III. Approximation Algorithms: Covering Problems

(Update on Final Exercise Question)

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Exercise: Consider the vertex cover problem, restricted to a graph where every vertex has exactly 3 neighbours. Which approximation ratio can we obtain?

1. 1 (i.e., I can solve it exactly!!!)
2. 2
3. $11/6 = 2 - 1/6$
4. $H(n) \leq \log(n)$

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- Unfortunately, this question is not well formulated and a bit fuzzy. A better formulation might be:

  Which approximation ratio can we obtain by a "simple" application of some of the results from the lectures on VERTEX-COVER and SET-COVER?

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Such graphs are called cubic graphs in the literature.

Some Research Articles on Vertex Cover on Cubic Graphs

- Vertex-Cover problem is NP-complete
  M.R. Garey, D.S. Johnson, L. Stockmeyer. "Some simplified NP-complete graph problems", Theoretical Computer Science, Volume 1, Issue 3, Pages 237–267, 1976.

- A poly-time algorithm with approximation ratio $3/2$ (based on 4-coloring)
  D. Hochbaum. "Efficient Bounds for the Stable Set, Vertex Cover and Set Packing Problems", Discrete Applied Mathematics, Volume 6, pages 243–254, 1983.

- A poly-time algorithm with approximation ratio of $7/6 + \epsilon$
  P. Berman and T. Fujito. "On Approximation Properties of the Independent Set Problem for Low Degree Graphs", Theory of Computing Systems, Volume 32, pages 115–132, 1999.

- Impossibility of a PTAS (unless $P = NP$)
  P. Alimonti and V. Kann. "Hardness of Approximating Problems on Cubic Graphs", Italian Conference on Algorithms and Algorithms, pages 288-298, 1997.