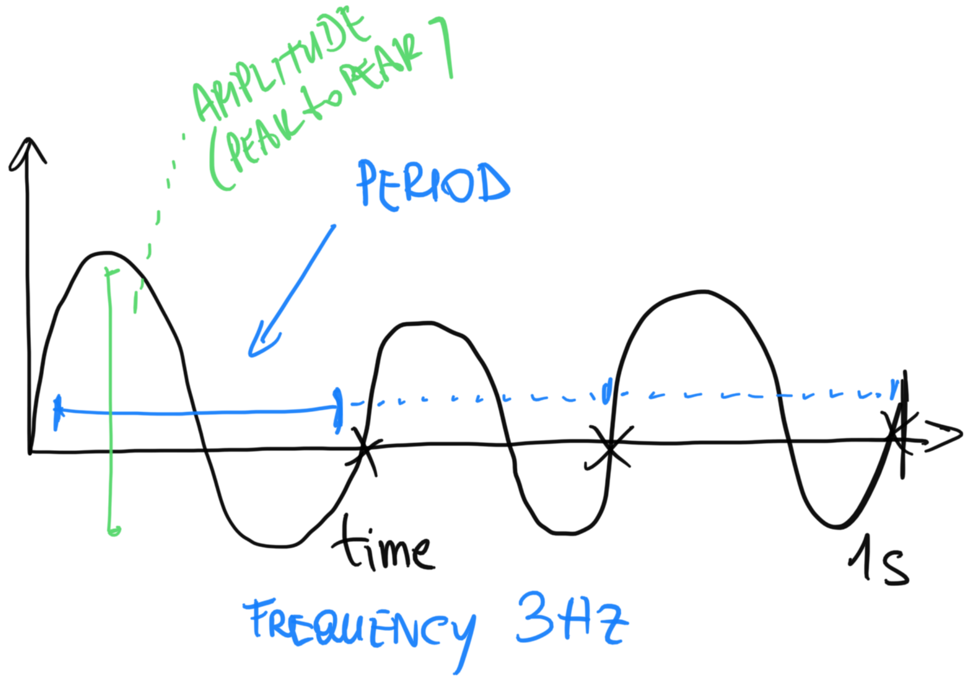
The Continuous Wave

- **Frequency of continuous signal**: how many times a second the wave repeats itself.
  - Measured in **Hertz**: 3Hz means the wave repeats itself three times in one second.
- **Period**: time required to produce a complete waveform.
- (Peak to Peak) **Amplitude**: max height of the wave.



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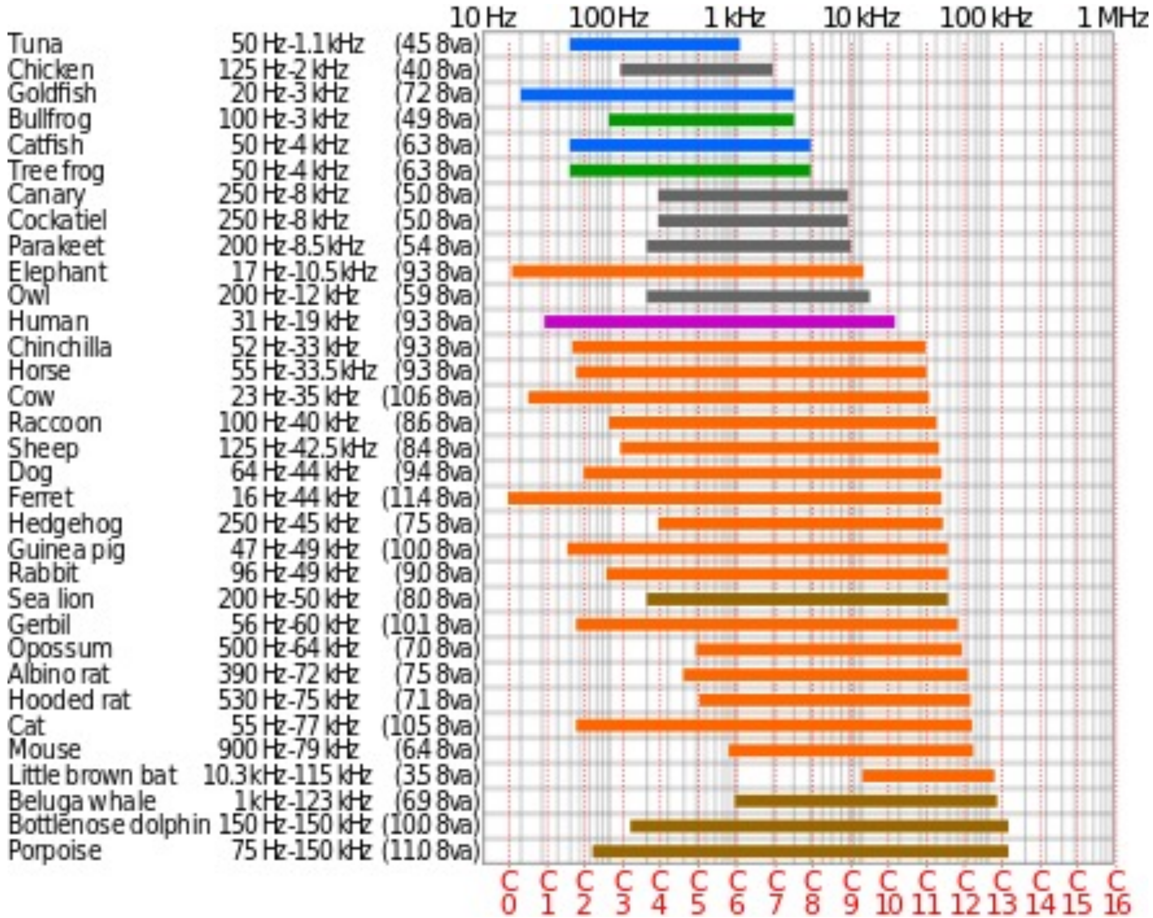
Frequency variations

- Humans hear 20Hz to 20kHz

- Cats hear 55Hz to 79kHz



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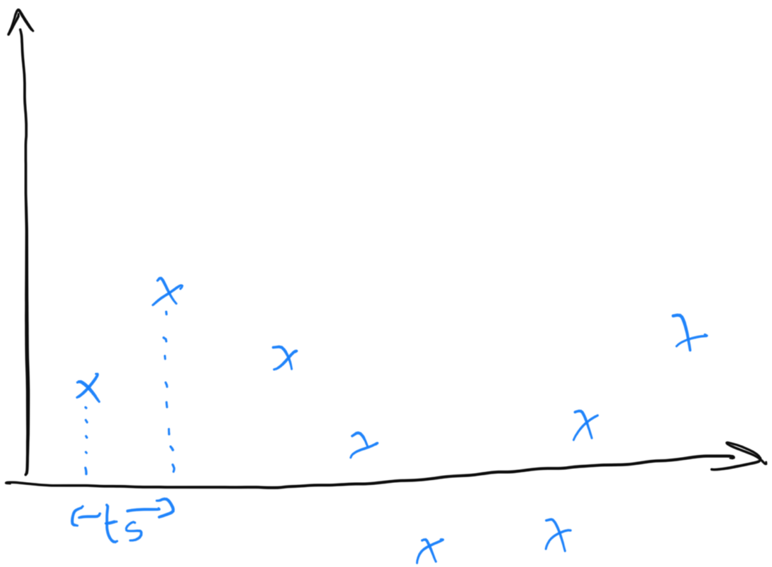
Digital Sampling of Continuous Signal

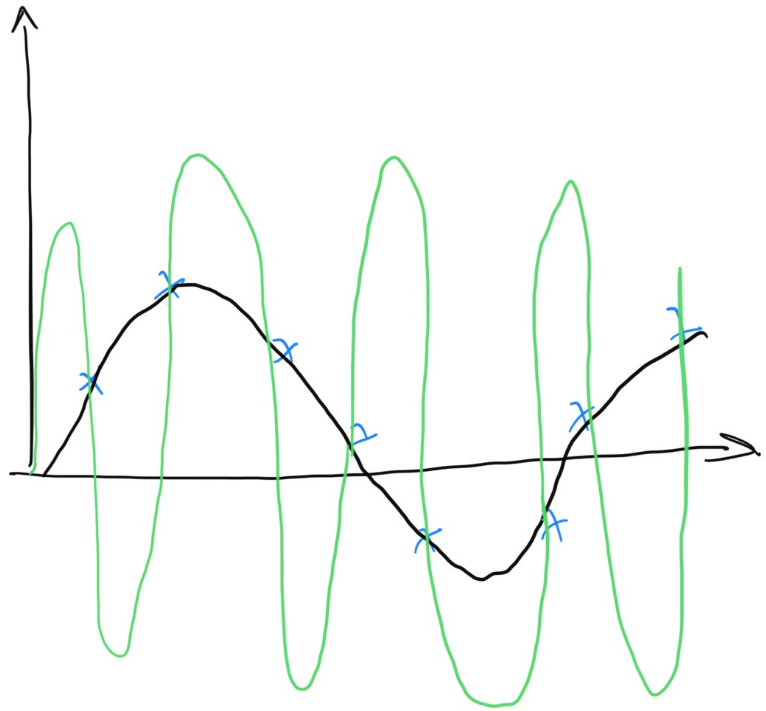
- How do we make sure our digital sampling of a continuous signal records “the important” characteristics of the signal?

- A little bit of signal processing recap ☺

Discrete Sampling of a Signal & Aliasing

t_s is the sampling rate can fit curves at two different frequencies (at least)





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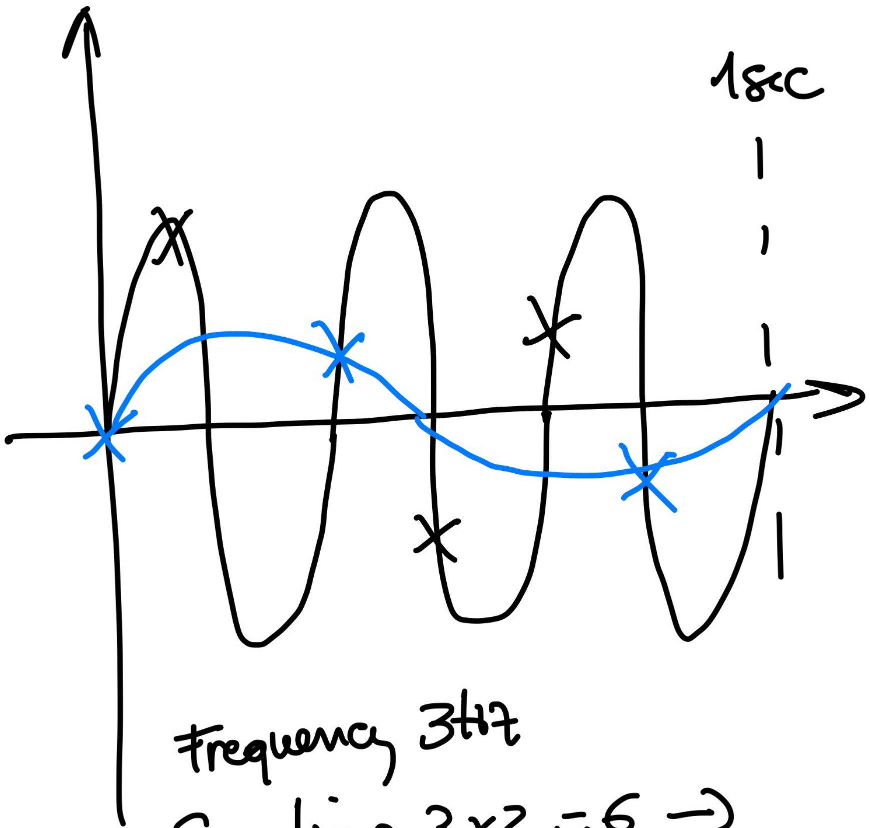
Nyquist Sampling Rate

- To avoid aliasing the sampling frequency should be **double** the maximum frequency to be captured in the signal.
    - **Nyquist sampling rate**: rate you need to sample (at least) to have no aliasing
    - Intuition: sample twice per period!
- What do we do when the sampling rate is fixed?
    - You can only trust frequencies up to half of the sampling rate.

Intuition of why the Nyquist rate works…



Frequency 3Hz
Sampling 3x2 = 6 → unique curve.

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Question for you!

- Audible frequencies of human voice are at max 8KHz
- What is the Nyquist rate we should sample at (at least)?

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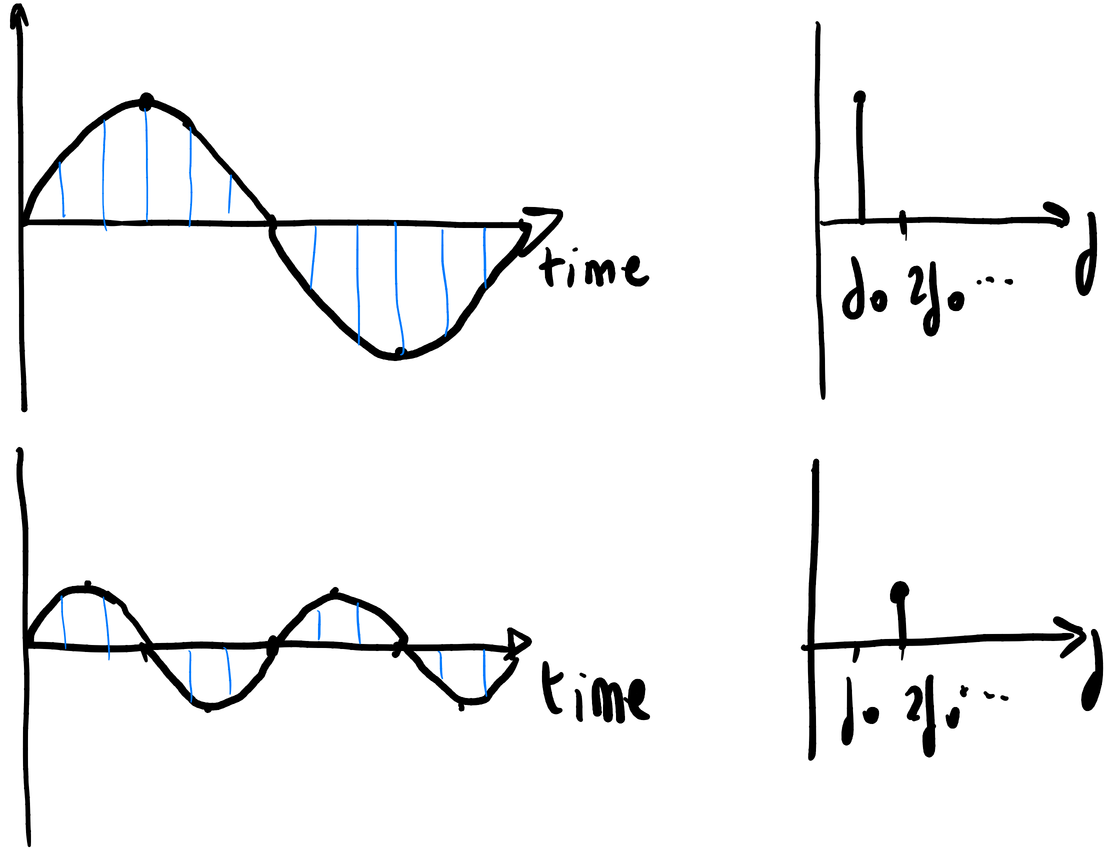
Representation of signal in frequency domain

- Signals can be represented in
    - Time domain
    - Frequency domain



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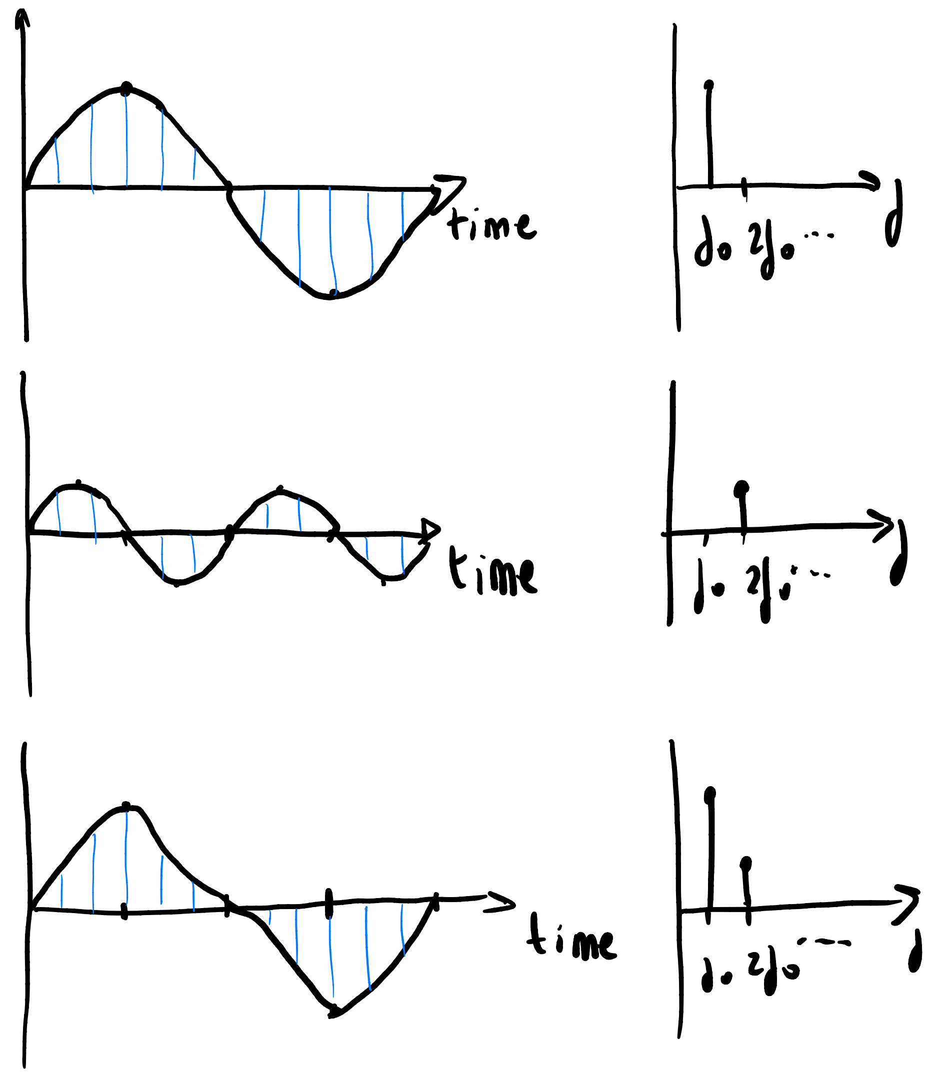
Complex Signals

- Signals are composed of more than one frequency
- Example sum of signals:



Discrete Fourier Transform (DFTs)

- DFTs map the time domain graph into a frequency domain graph
  - They tell us which frequencies are important in the signal
- How? By correlating with various sin/cos waves at different frequencies!

Correlation

$$\sum_{i=0}^{N} x(i)\, y(i)$$

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Intuition on DFTs

- DFTs **correlate** a wave with sin() and cos() waves at different frequencies.

function

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i 2\pi k n / N}$$

analyzing function

$$e^{-i\theta} = \cos\theta - i \sin\theta$$

$$\theta = 2\pi k n / N$$

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DFTs as Correlations of Sinusoids

$$X(k) = \underbrace{\sum_{n=0}^{N-1} x(n)\cos\left(\frac{2\pi kn}{N}\right)}_{\text{CORRELATION with COS}} - i\underbrace{\sum_{n=0}^{N-1} x(n)\sin\left(\frac{2\pi kn}{N}\right)}_{\text{CORRELATION WITH SIN}}$$

Workings of DFTs (assume N=4)

- The result for each X(k) is a complex number which represents a vector

- The magnitude of the vector (ie of the two components) is the amplitude in the frequency domain

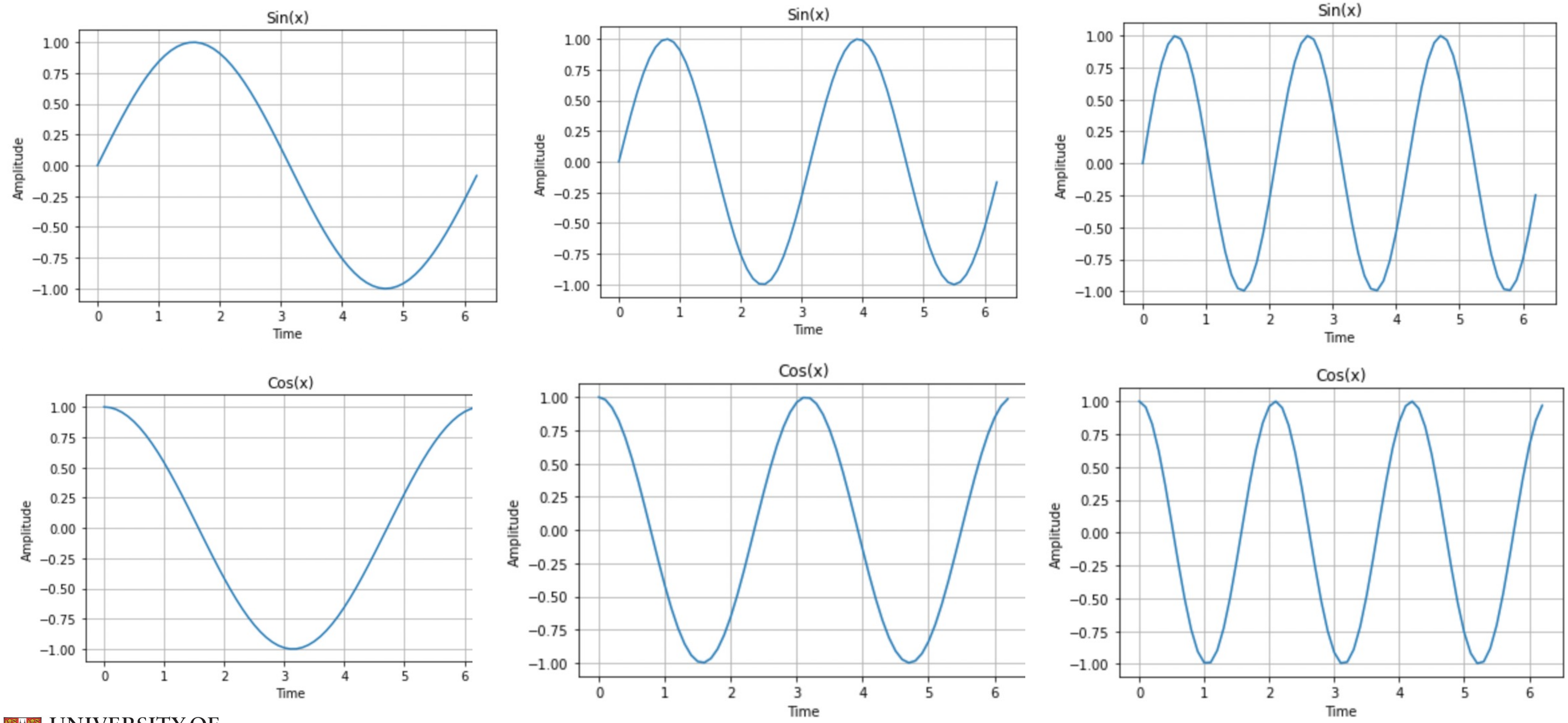
$$
\begin{aligned}
X(0) = {} & x(0)\cos(2\pi\cdot 0\cdot 0/4) - j\,x(0)\sin(2\pi\cdot 0\cdot 0/4) \\
& + x(1)\cos(2\pi\cdot 1\cdot 0/4) - j\,x(1)\sin(2\pi\cdot 1\cdot 0/4) \\
& + x(2)\cos(2\pi\cdot 2\cdot 0/4) - j\,x(2)\sin(2\pi\cdot 2\cdot 0/4) \\
& + x(3)\cos(2\pi\cdot 3\cdot 0/4) - j\,x(3)\sin(2\pi\cdot 3\cdot 0/4)
\end{aligned}
$$

$$
\begin{aligned}
X(1) = {} & x(0)\cos(2\pi\cdot 0\cdot 1/4) - j\,x(0)\sin(2\pi\cdot 0\cdot 1/4) \\
& + x(1)\cos(2\pi\cdot 1\cdot 1/4) - j\,x(1)\sin(2\pi\cdot 1\cdot 1/4) \\
& + x(2)\cos(2\pi\cdot 2\cdot 1/4) - j\,x(2)\sin(2\pi\cdot 2\cdot 1/4) \\
& + x(3)\cos(2\pi\cdot 3\cdot 1/4) - j\,x(3)\sin(2\pi\cdot 3\cdot 1/4)
\end{aligned}
$$

$$
\begin{aligned}
X(2) = {} & x(0)\cos(2\pi\cdot 0\cdot 2\ 4) - j\,x(0)\sin(2\pi\cdot 0\cdot 2/4) \\
& + x(1)\cos(2\pi\cdot 1\cdot 2\ 4) - j\,x(1)\sin(2\pi\cdot 1\cdot 2/4) \\
& + x(2)\cos(2\pi\cdot 2\cdot 2/4) - j\,x(2)\sin(2\pi\cdot 2\cdot 2/4) \\
& + x(3)\cos(2\pi\cdot 3\cdot 2/4) - j\,x(3)\sin(2\pi\cdot 3\cdot 2/4)
\end{aligned}
$$

$$
\begin{aligned}
X(3) = {} & x(0)\cos(2\pi\cdot 0\cdot 3/4) - j\,x(0)\sin(2\pi\cdot 0\cdot 3/4) \\
& + x(1)\cos(2\pi\cdot 1\cdot 3/4) - j\,x(1)\sin(2\pi\cdot 1\cdot 3/4) \\
& + x(2)\cos(2\pi\cdot 2\cdot 3/4) - j\,x(2)\sin(2\pi\cdot 2\cdot 3/4) \\
& + x(3)\cos(2\pi\cdot 3\cdot 3/4) - j\,x(3)\sin(2\pi\cdot 3\cdot 3/4)
\end{aligned}
$$

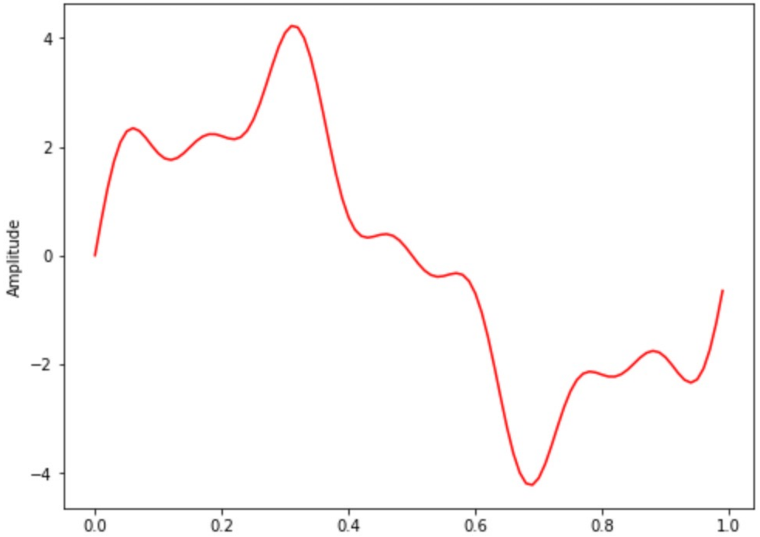
Sin and Cos Waves at different frequencies

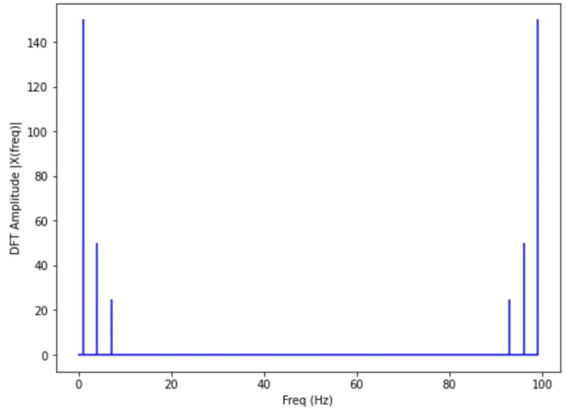


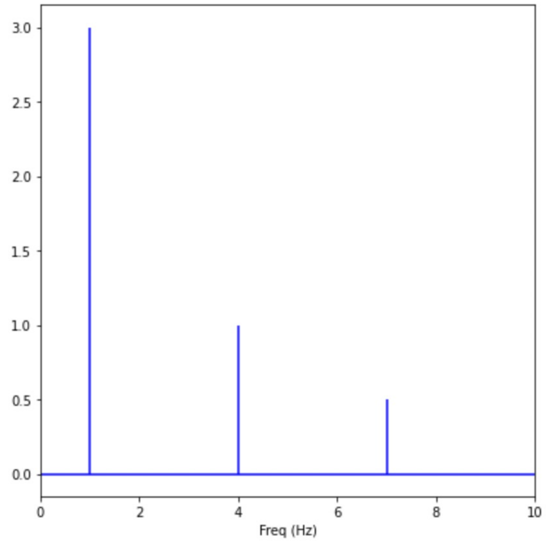
DFT plot

DFT for this wave showing the three frequencies and amplitude.

Wave composed of 3 sine waves with frequencies 1 Hz, 4 Hz, and 7 Hz, amplitudes 3, 1 and 0.5
Sampling rate 100Hz







Example from https://pythonnumericalmethods.berkeley.edu/notebooks/chapter24.02-Discrete-Fourier-Transform.html

Fast Fourier Transforms

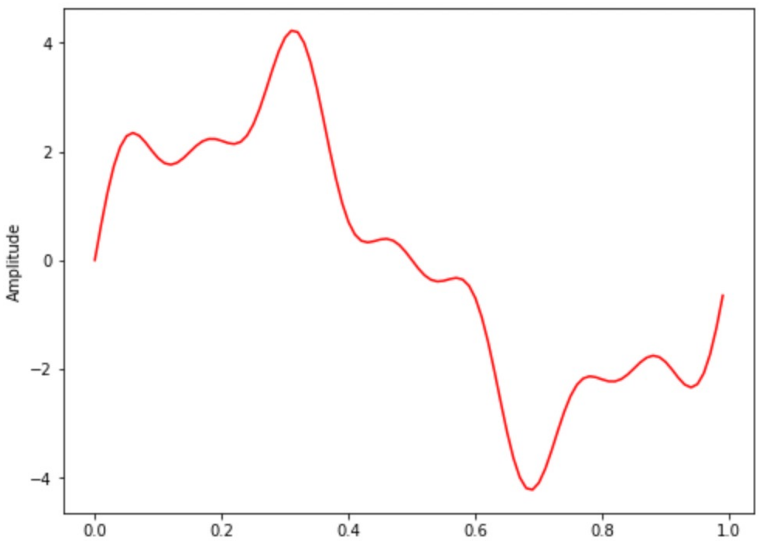
- Fast way of calculating DFT
  - Intuition: Splitting the computation of even and odd n in the formula recursively and parallelize

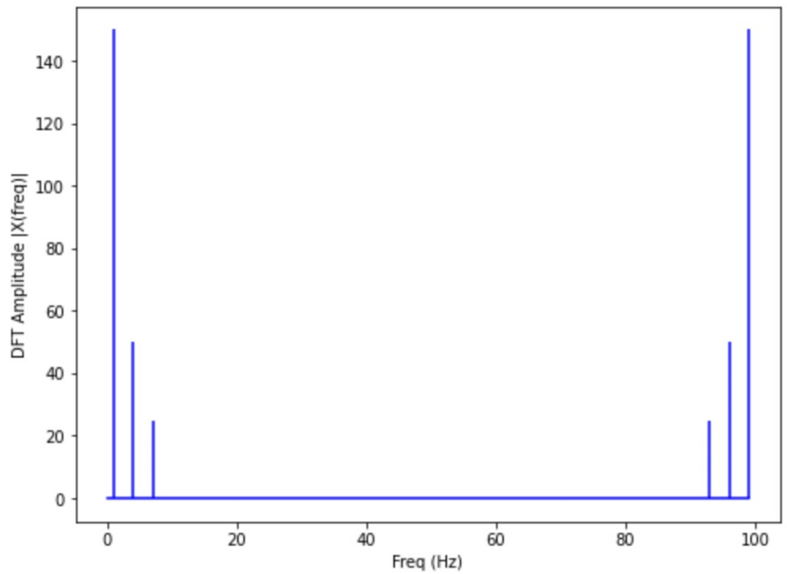
  - DFT ~ O(N^2)

  - FFT ~ O(NlogN)

From time and frequency representations to Spectrograms

- Representation in time and frequency (separately)





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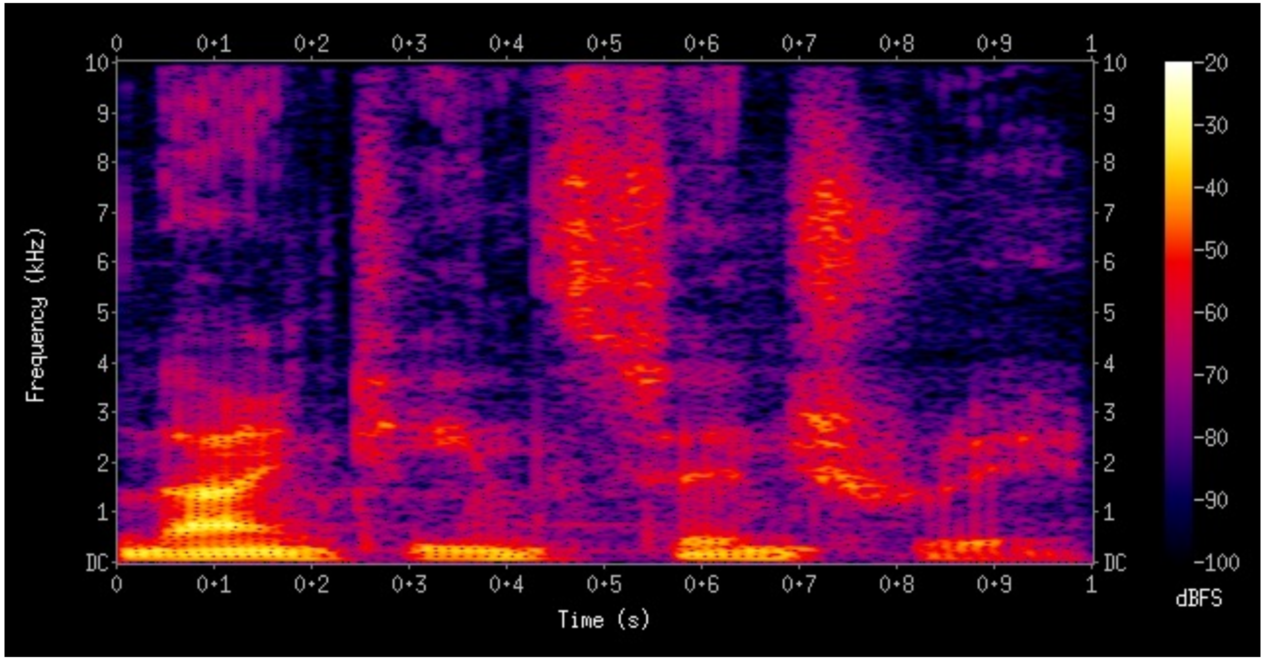
Spectrograms

- Representation in frequency and time



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From Wikipedia. Word: nineteenth century

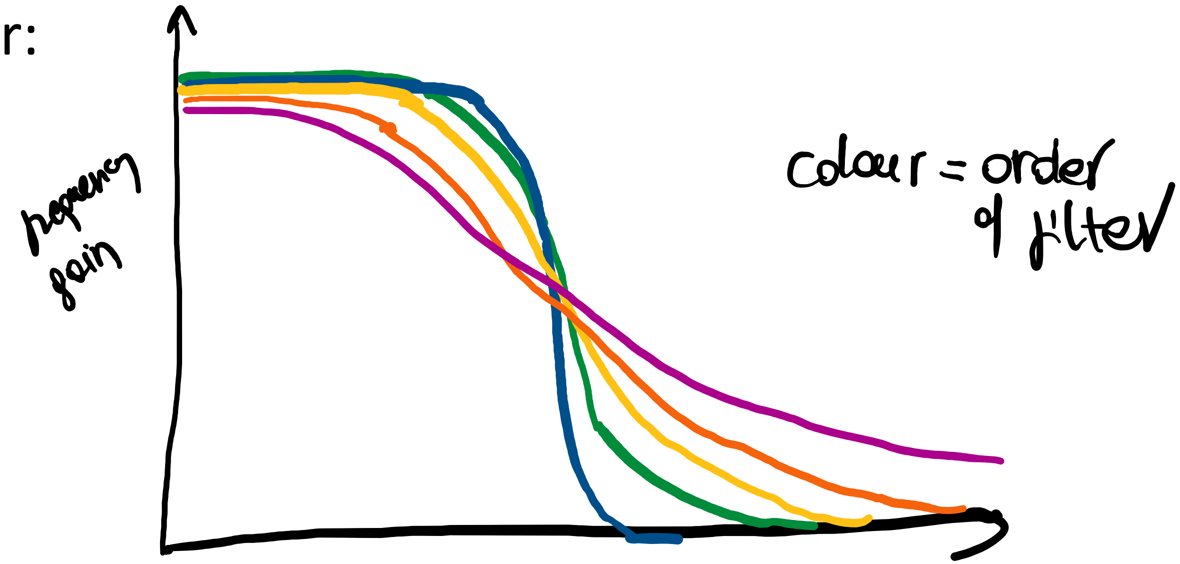
How to generate a spectrogram

- For each time window a DFT is calculated and frequency intensity is represented with ”colours” which indicate the level.

- The X axis is the time
- The Y axis is the frequency

Filtering of the signal

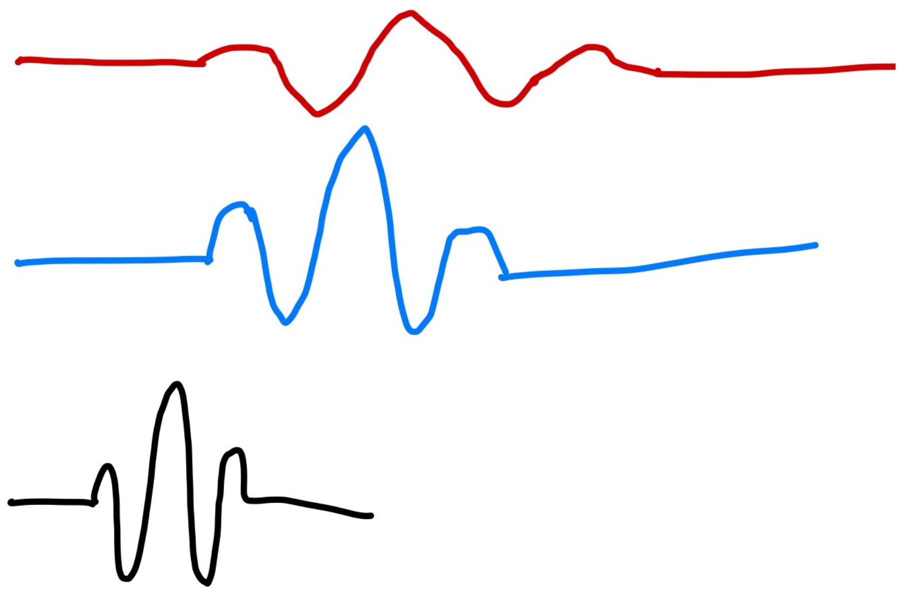
- Band-pass filter: an operation which ensures that only certain frequencies are kept in the signal
    - Often used to eliminate specific signals: eg heart signals from respiration signals or noise reduction.
- Butterworth Filter:



DWT: Discrete Wavelet Transforms

- DFT or STFT (Short-time Fourier Transforms: FTs on a portion of the time segment) fail to capture time and frequency dependencies well.
  - Only known what frequency in an interval.
  - DFTs decompose the signal into sinusoidal basis functions of different frequencies.
- Discrete Wavelet Transforms (DWT) decompose a signal in orthogonal wavelet basis functions. These functions are nonzero over only part of the total signal length.
- DWTs are dilated, translated and scaled versions of a ”base” function (wavelet).

DWT: Discrete Wavelet Transforms

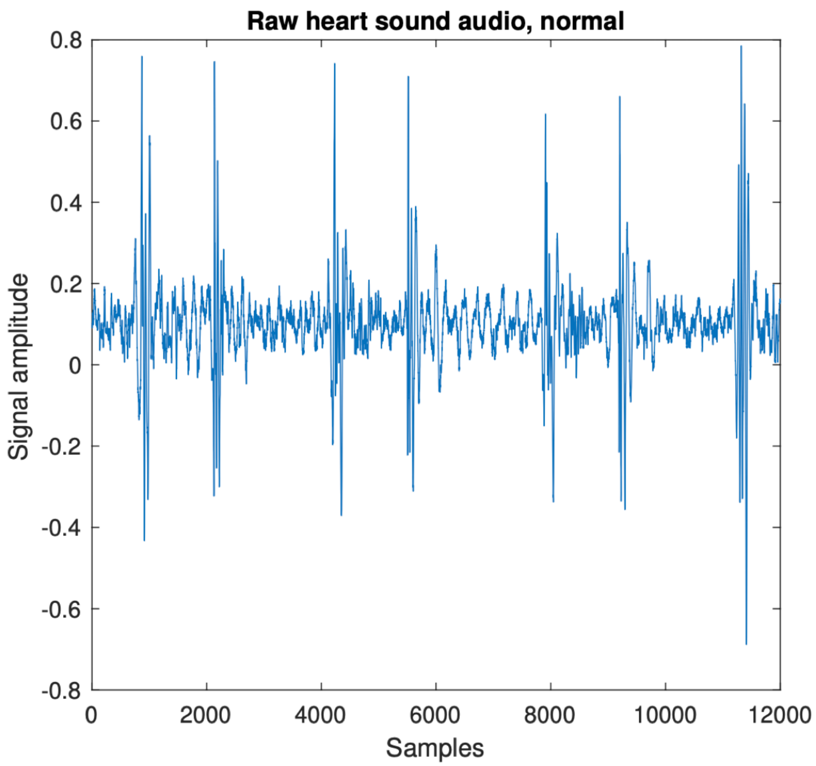


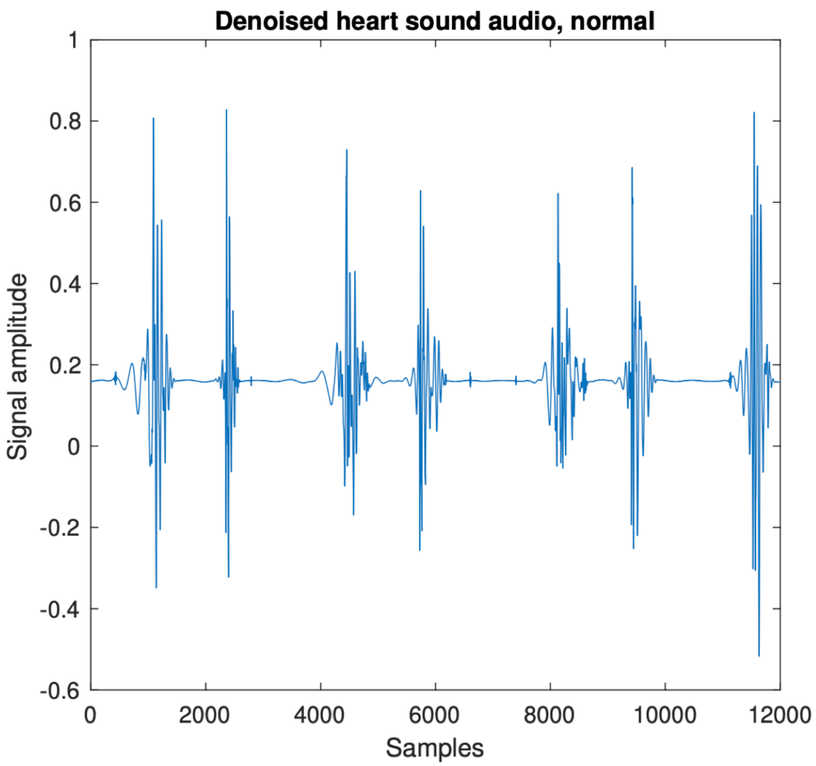
Wavelets: intuition

- The various wavelets (with different frequency) are passed through a signal and highlight different regions of the signal, respectively.

- This process is very good for denoising a signal (highlighting the important frequencies).

Denoising with DWT





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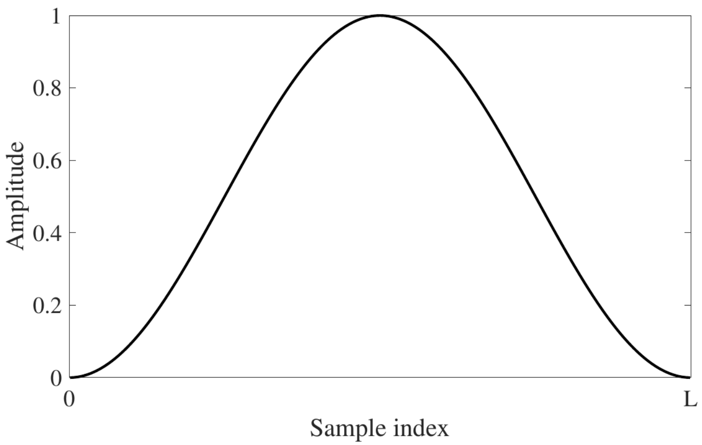
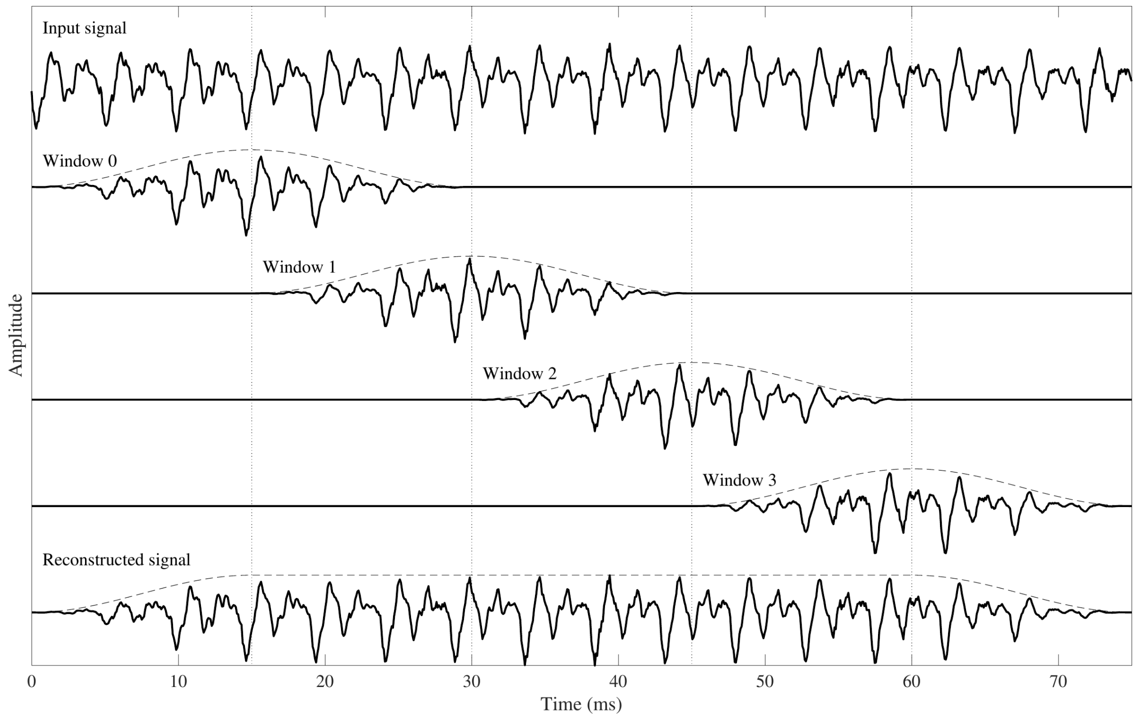
Bondareva, E., Han, J., Bradlow, W., & Mascolo, C. (2021). Segmentation-free Heart Pathology Detection Using Deep Learning. Graphs from presentation at 2021 43rd Annual International Conference of the IEEE Engineering in Medicine & Biology Society (EMBC).

Windowing

- Used to isolate a small portion of the signal.
- Often a filer is applied to smoothen start and end discontinuities.
- Example of filter function (Hann).





Figures from https://wiki.aalto.fi/display/ITSP/Windowing CC

Questions