

II. Linear Programming (Illustration of the Auxiliary LP)

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UNIVERSITY OF
CAMBRIDGE

Introduction



- Let us illustrate the role of x_0 as “distance from feasibility”



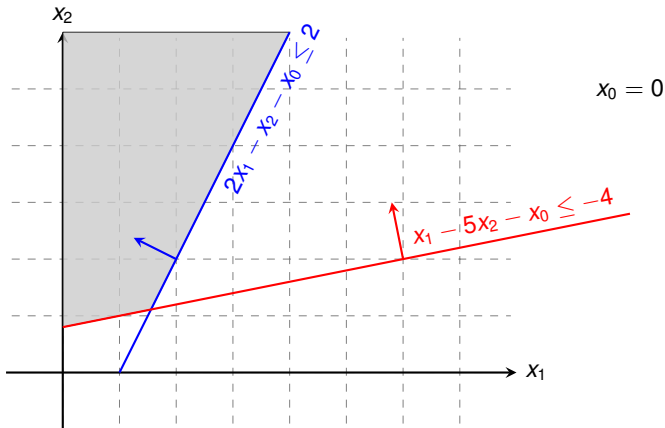
- Let us illustrate the role of x_0 as “distance from feasibility”
- We will also see that increasing x_0 enlarges the feasible region.



Geometric Illustration

maximize $-x_0$
subject to

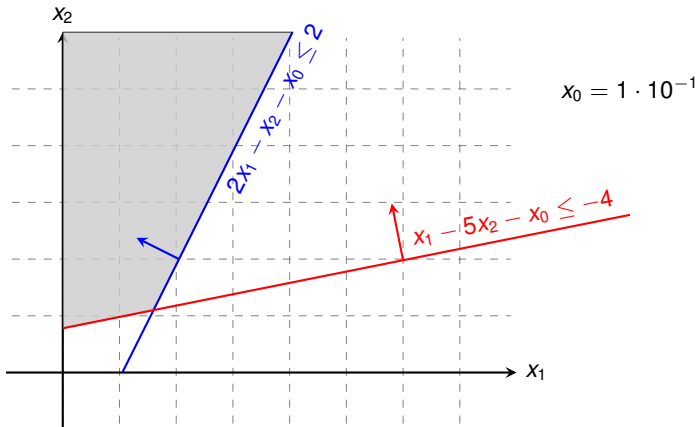
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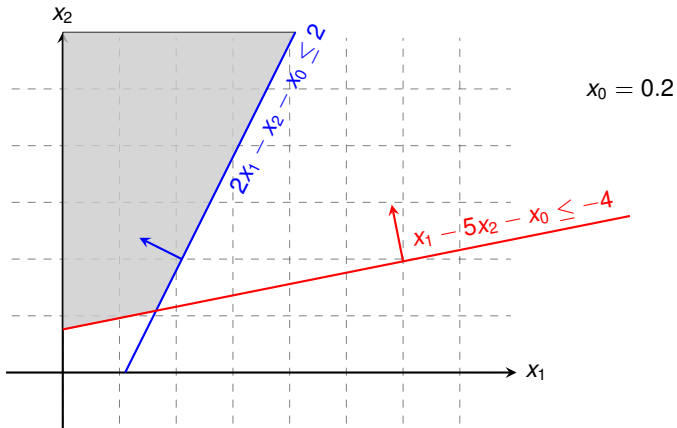
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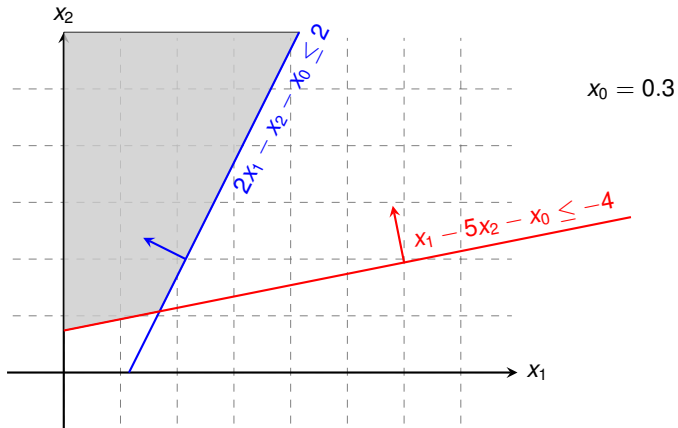
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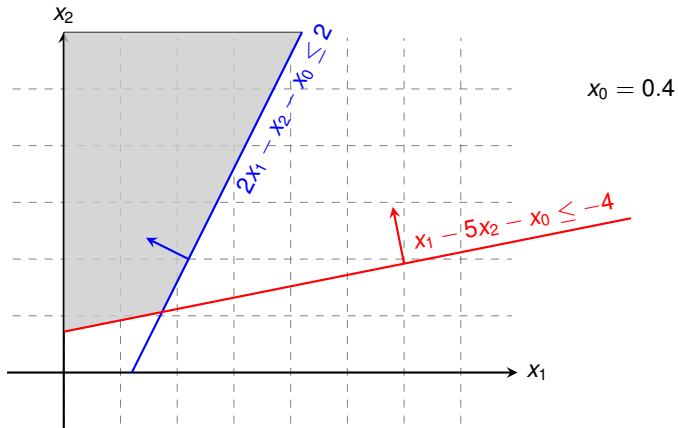
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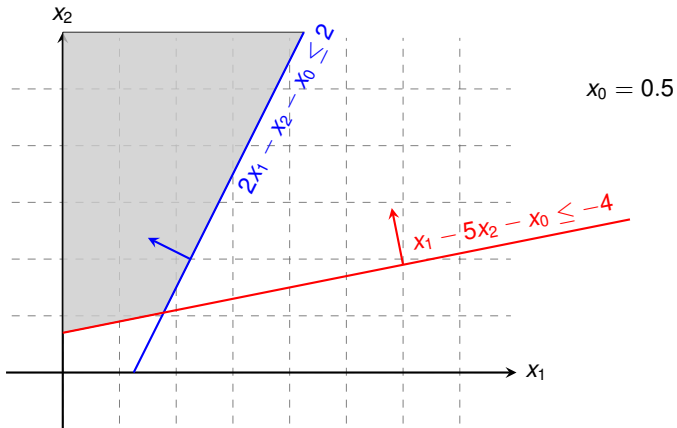
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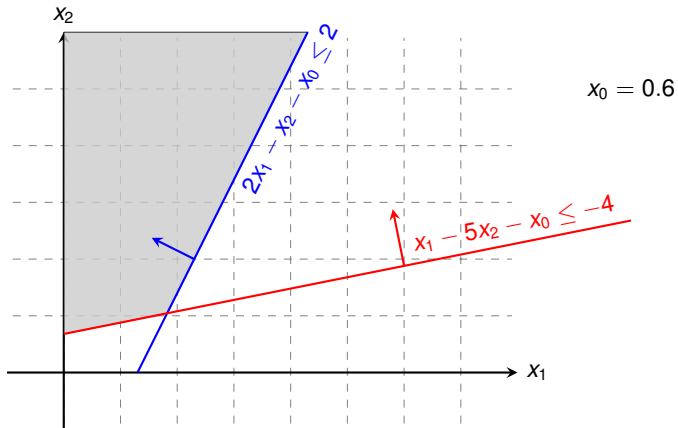
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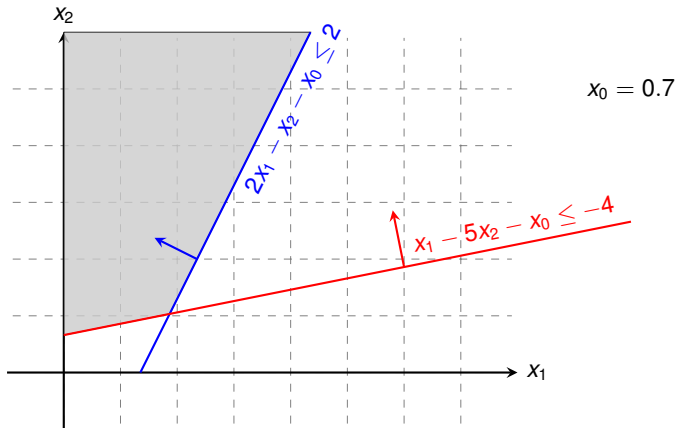
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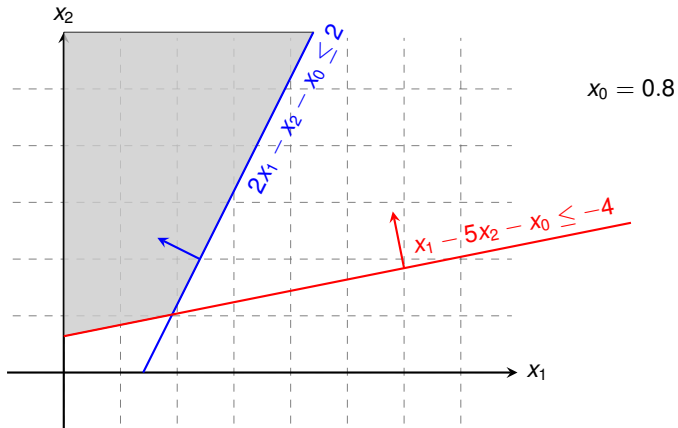
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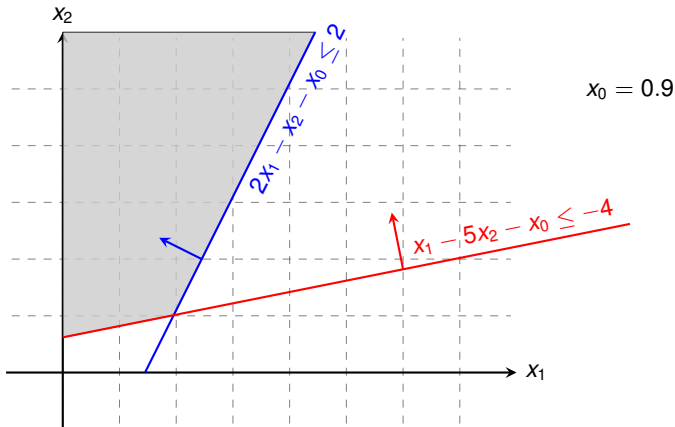
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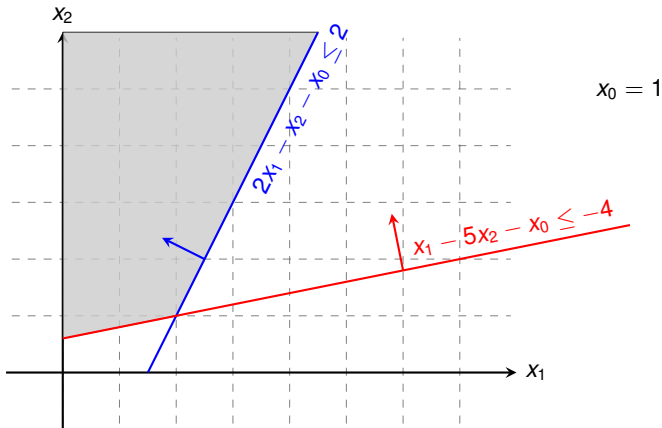
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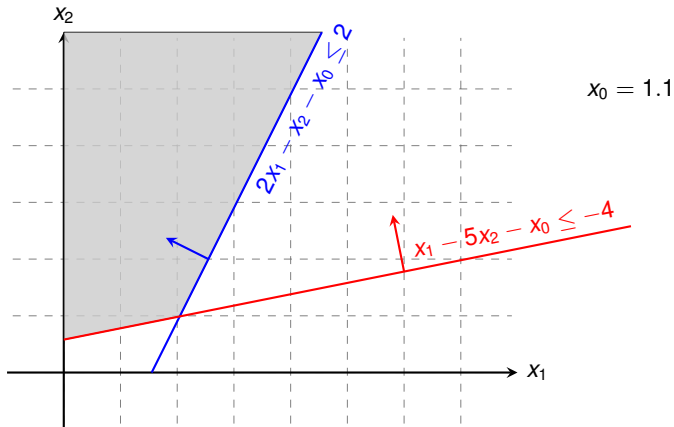
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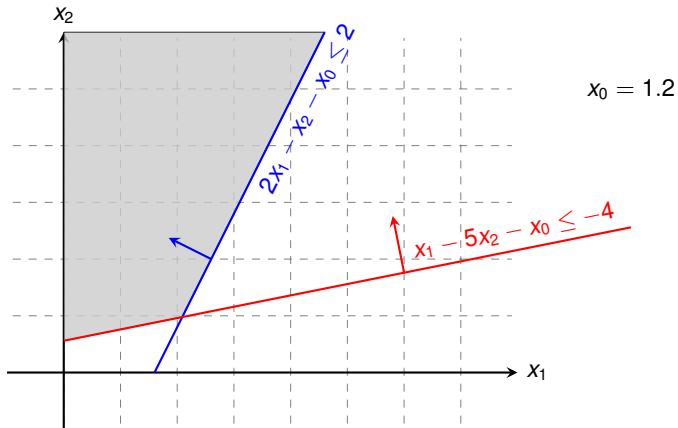
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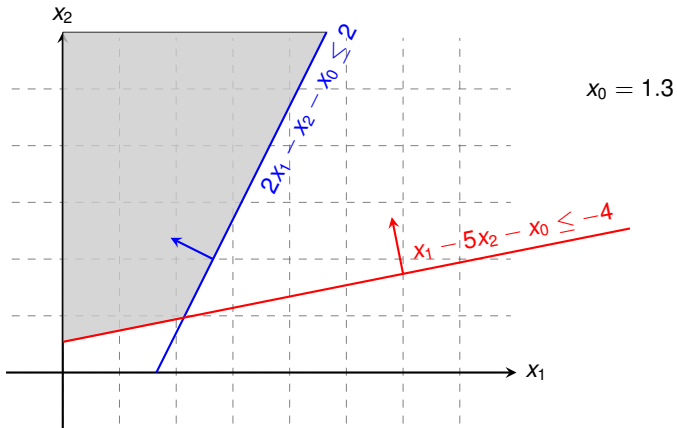
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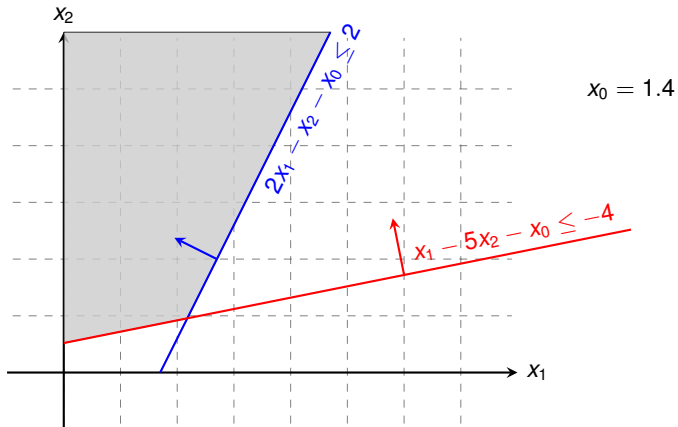
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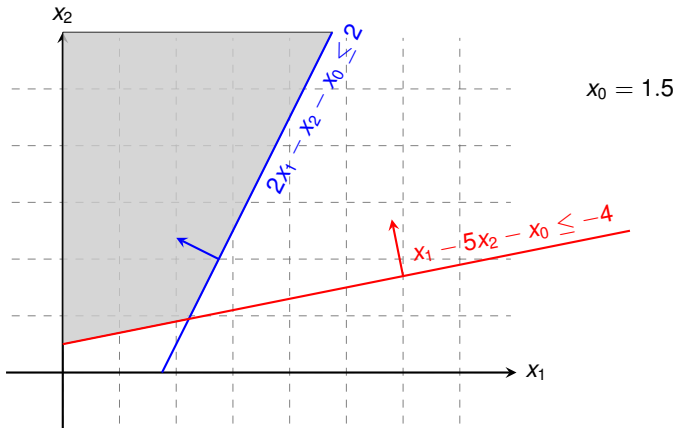
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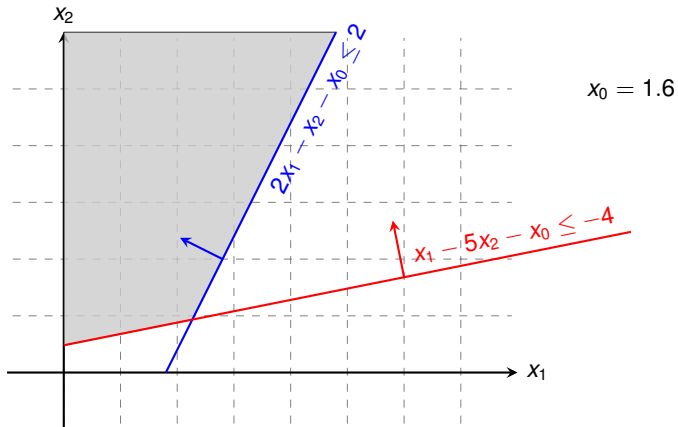
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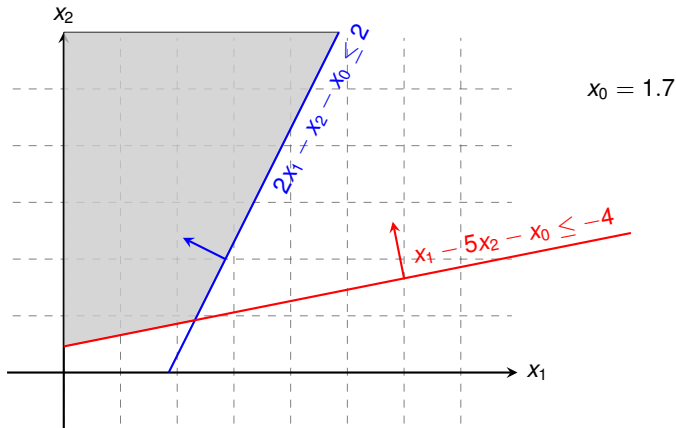
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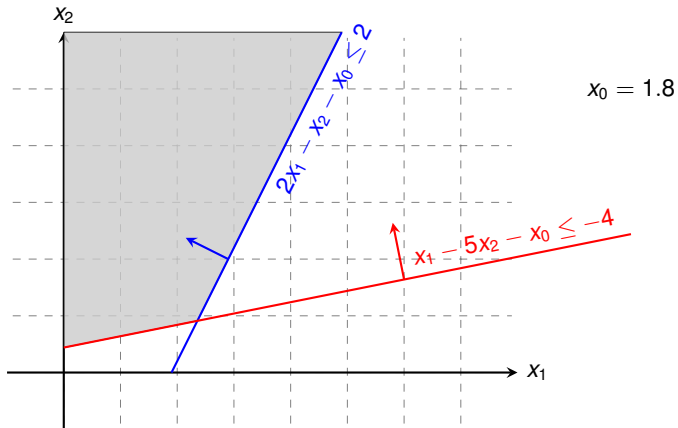
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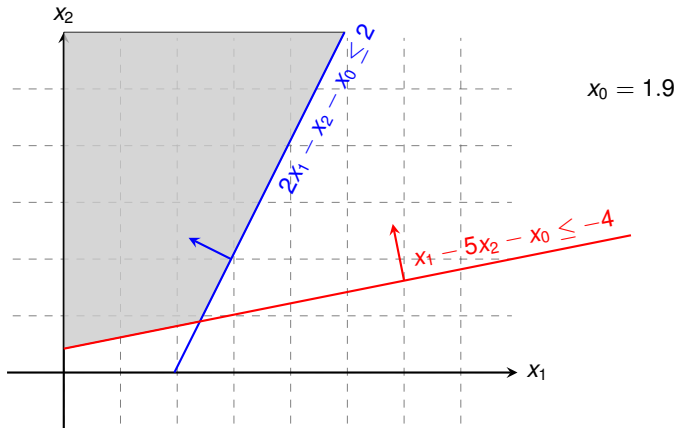
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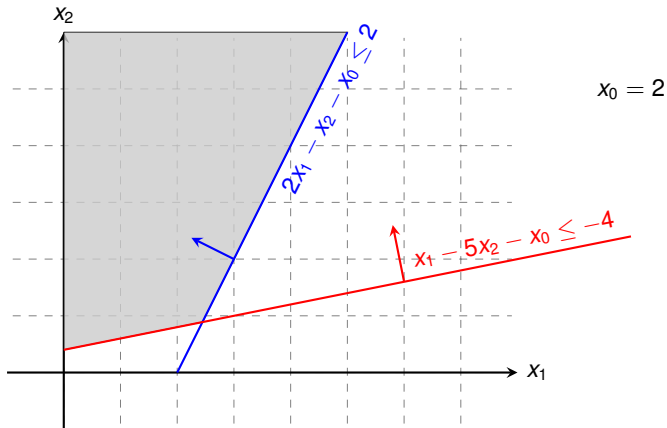
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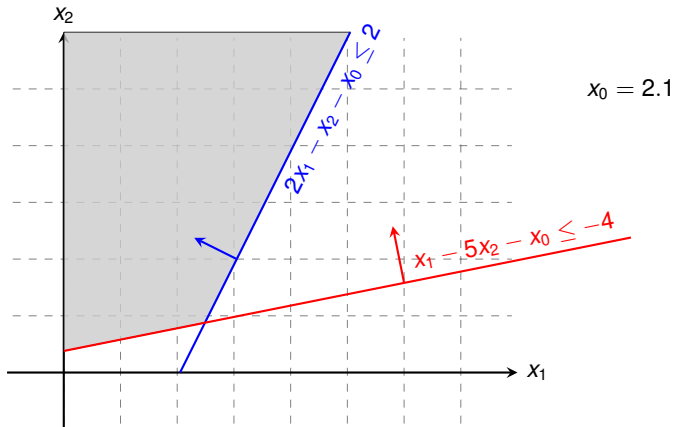
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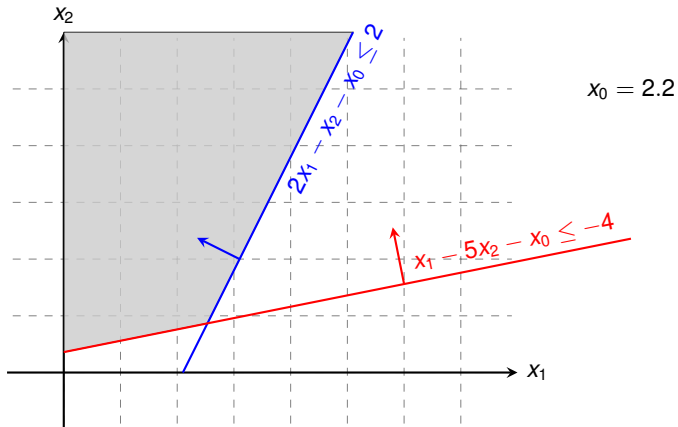
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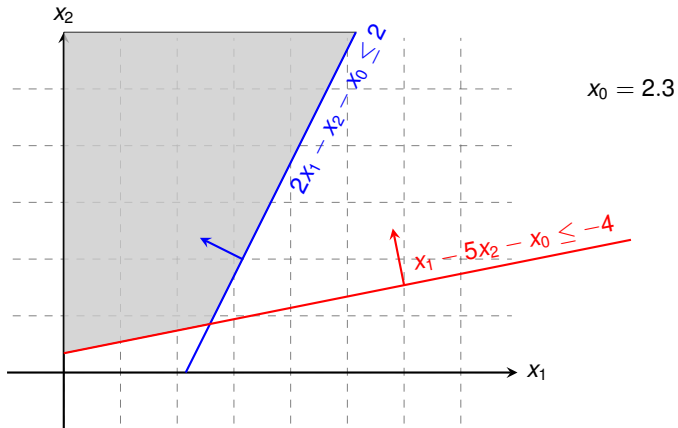
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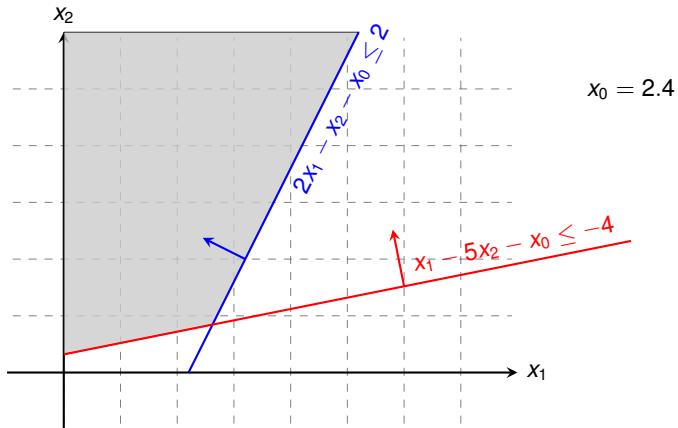
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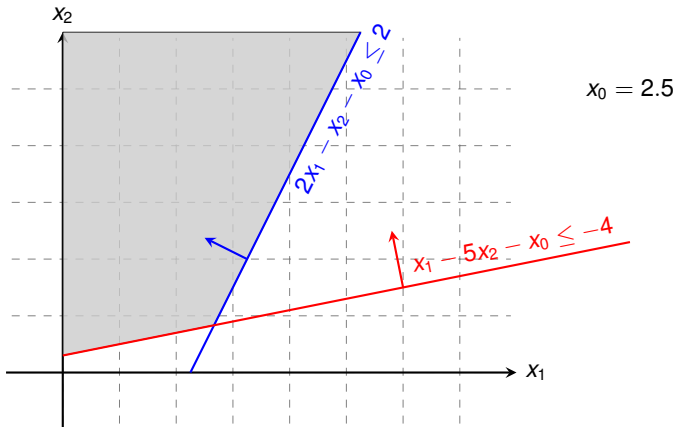
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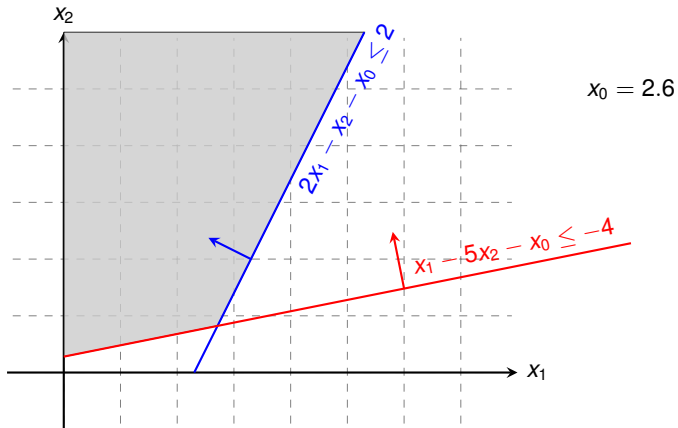
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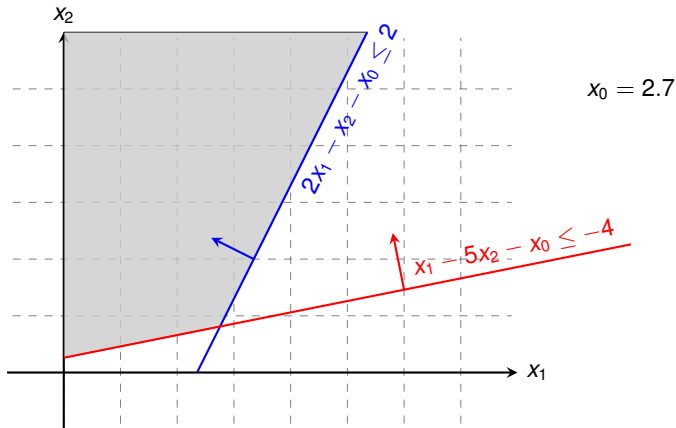
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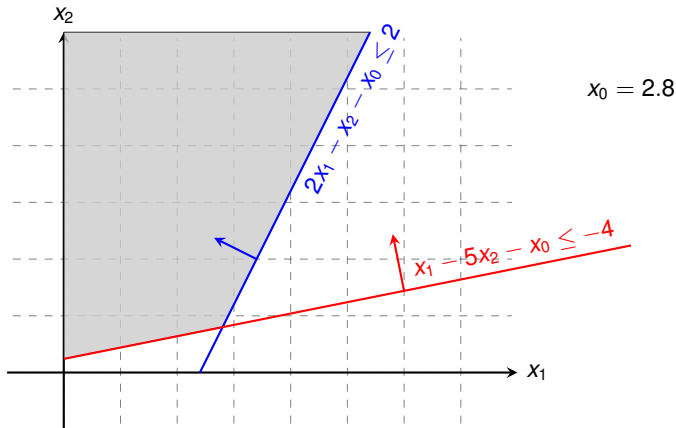
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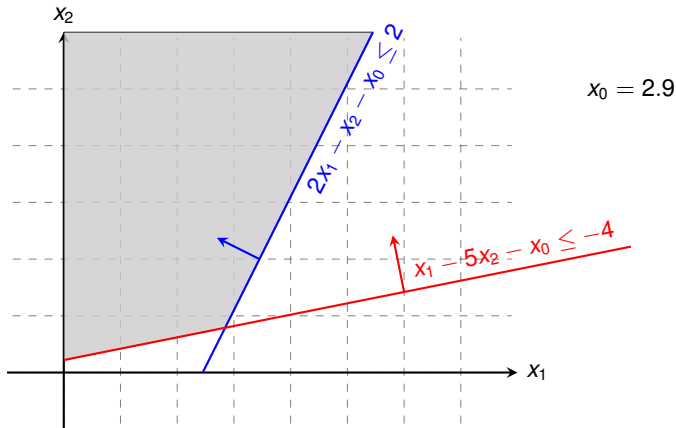
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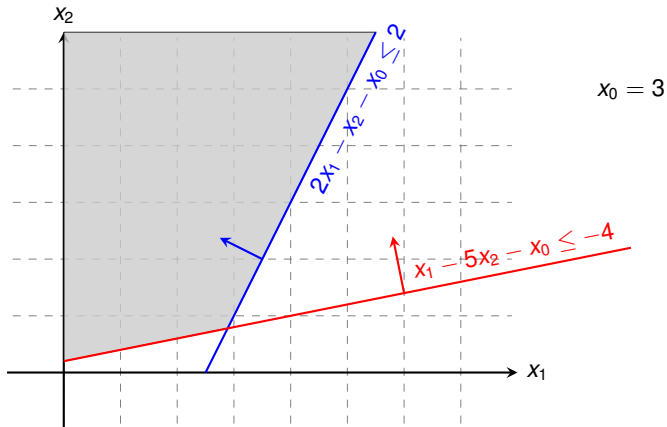
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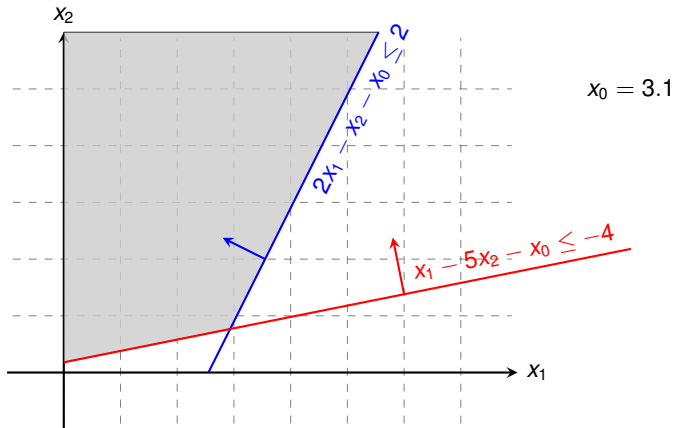
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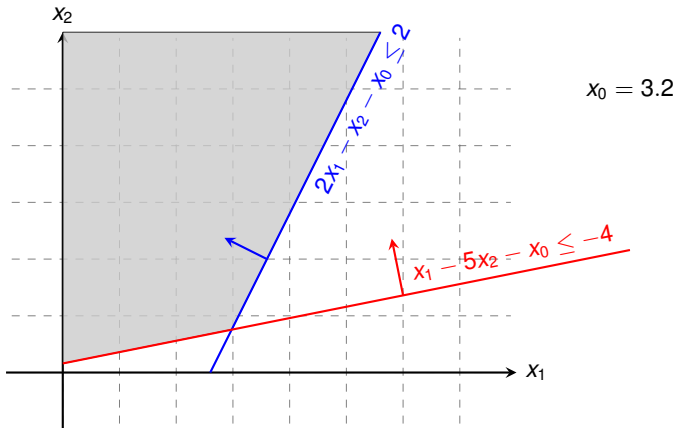
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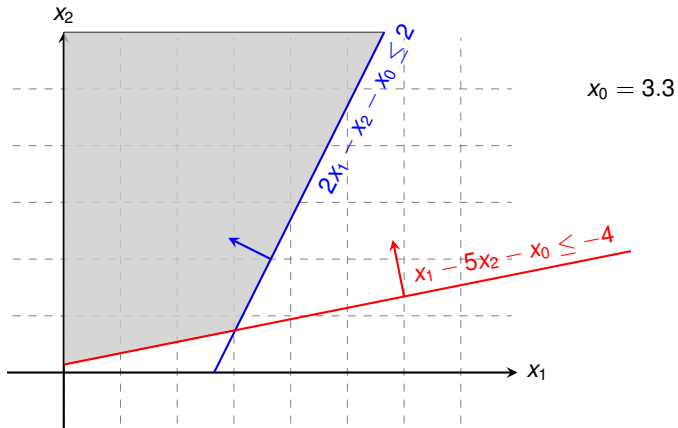
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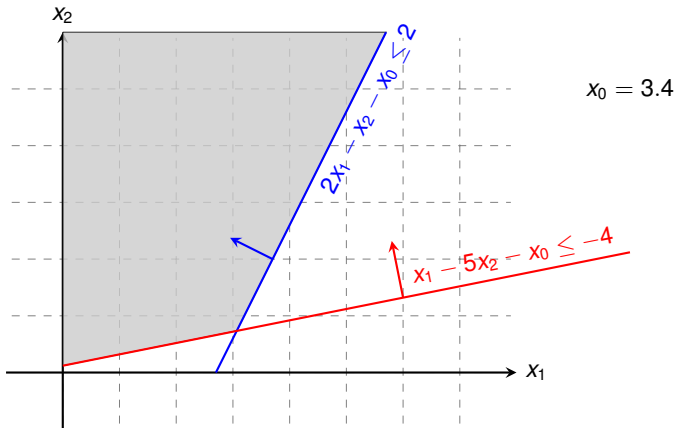
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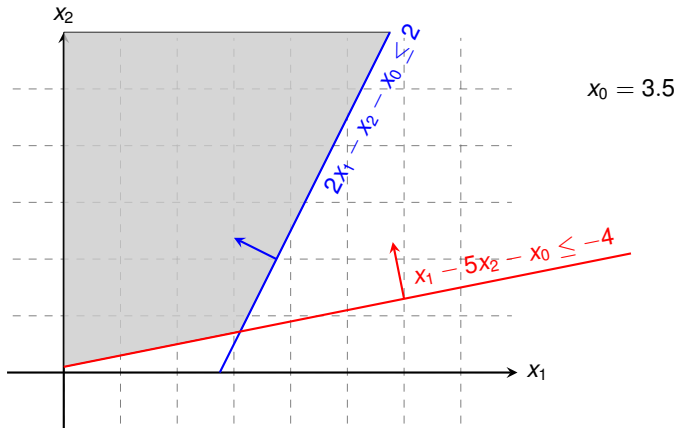
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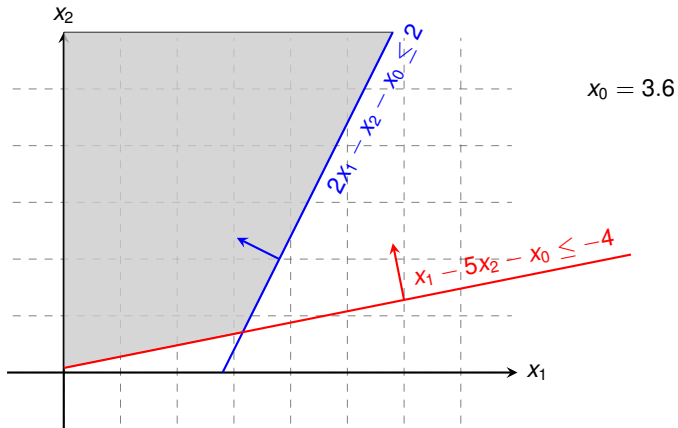
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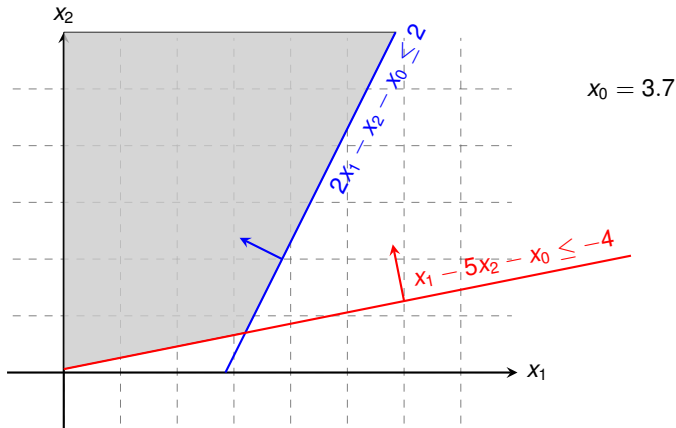
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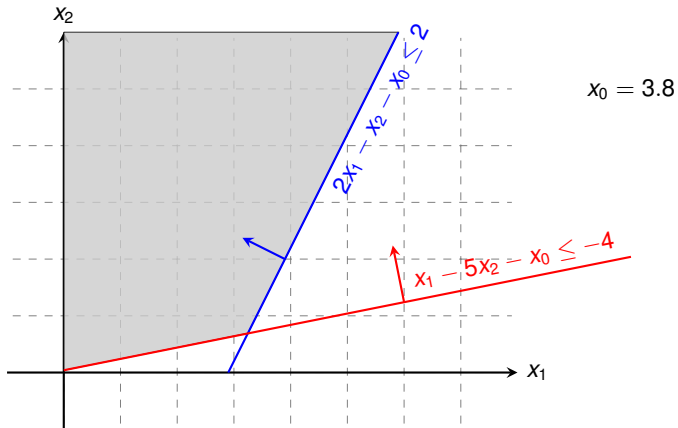
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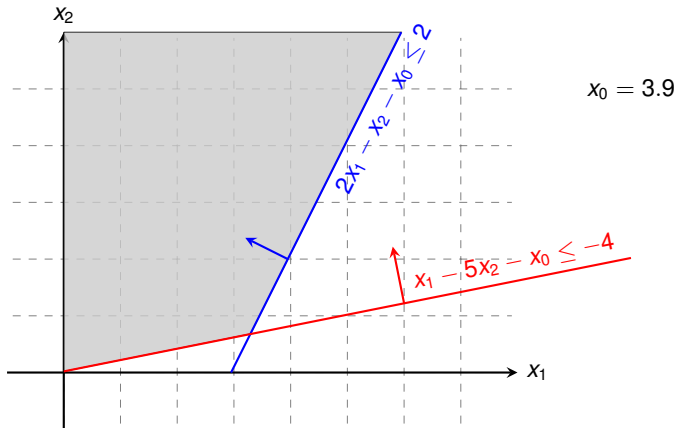
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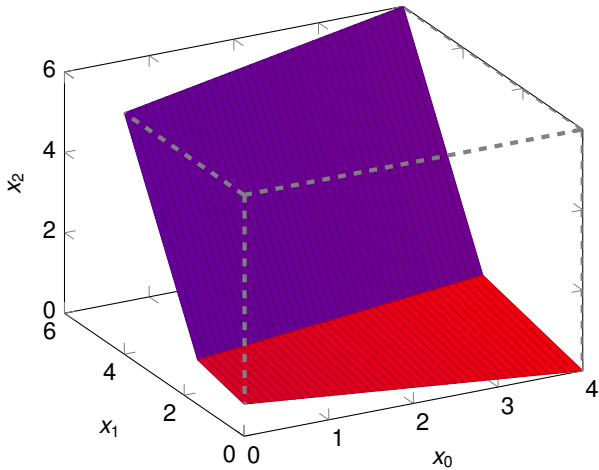
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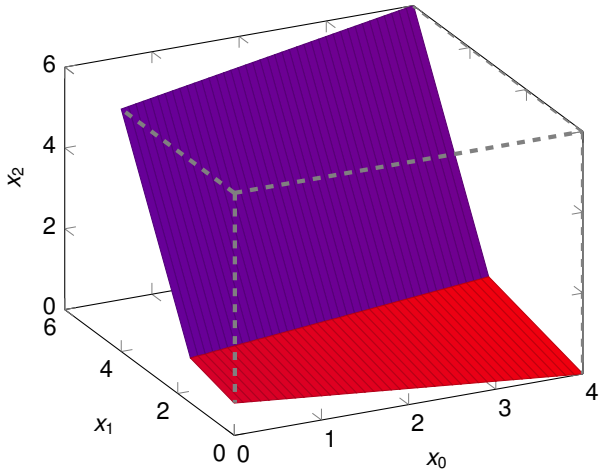
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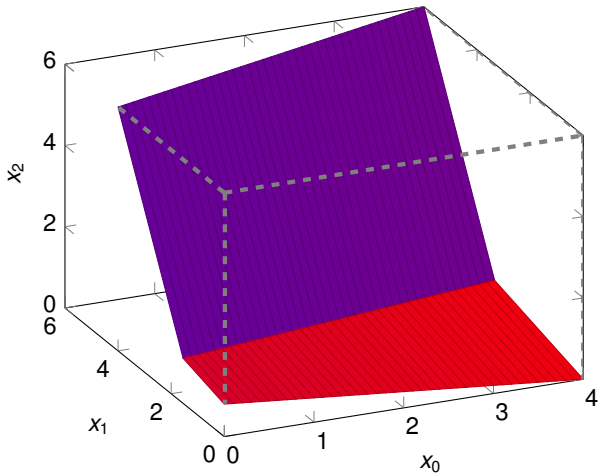


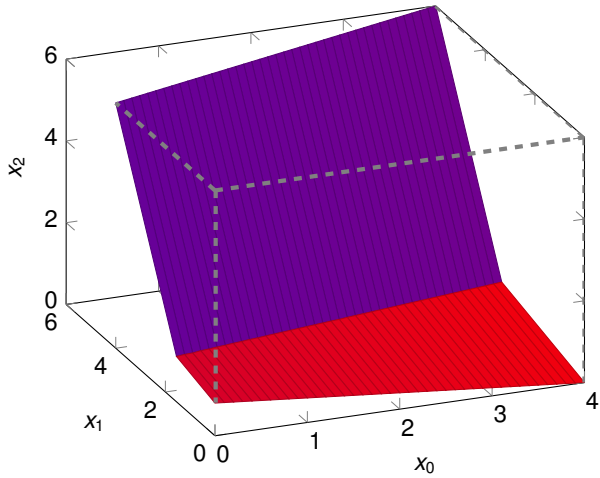
Now the Feasible Region of the Auxiliary LP in 3D

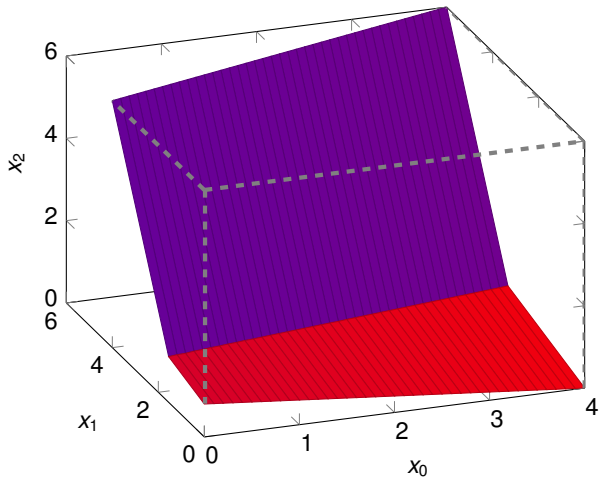


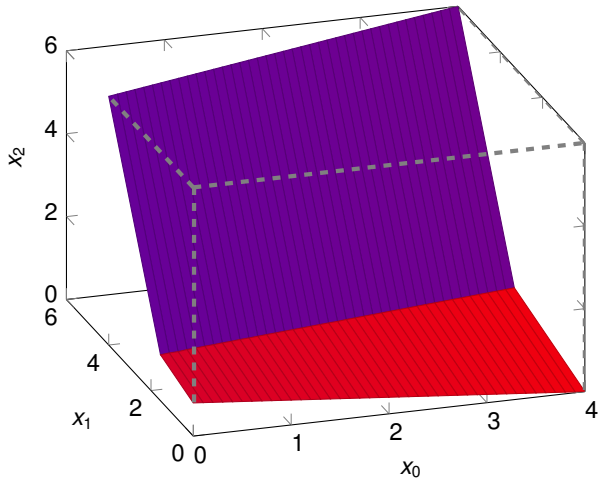


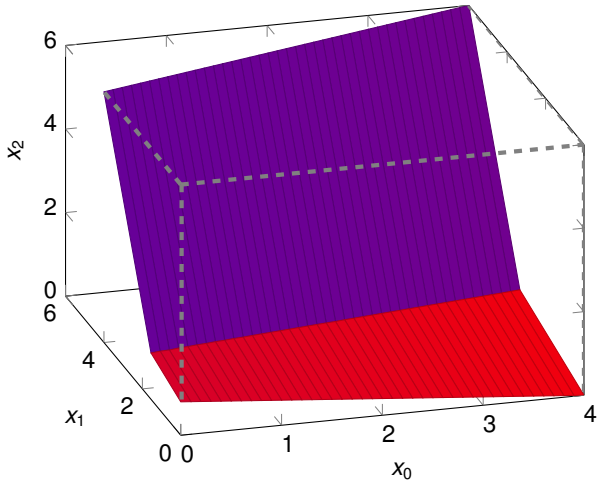


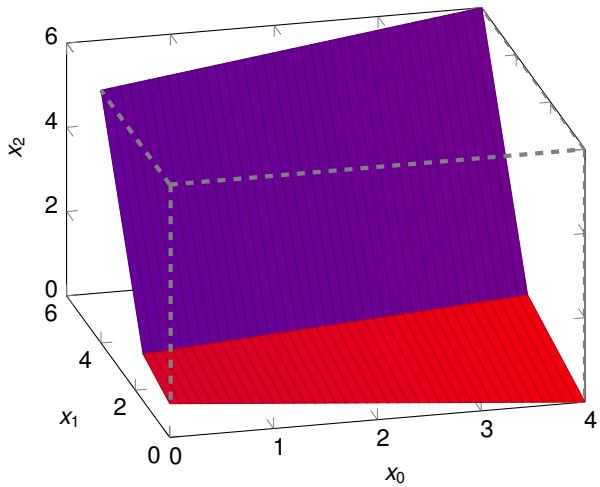


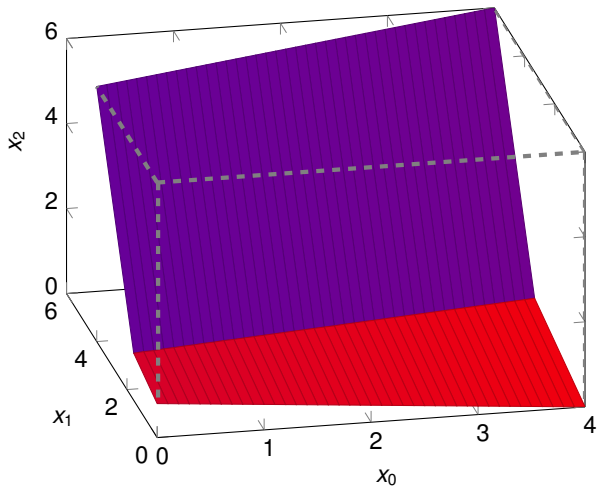


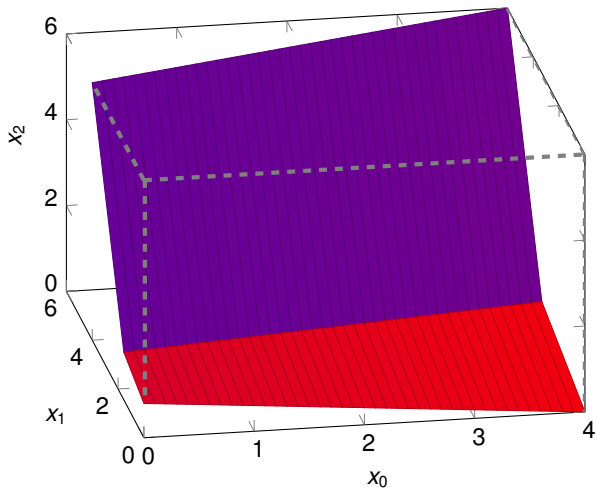


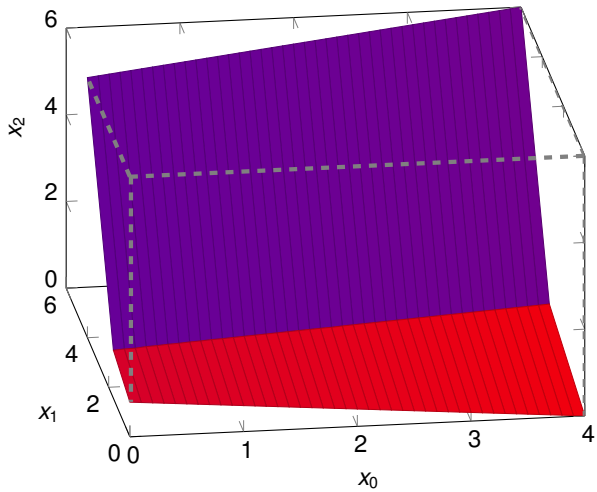


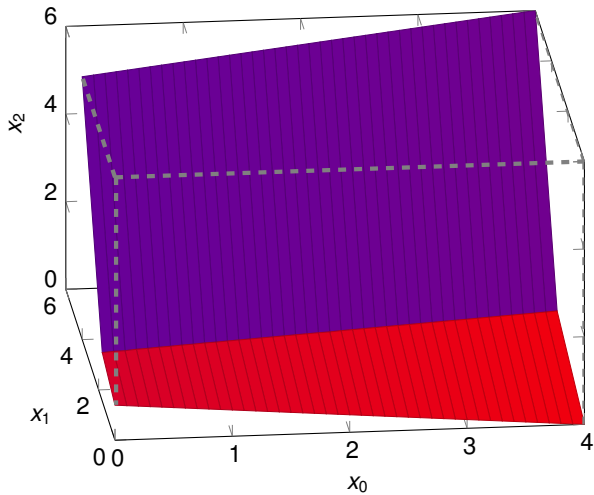


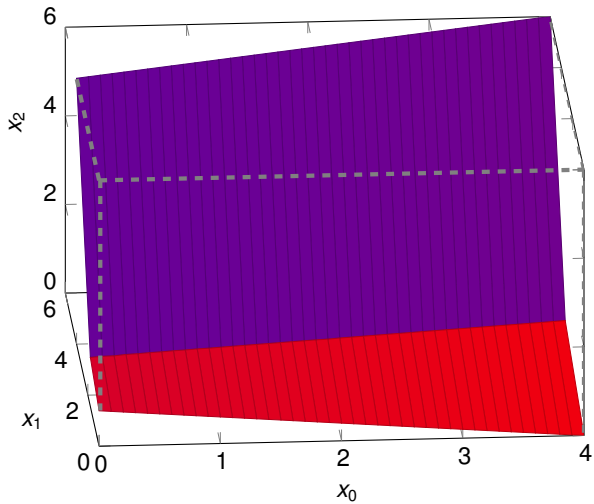


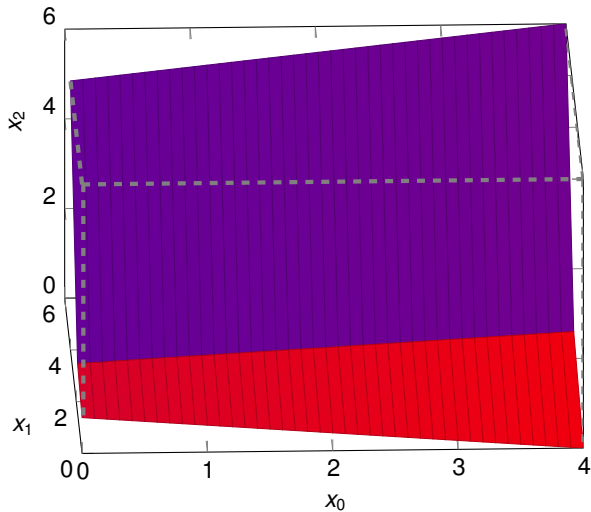


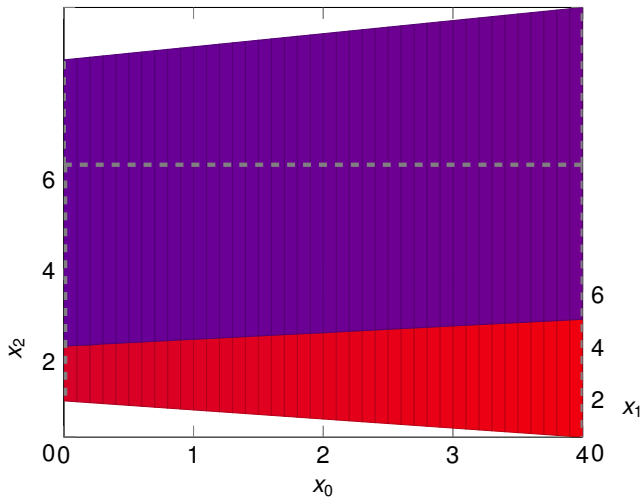


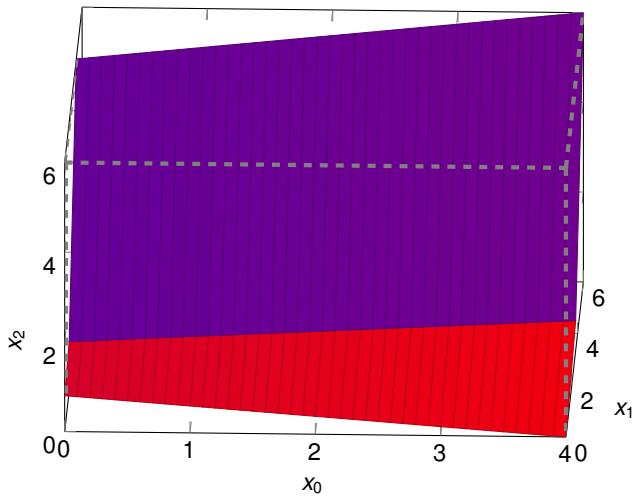


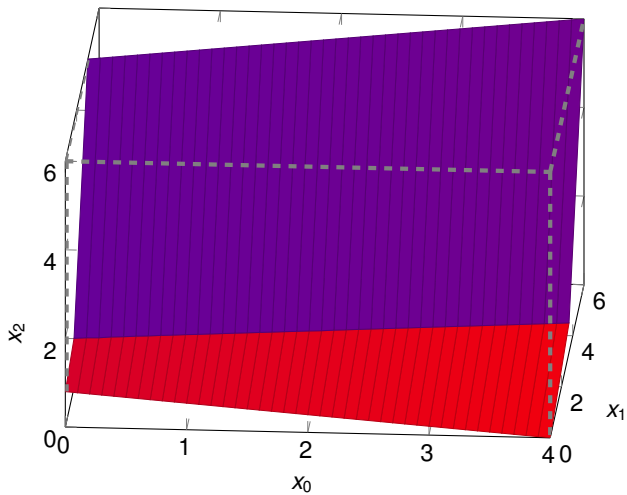


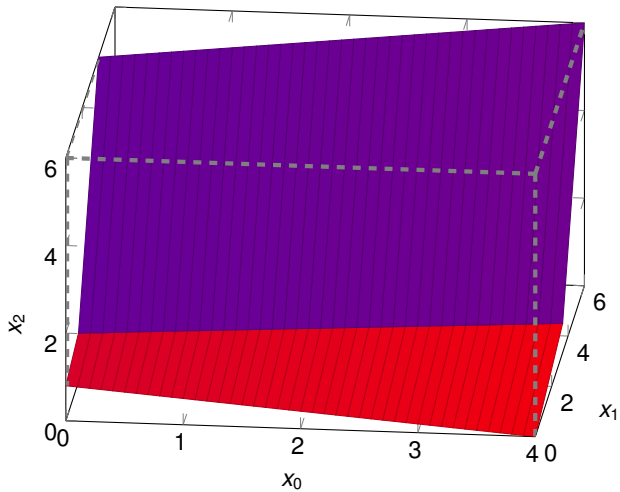


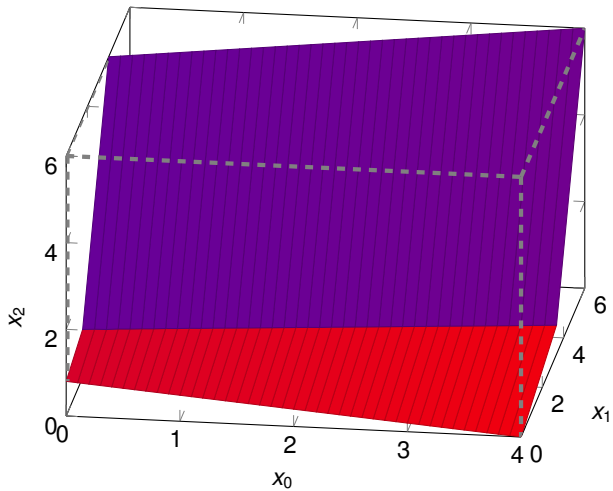


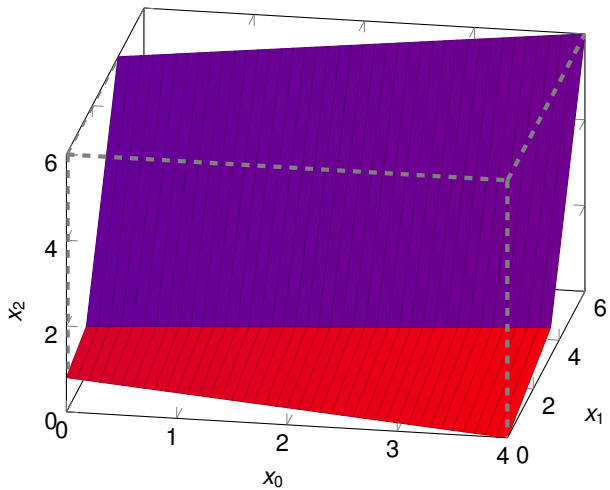


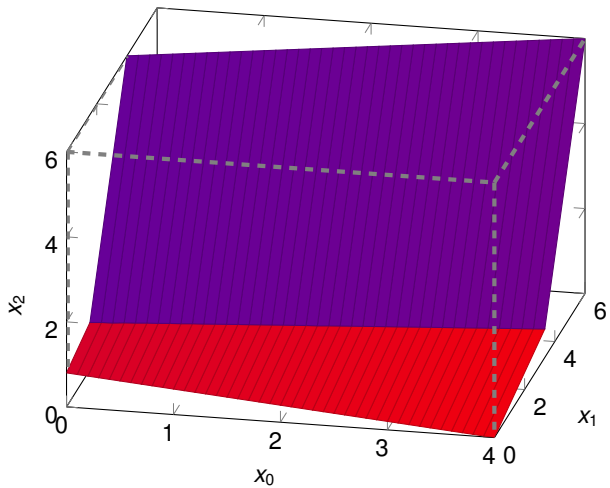


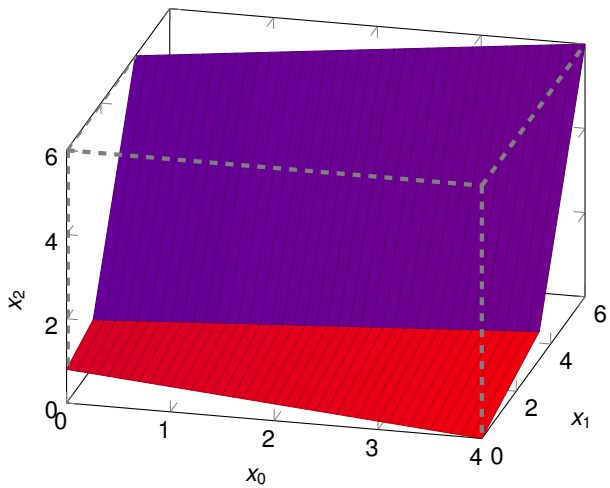


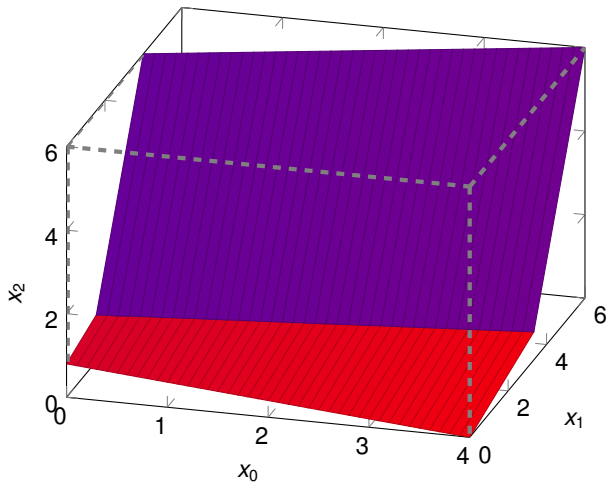


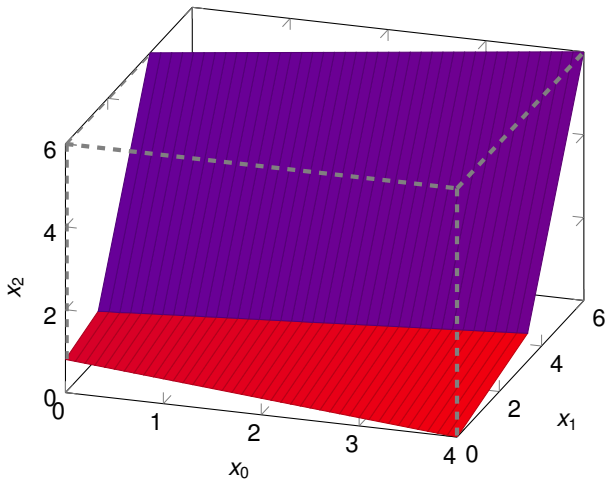


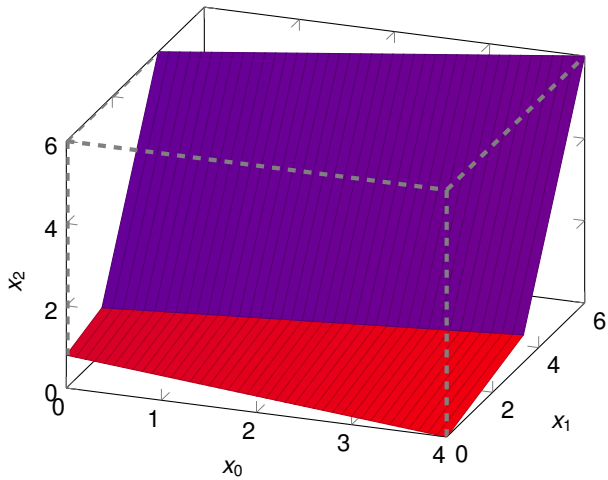


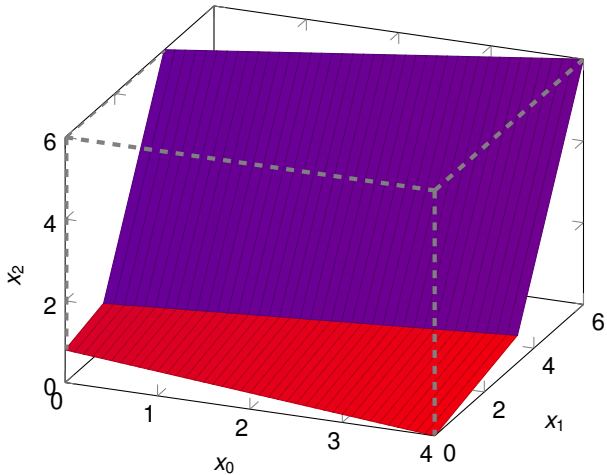


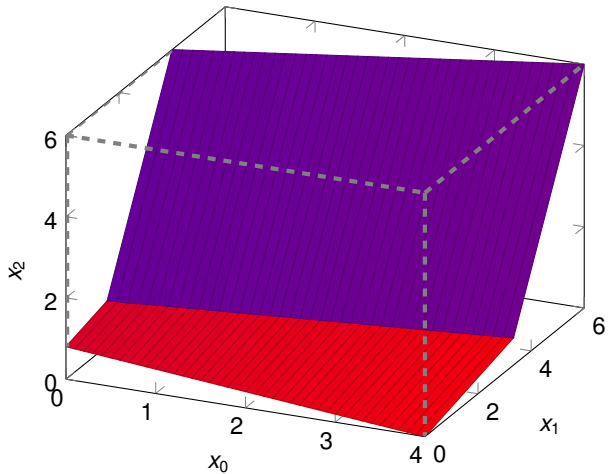


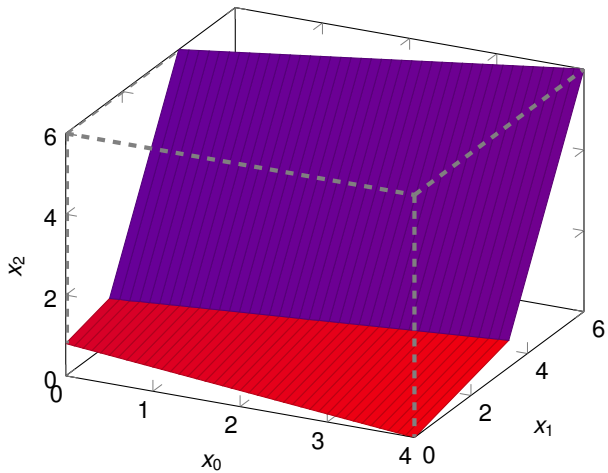


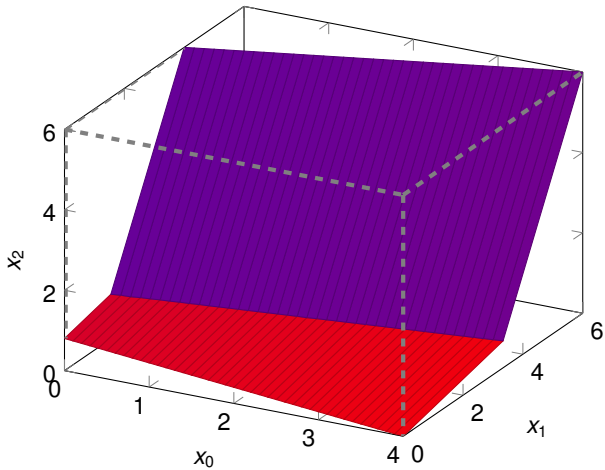


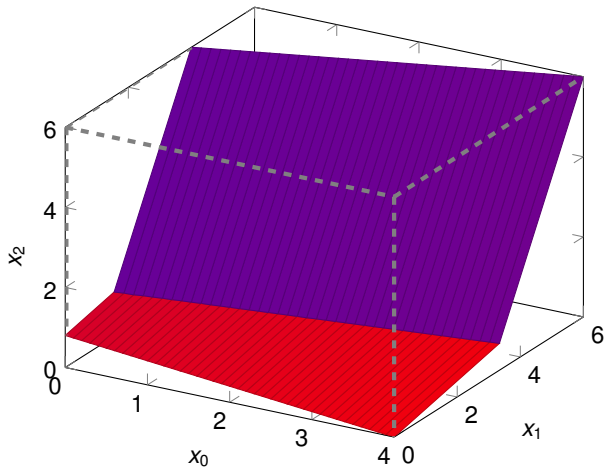


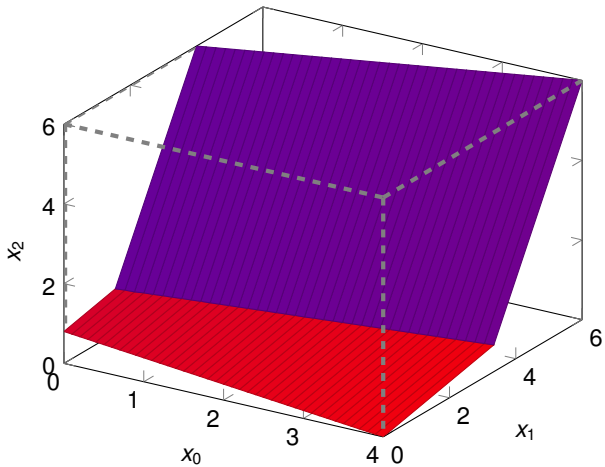


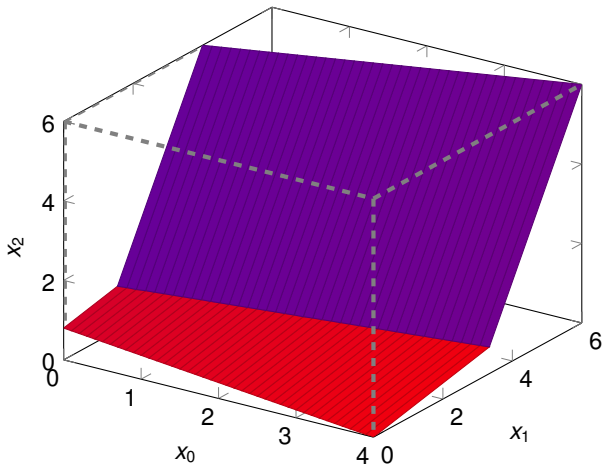


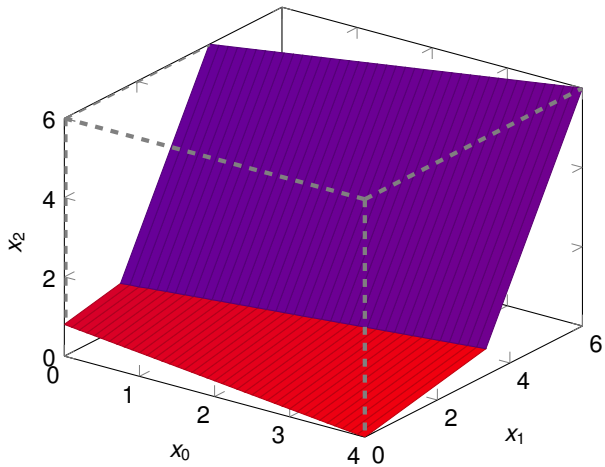


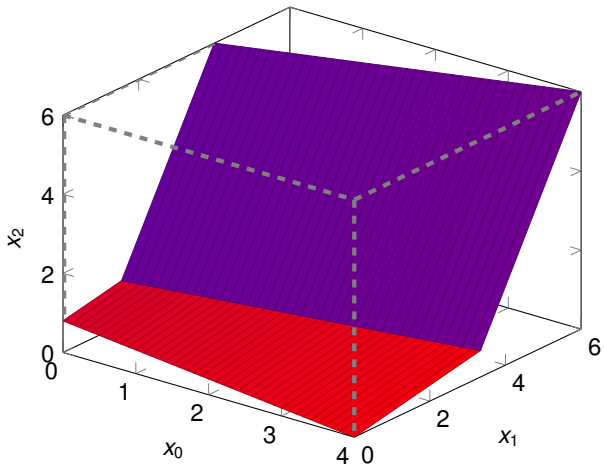


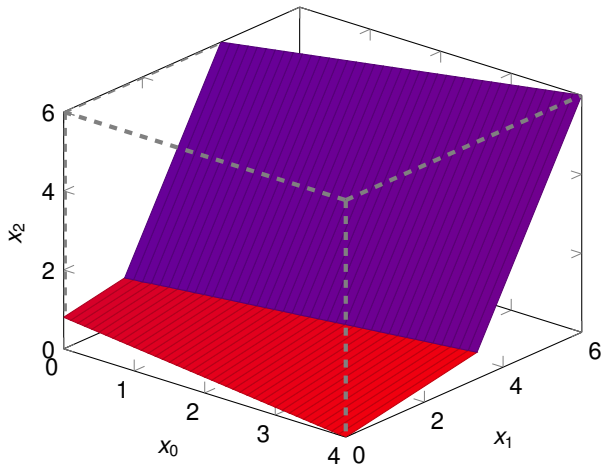


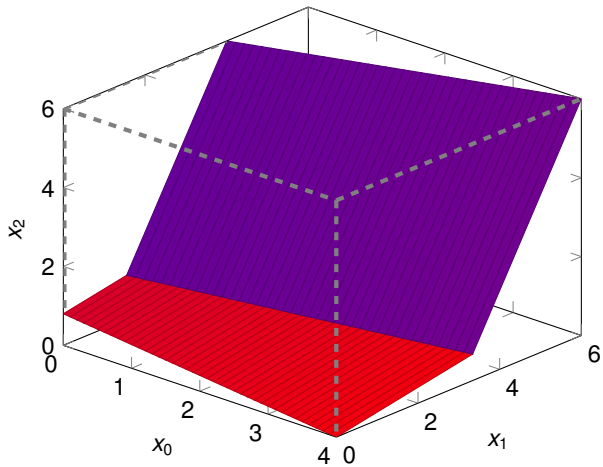


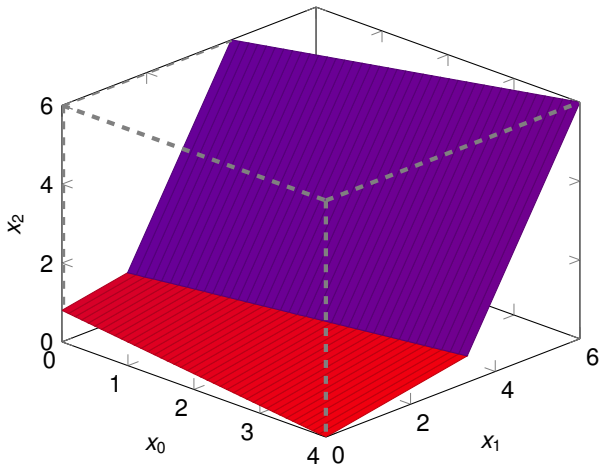


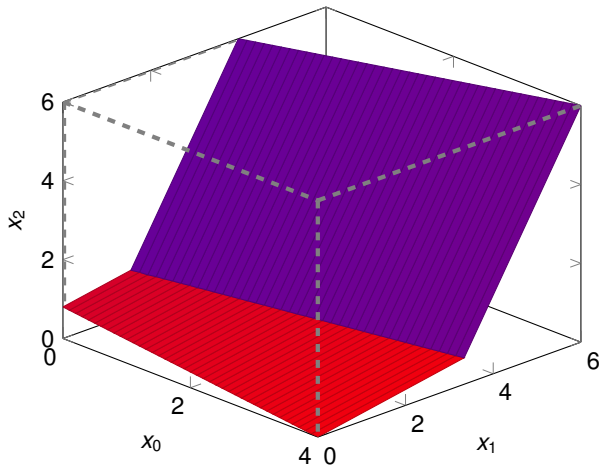


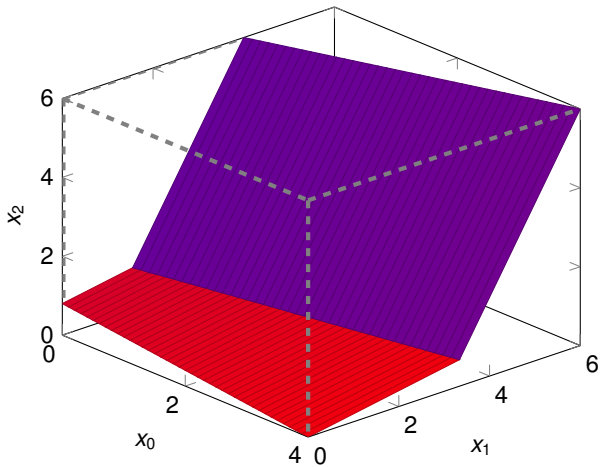


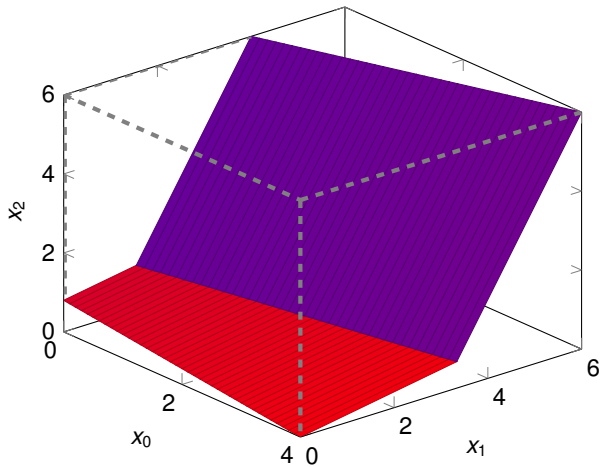


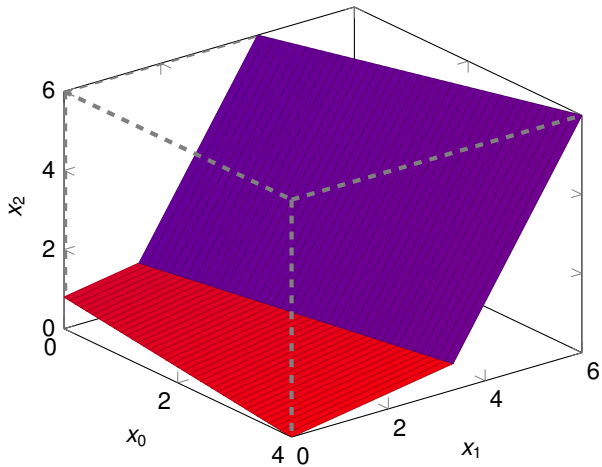


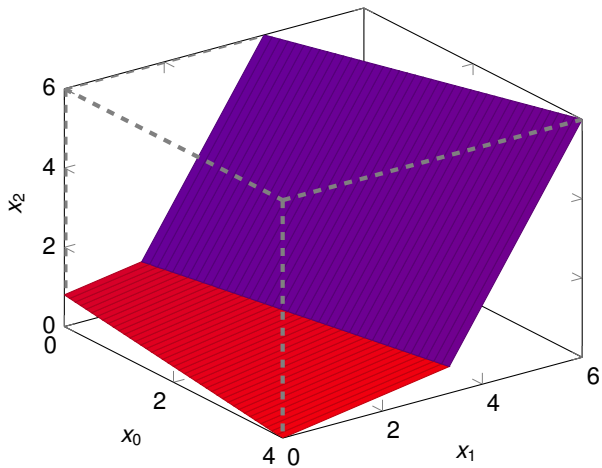


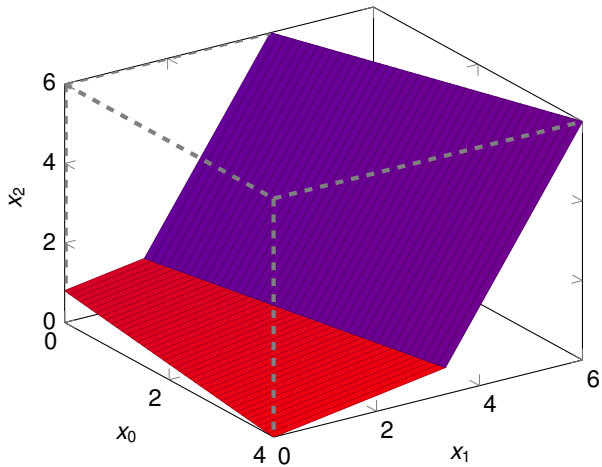


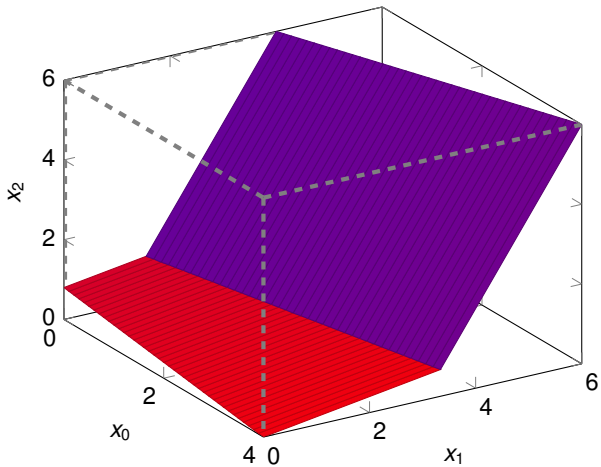


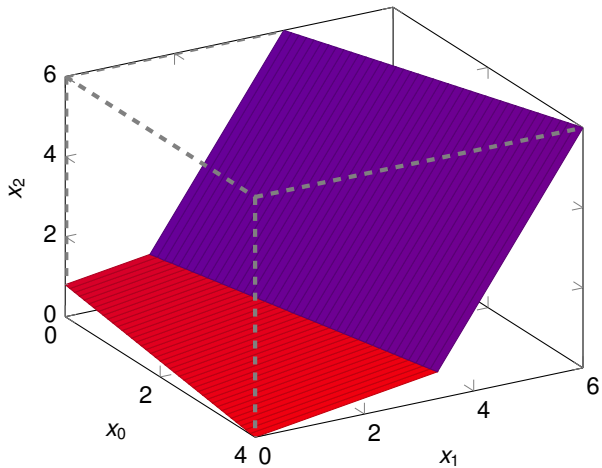


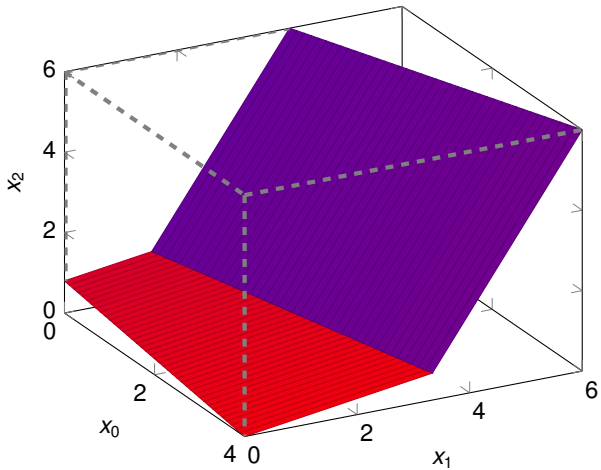


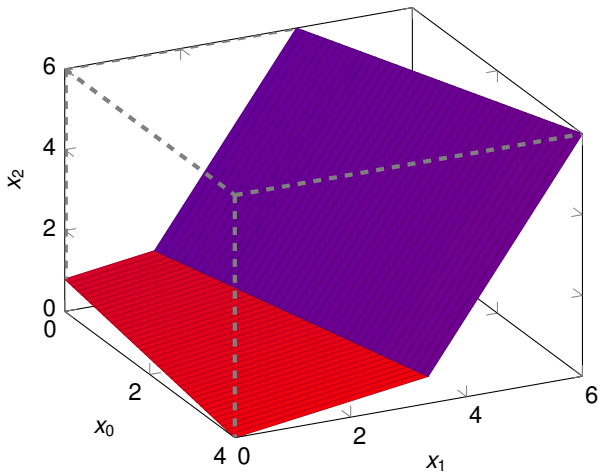


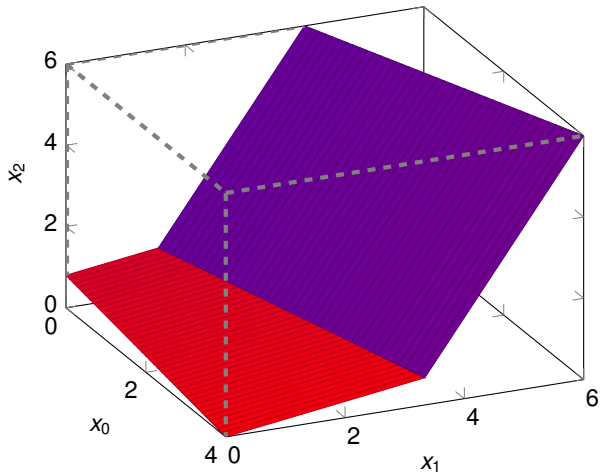


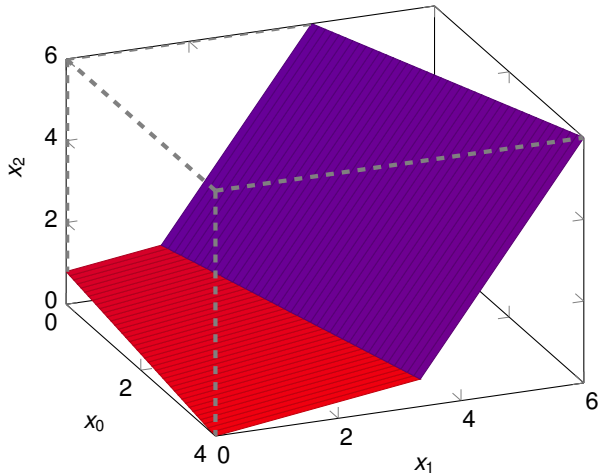


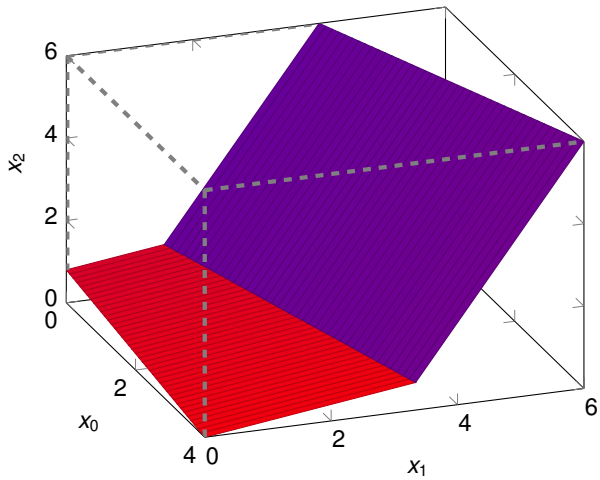


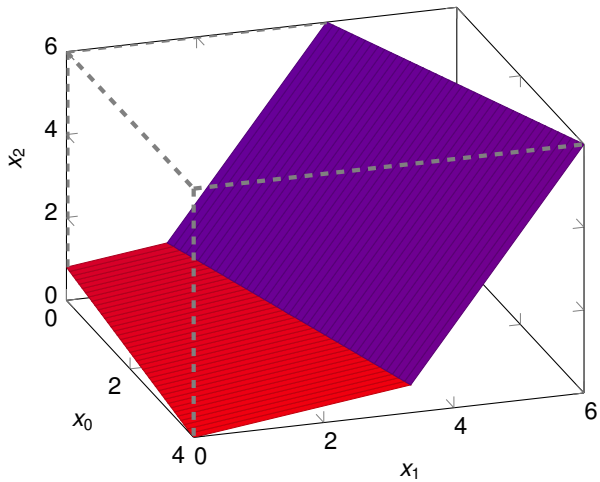


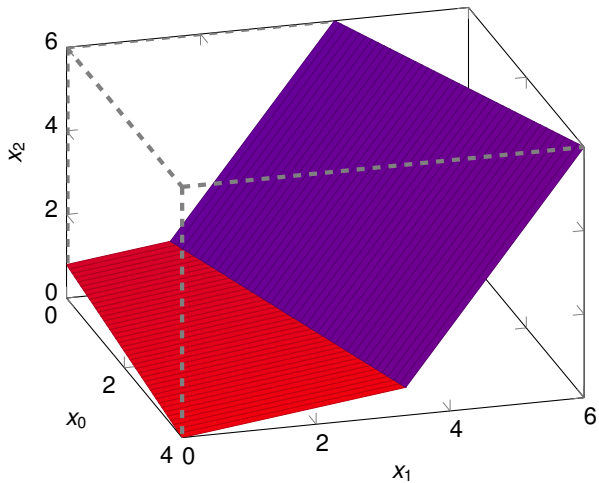


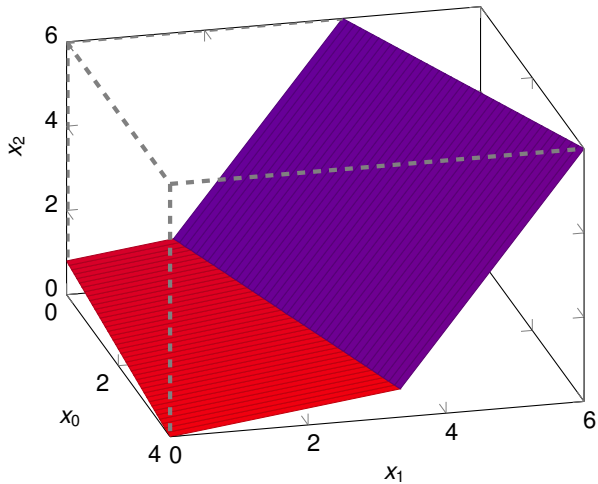


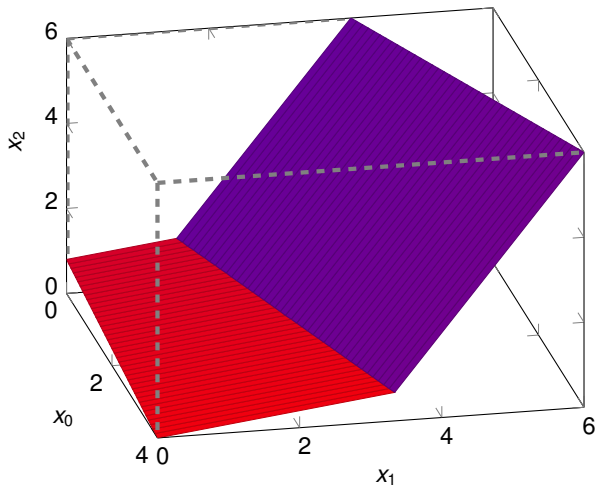


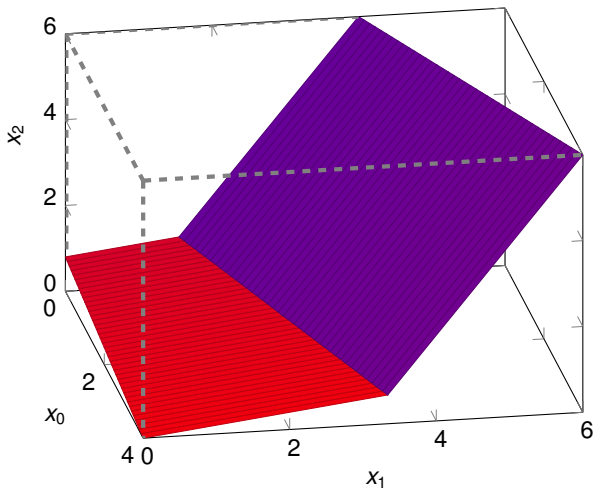


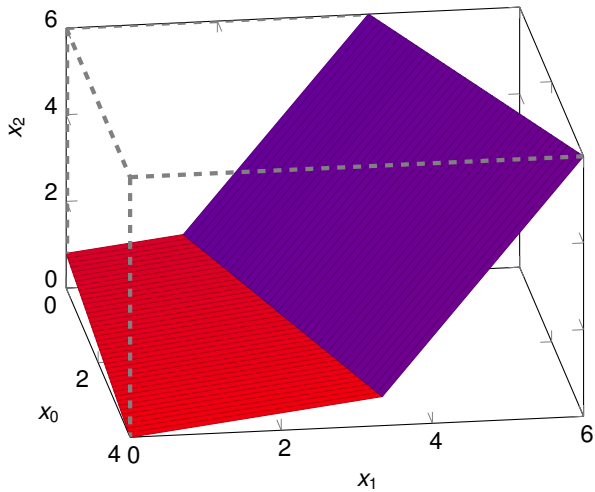


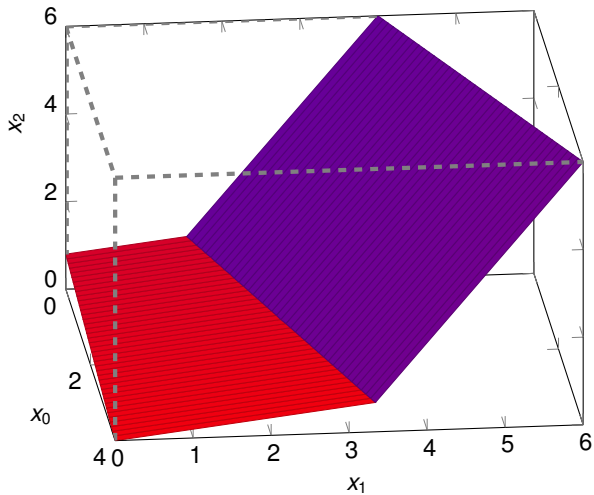


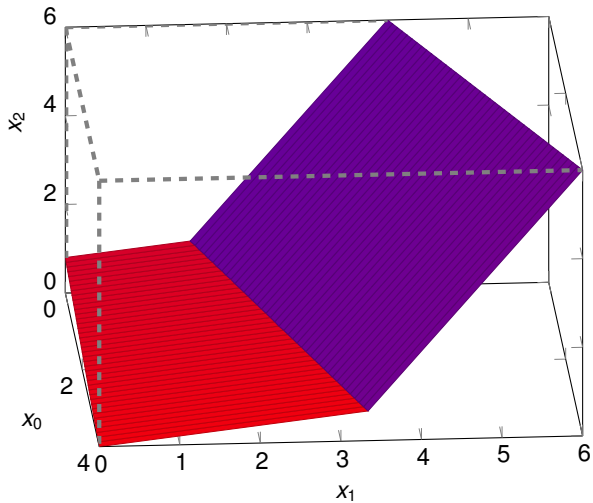


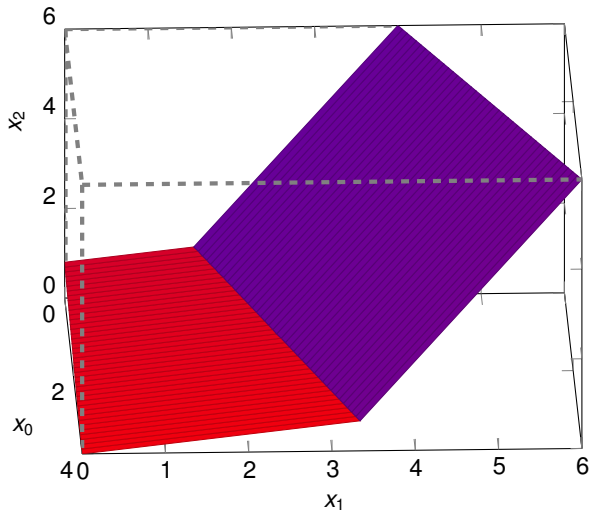


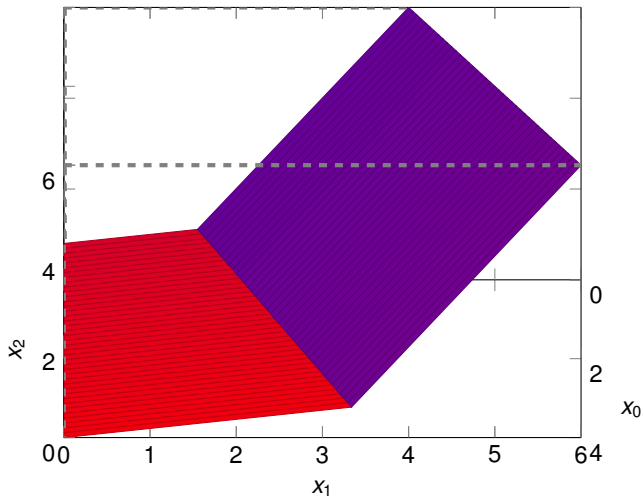


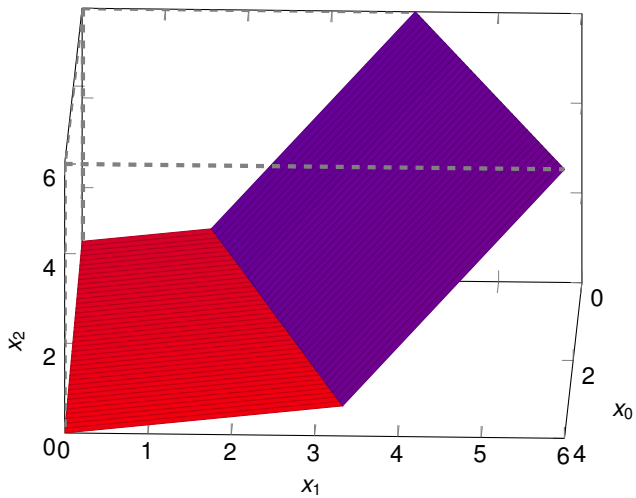


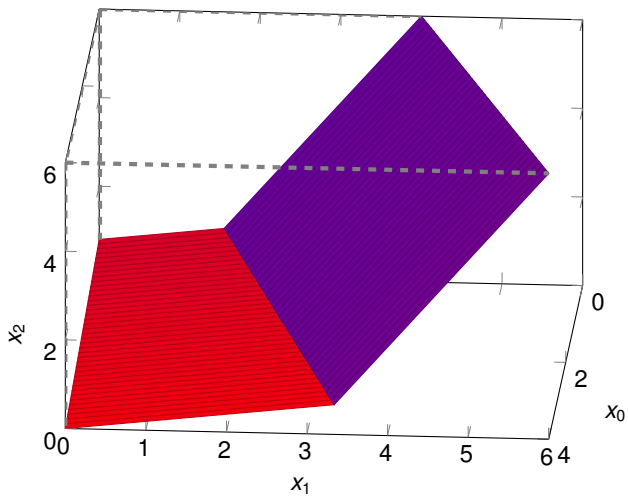


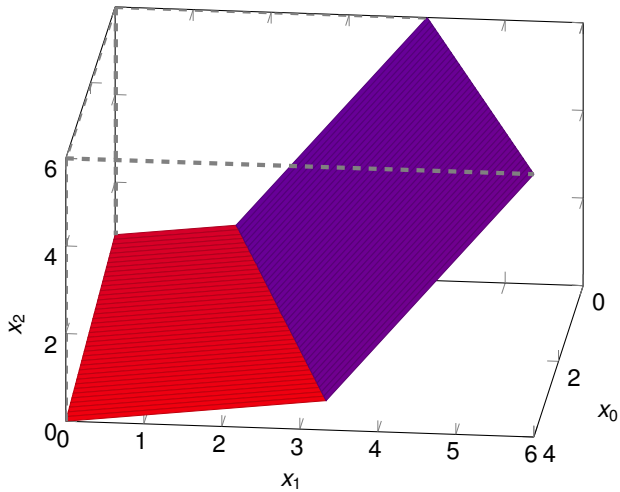


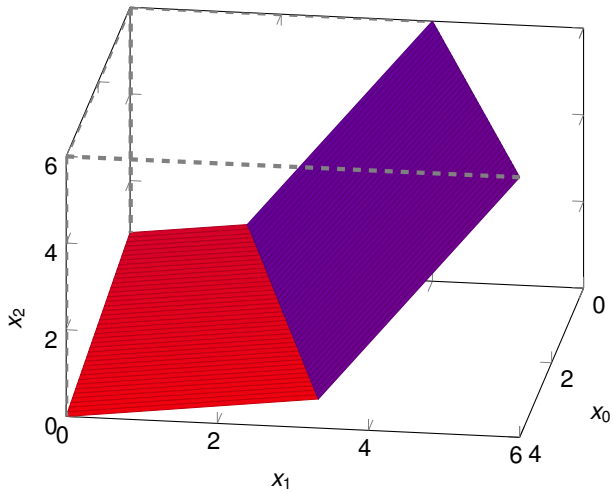


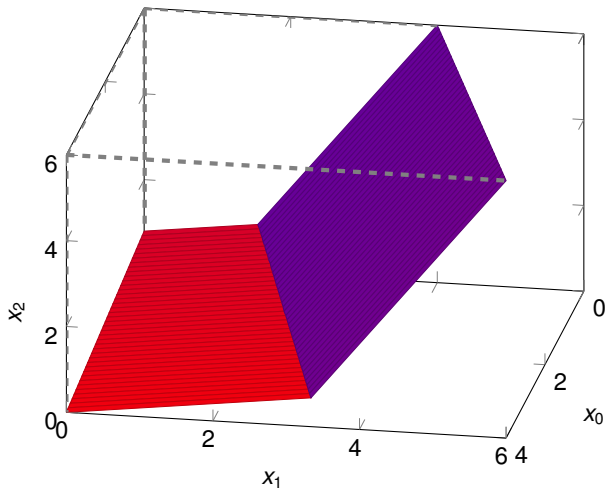


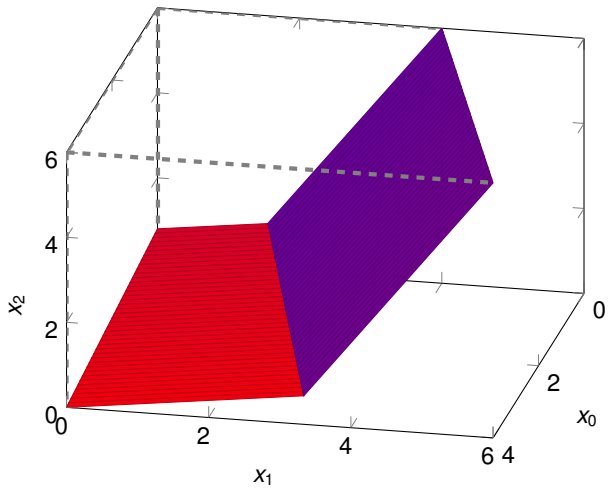


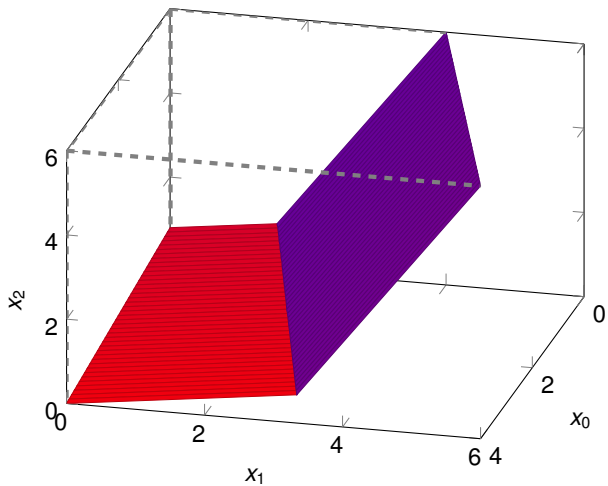


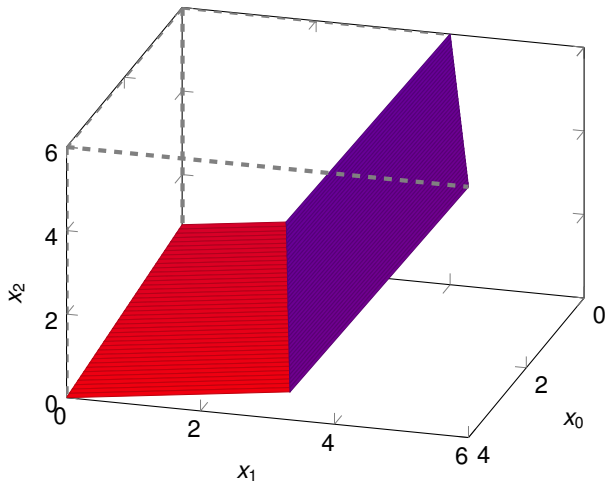


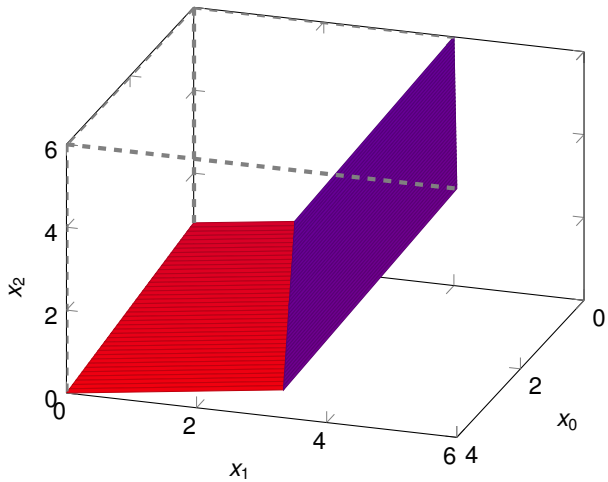


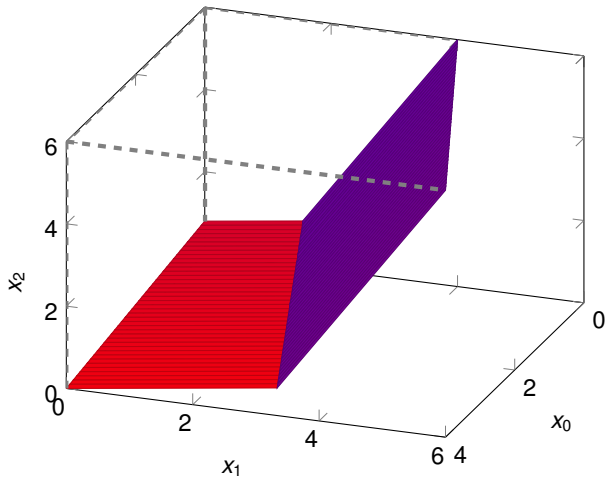


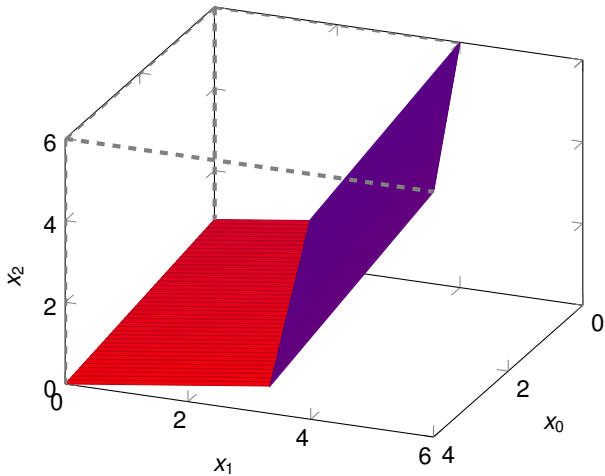


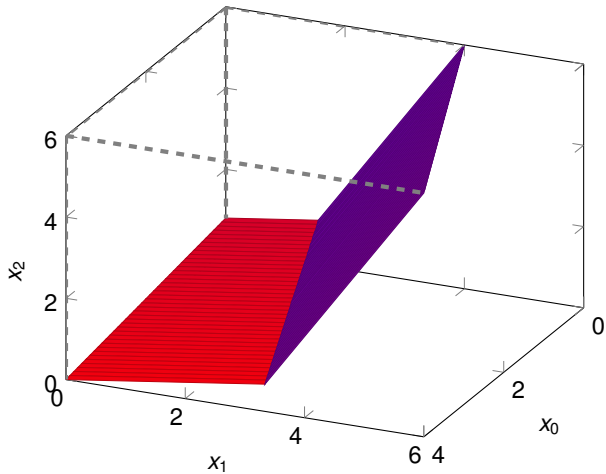


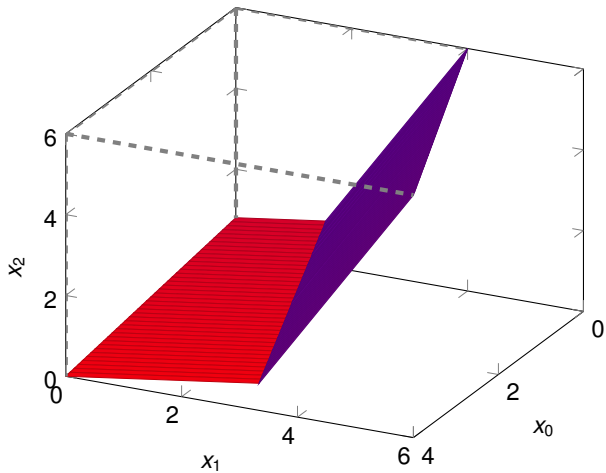


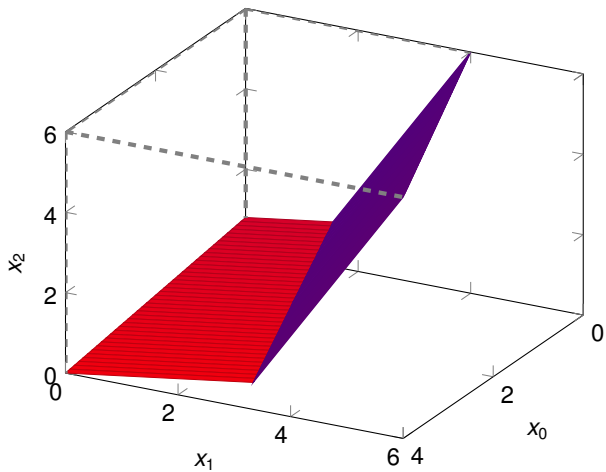


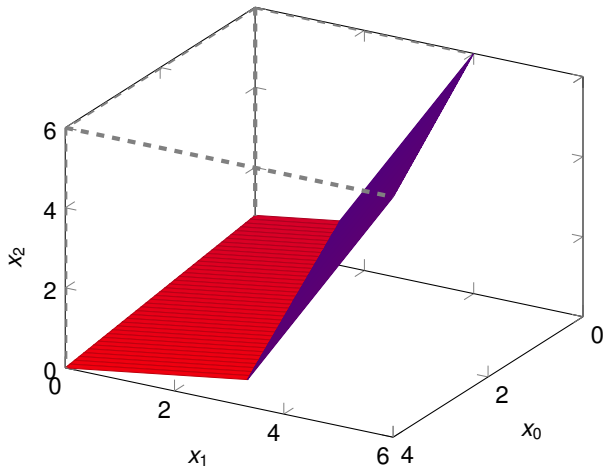


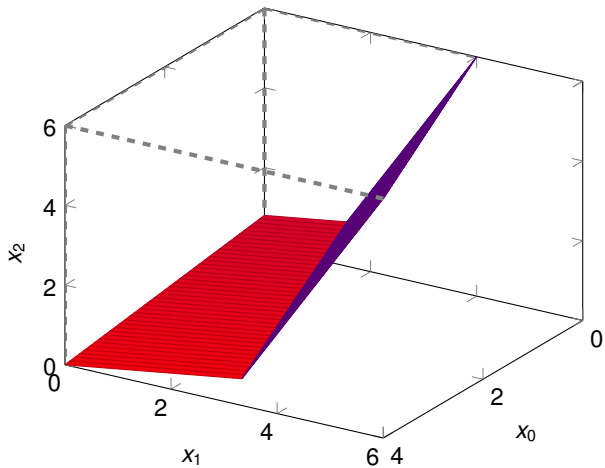


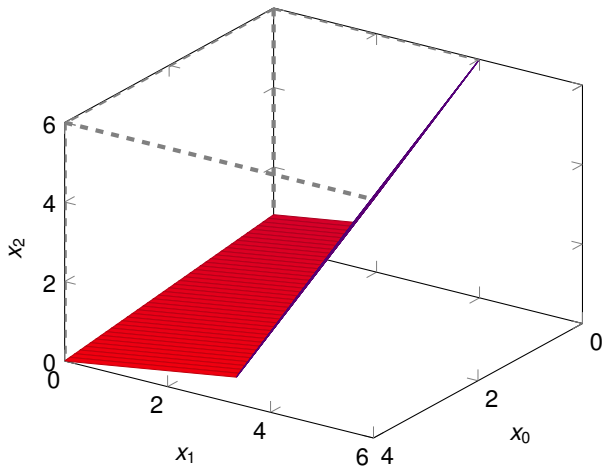


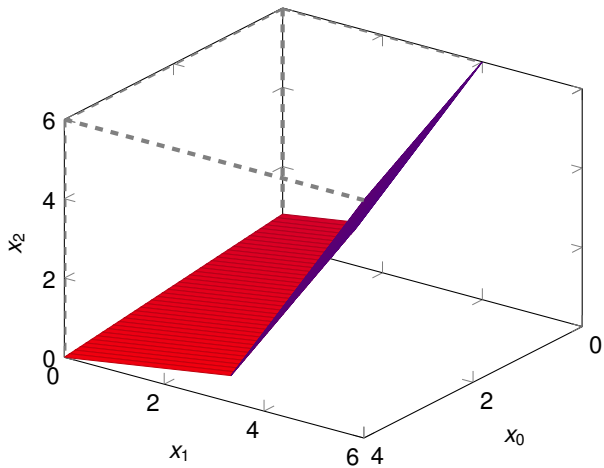


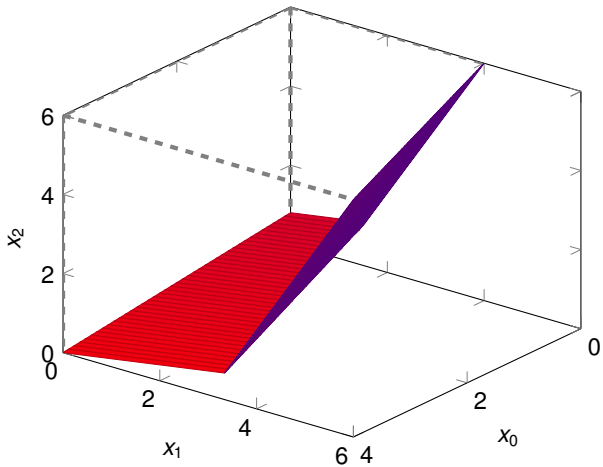


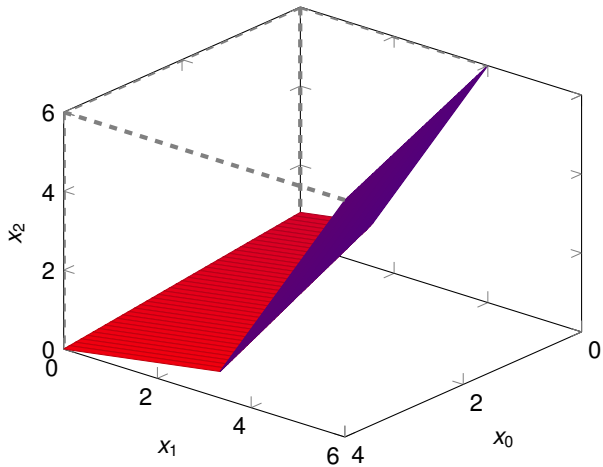


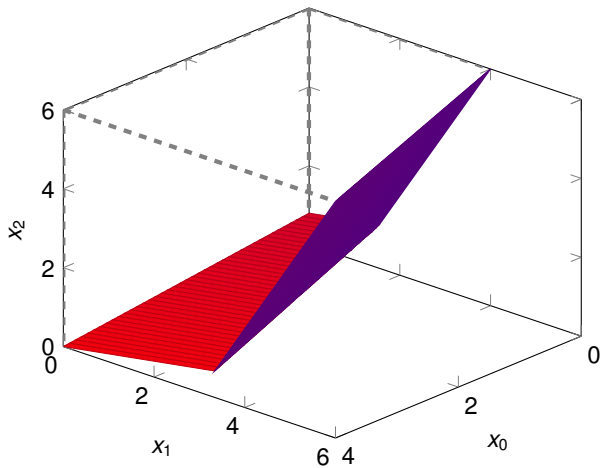


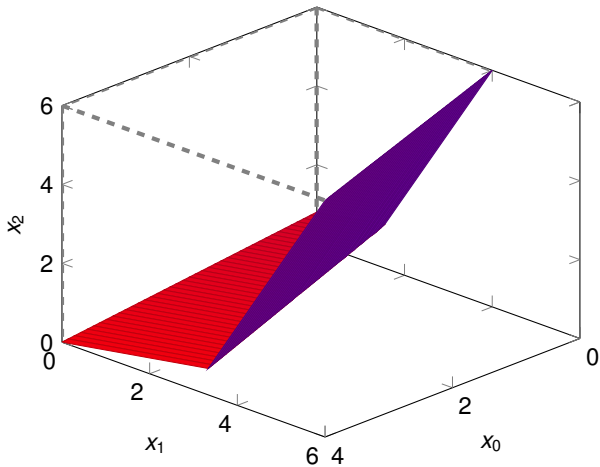


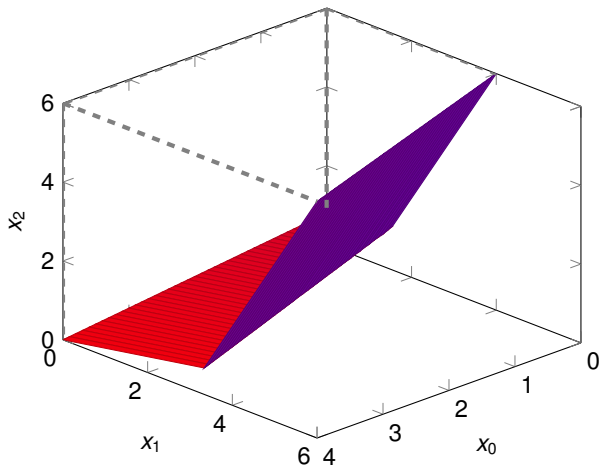


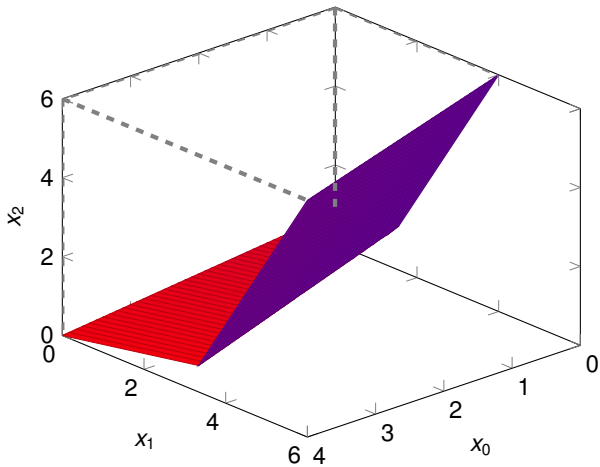


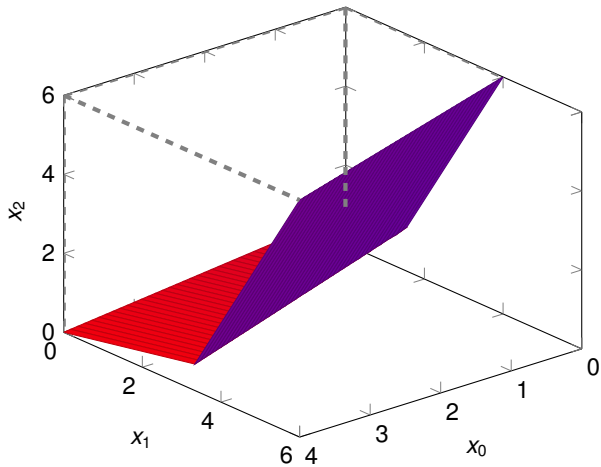


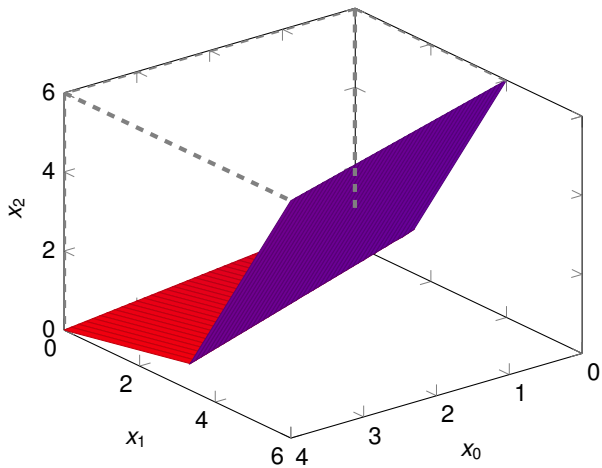


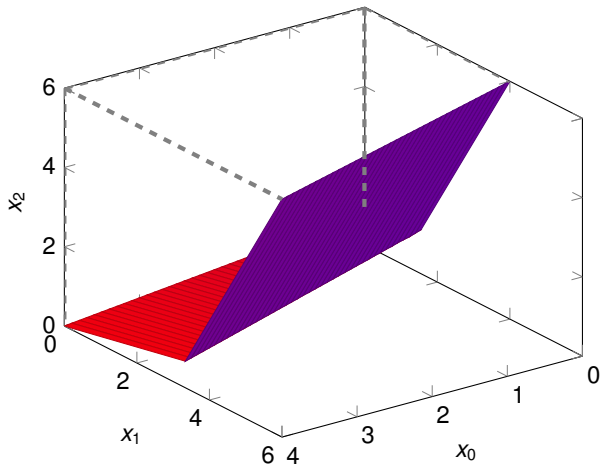


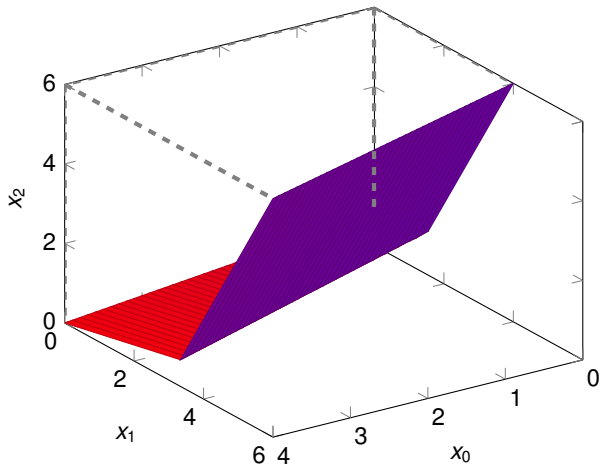


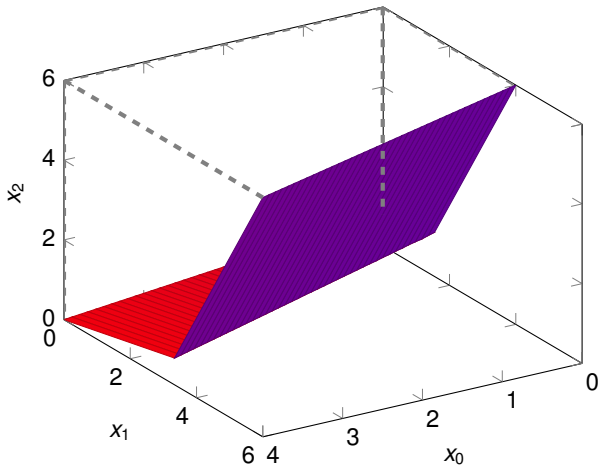


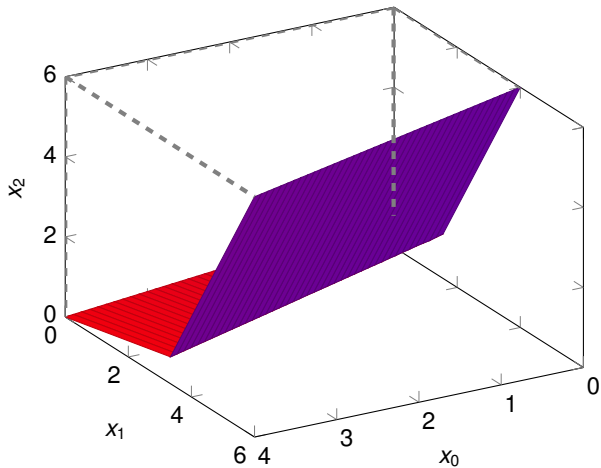


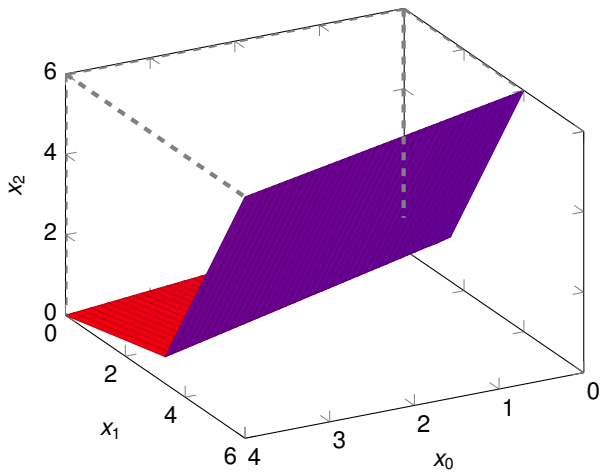


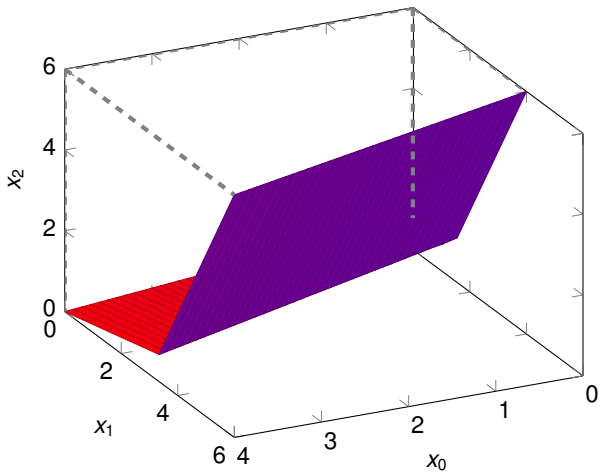


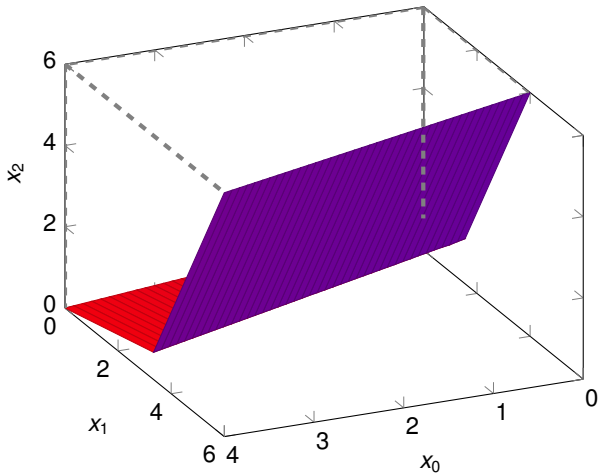


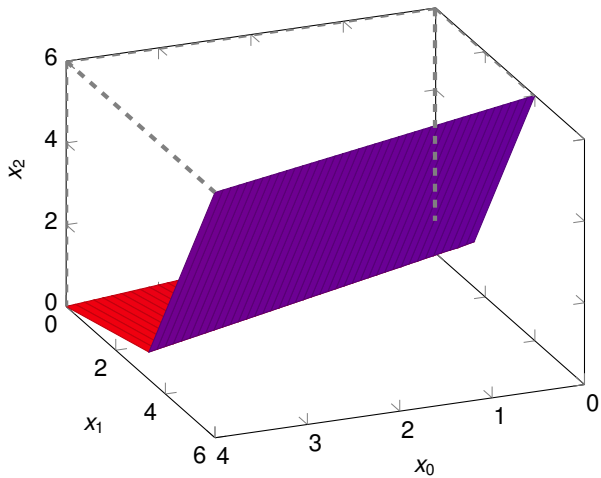


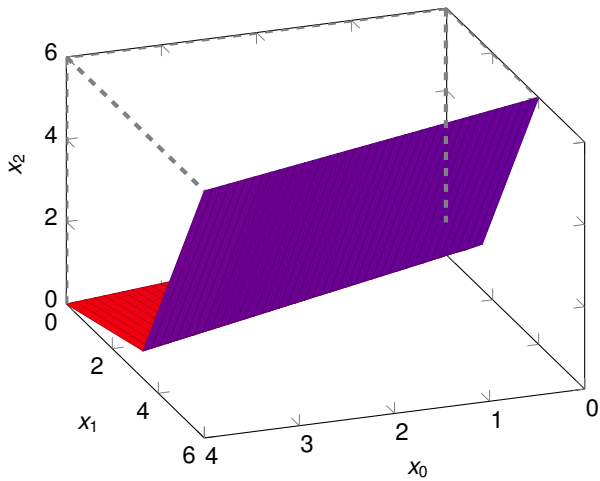


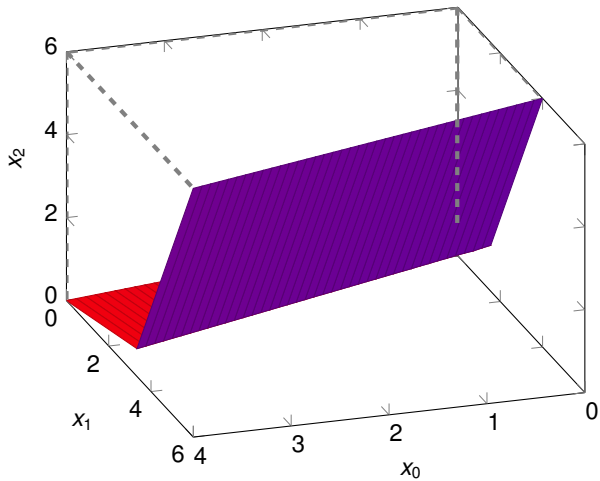


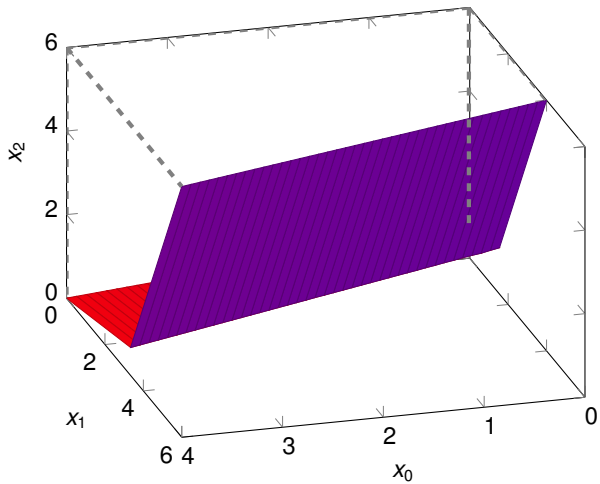


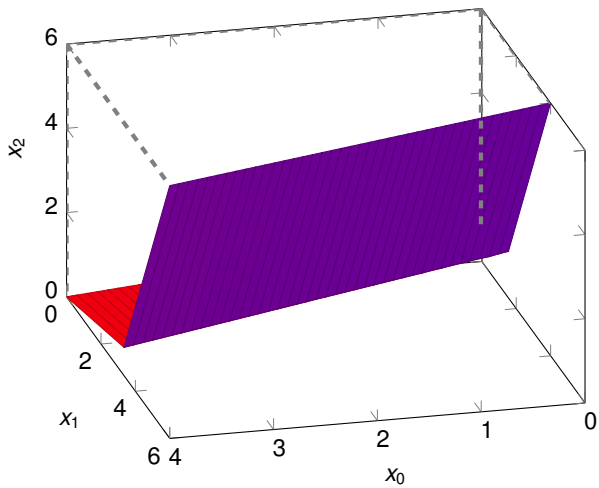


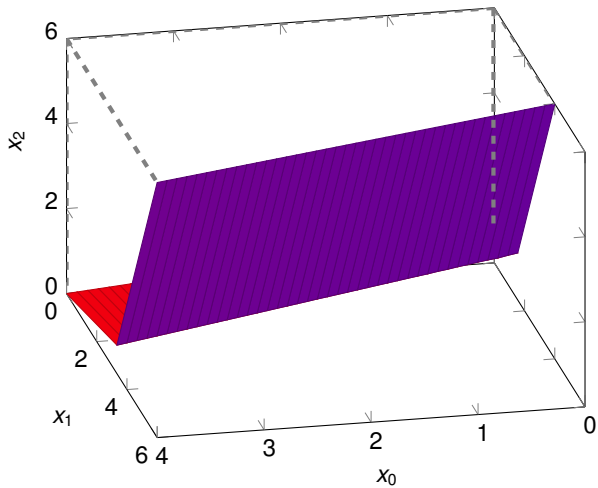


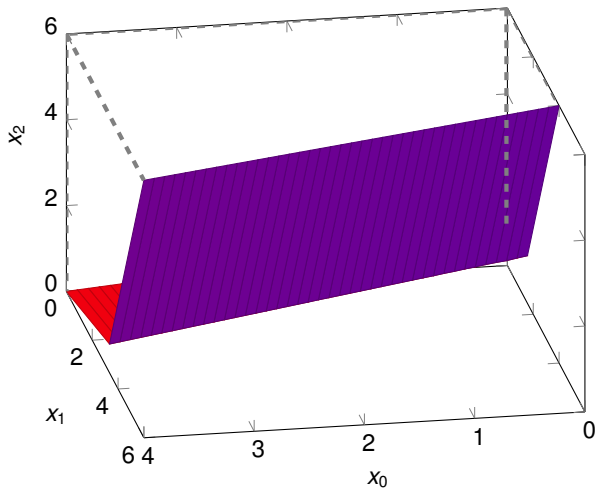












- Let us now modify the original linear program so that it is **not feasible**



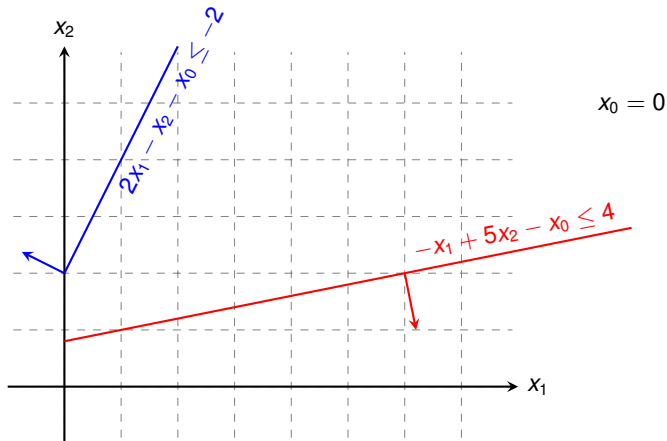
- Let us now modify the original linear program so that it is **not feasible**
- ⇒ Hence the auxiliary linear program has only a solution for a sufficiently large $x_0 > 0$!



Geometric Illustration

maximize $-x_0$
subject to

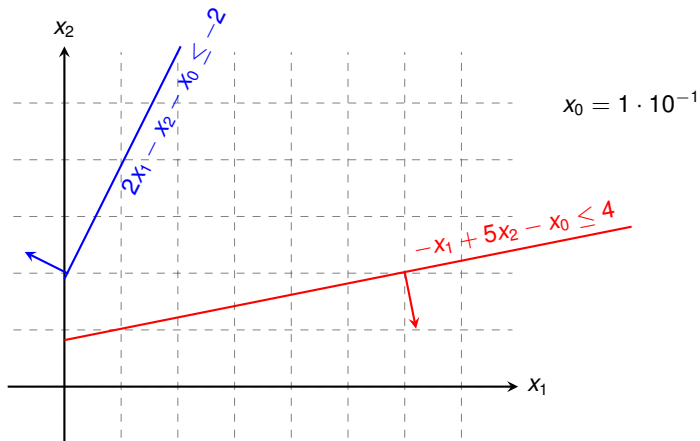
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

maximize $-x_0$
subject to

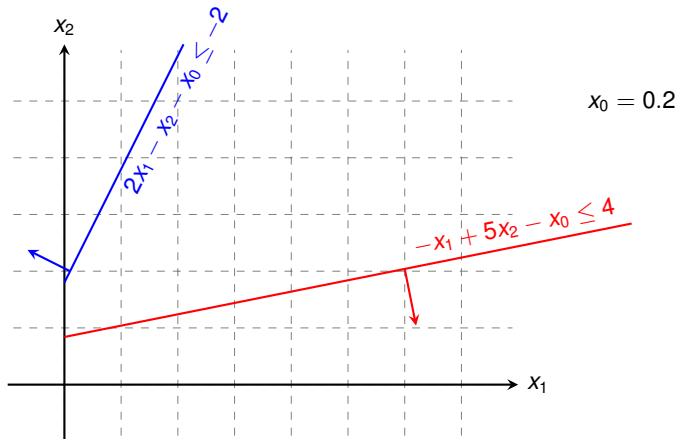
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

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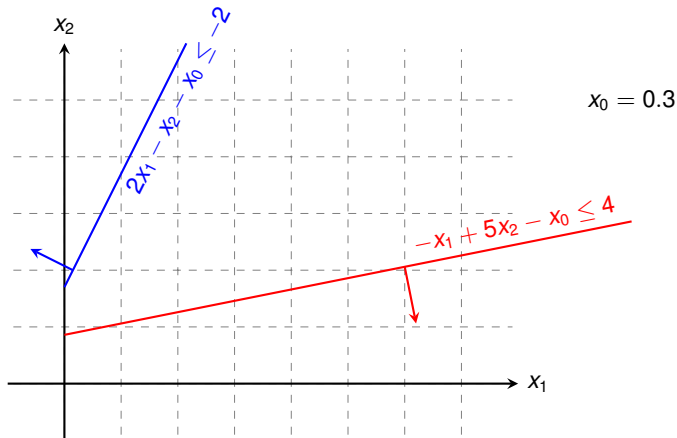
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

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subject to

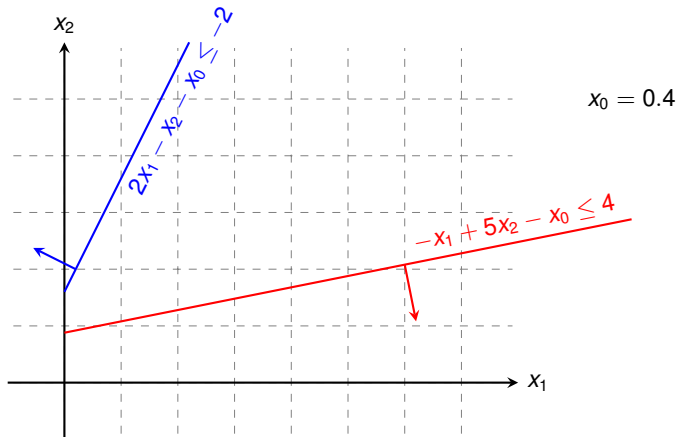
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Geometric Illustration

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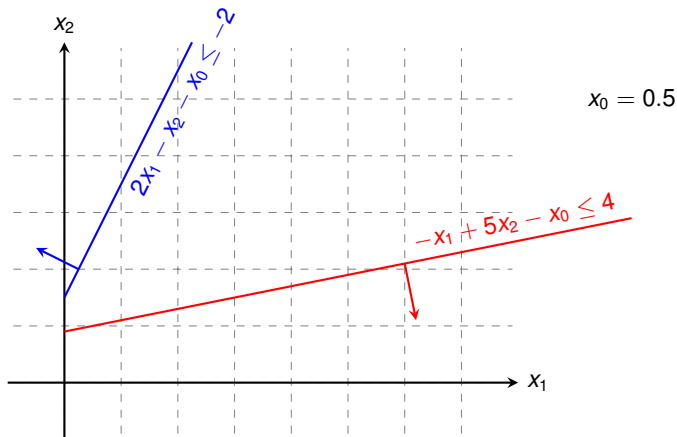
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

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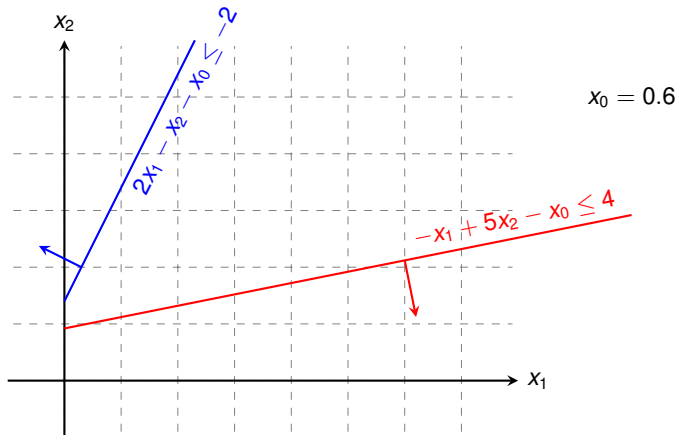
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

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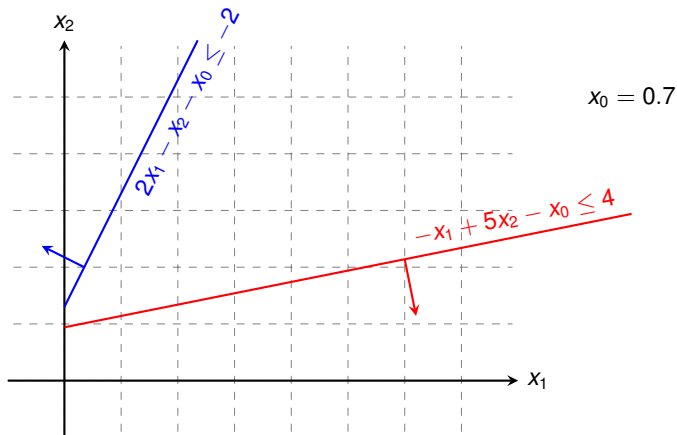
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Geometric Illustration

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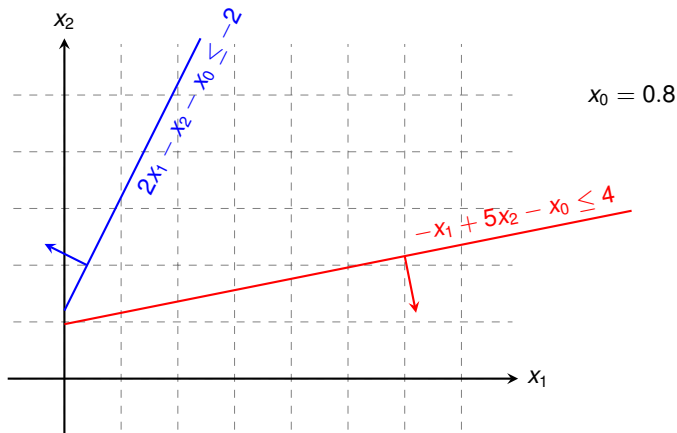
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Geometric Illustration

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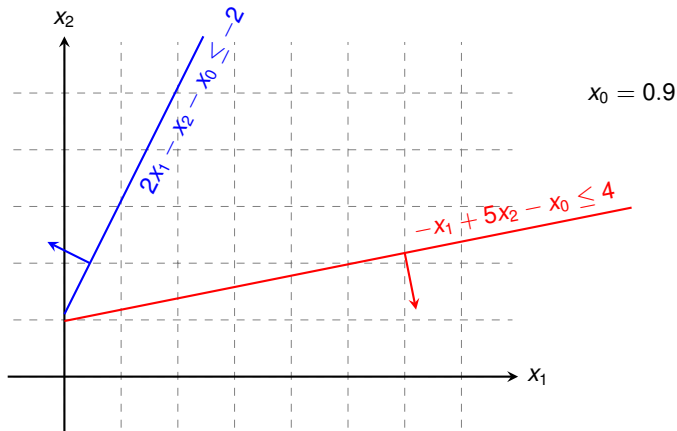
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Geometric Illustration

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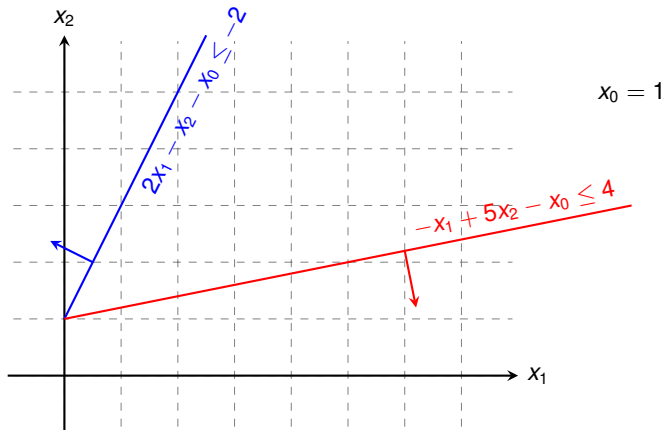
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Geometric Illustration

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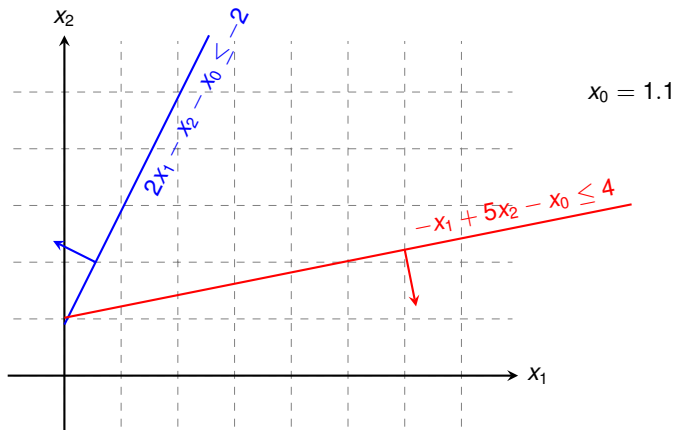
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Geometric Illustration

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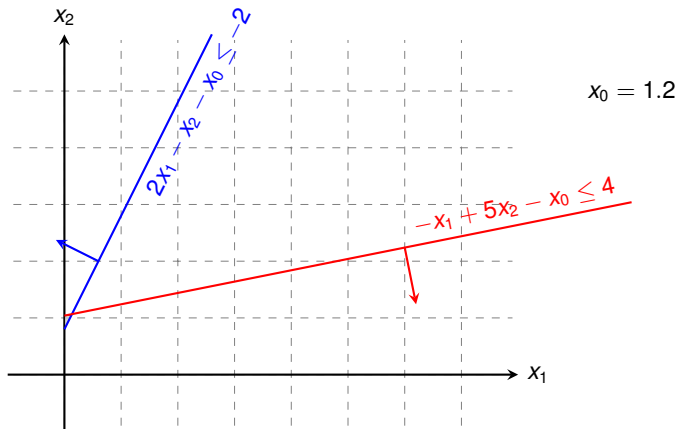
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Geometric Illustration

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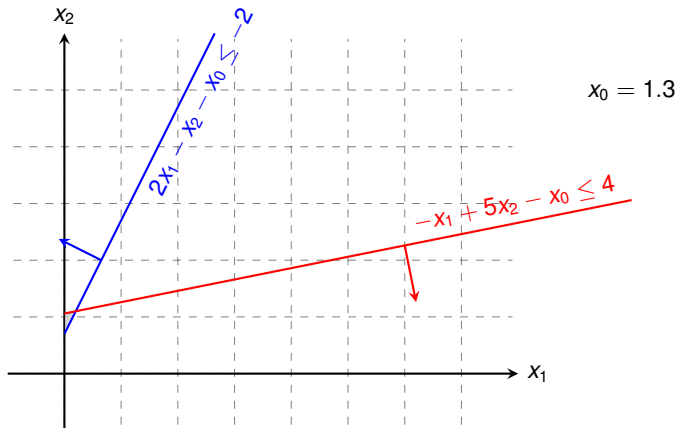
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Geometric Illustration

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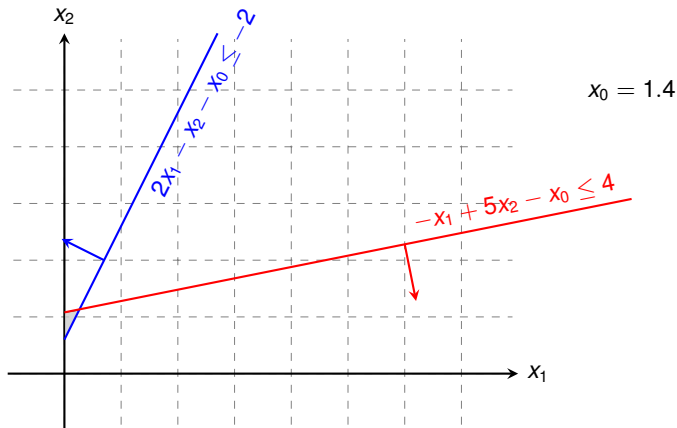
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Geometric Illustration

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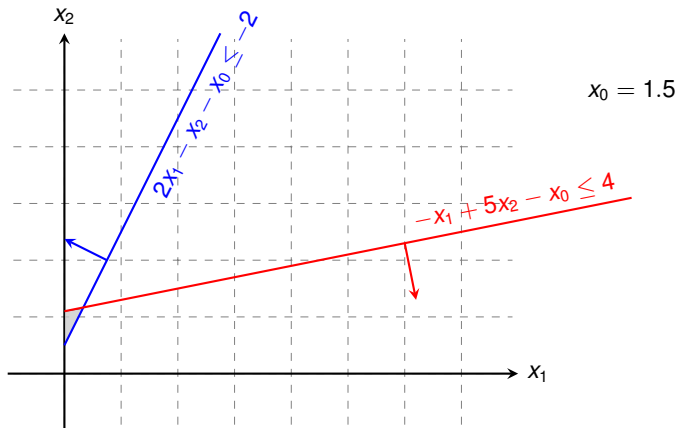
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Geometric Illustration

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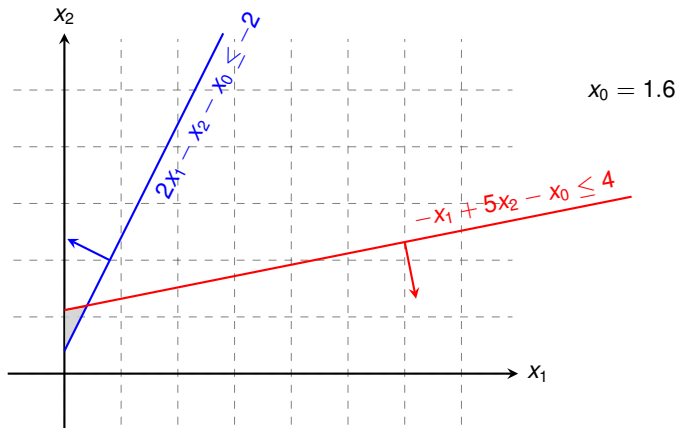
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Geometric Illustration

maximize $-x_0$
subject to

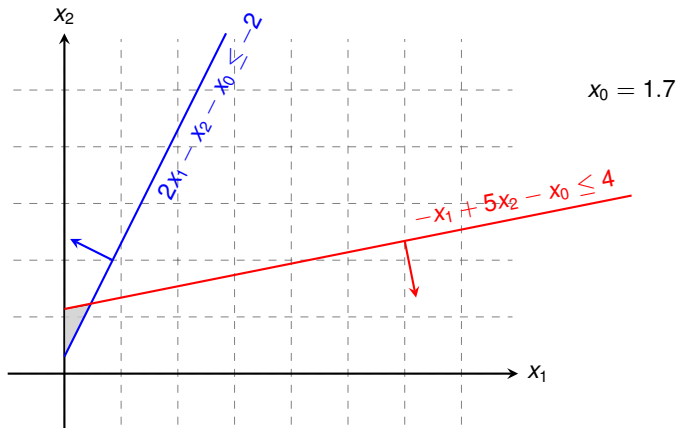
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Geometric Illustration

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subject to

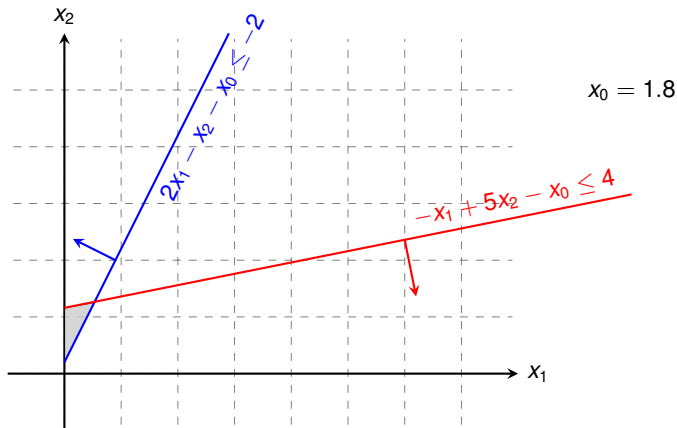
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Geometric Illustration

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subject to

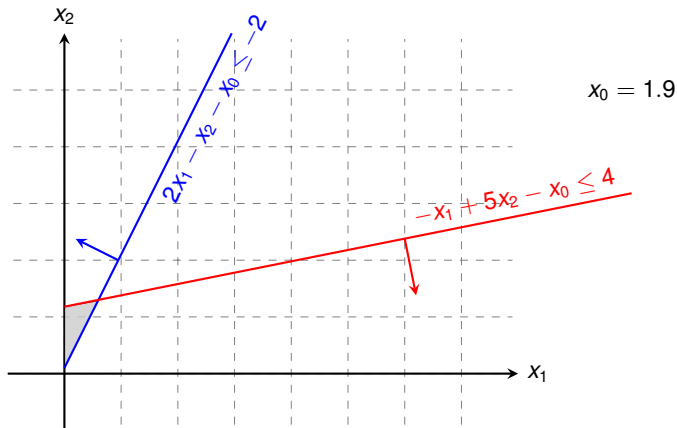
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Geometric Illustration

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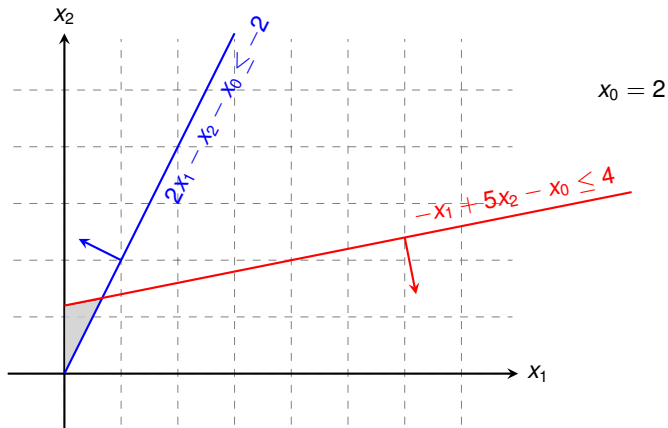
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Geometric Illustration

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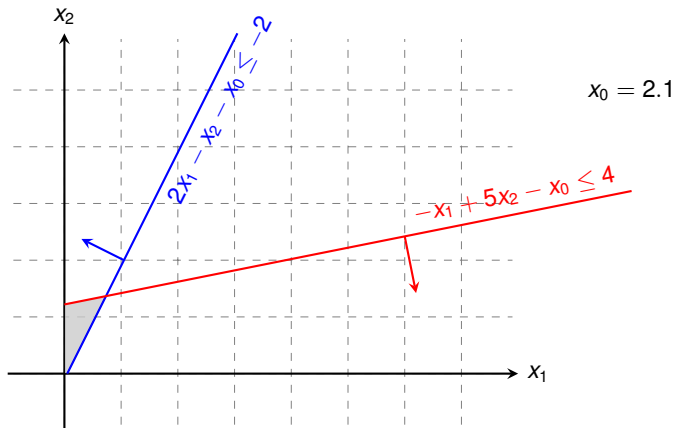
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Geometric Illustration

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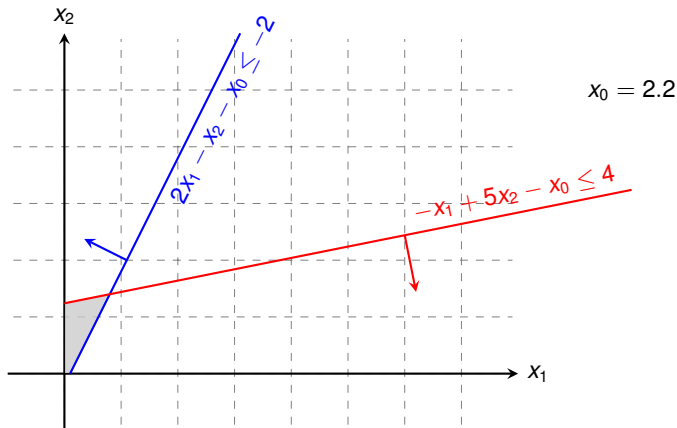
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Geometric Illustration

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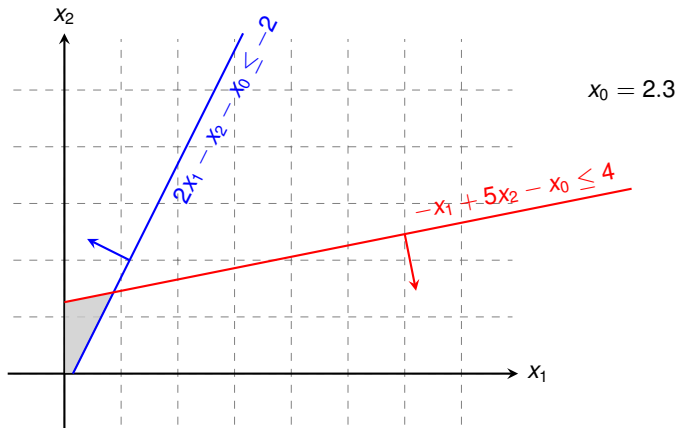
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Geometric Illustration

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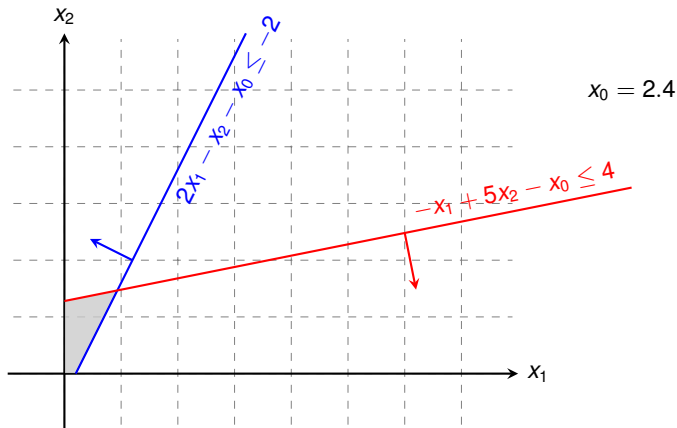
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Geometric Illustration

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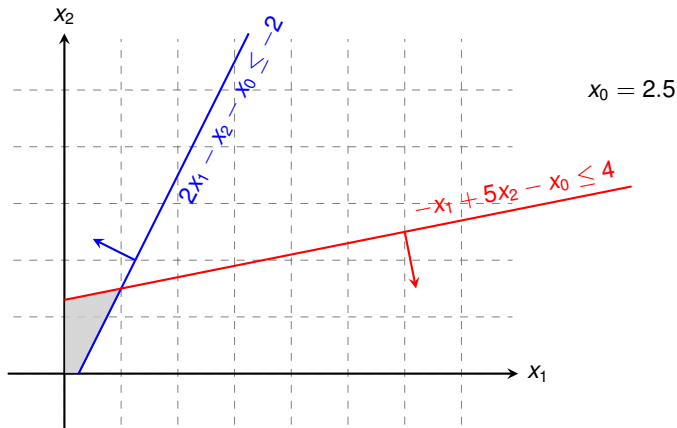
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Geometric Illustration

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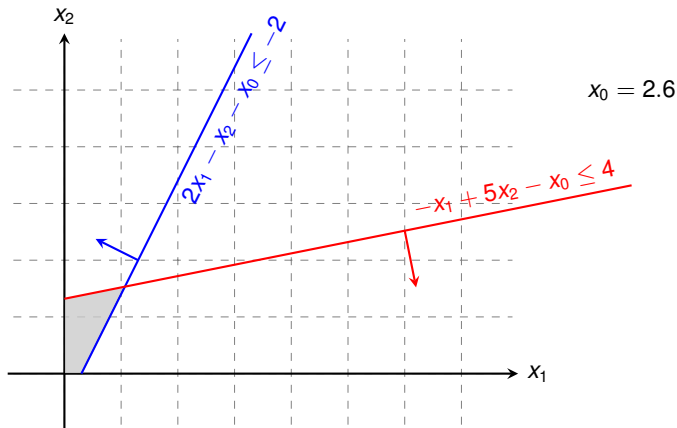
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Geometric Illustration

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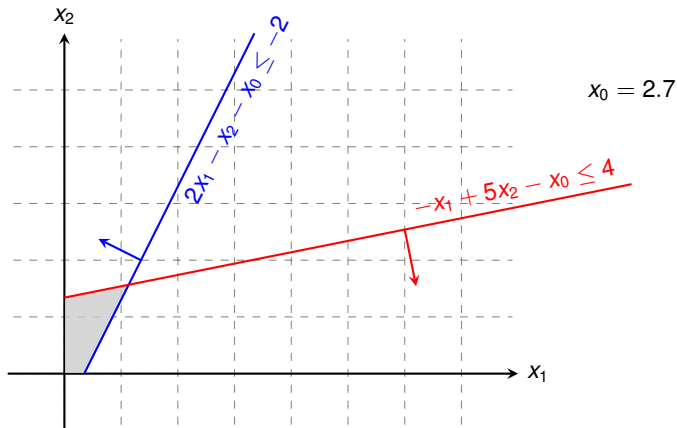
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Geometric Illustration

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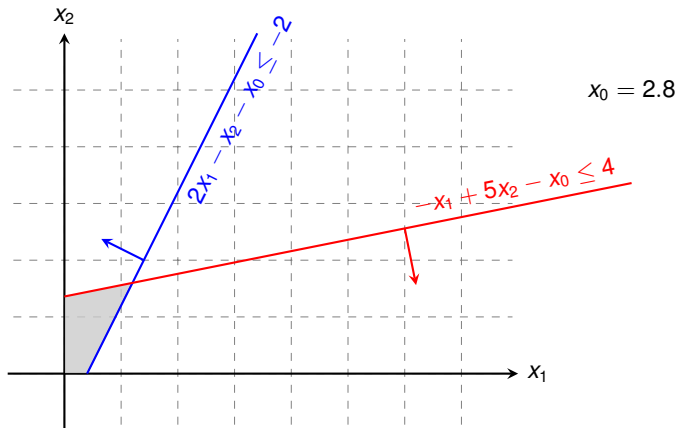
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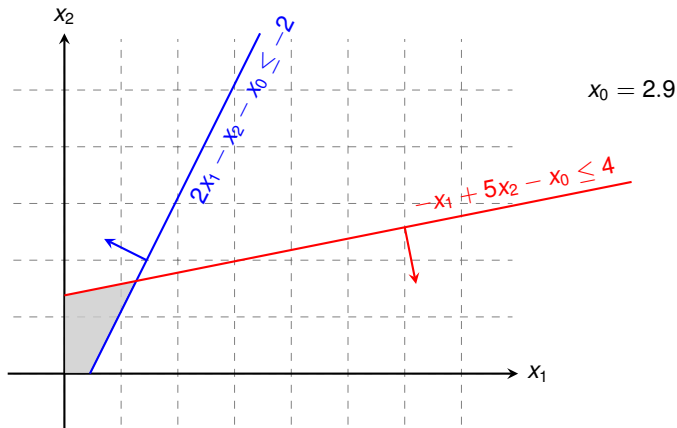
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Geometric Illustration

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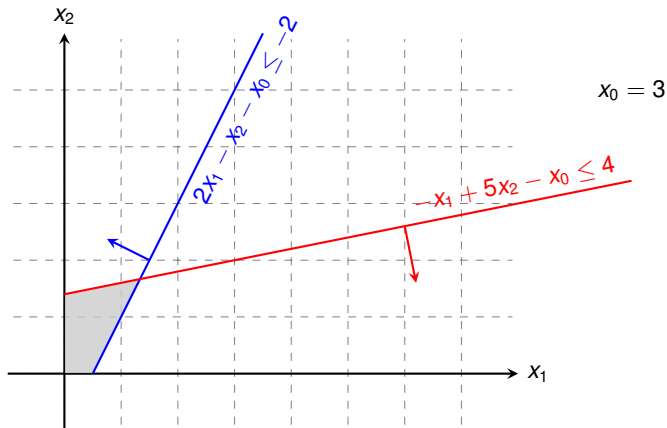
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Geometric Illustration

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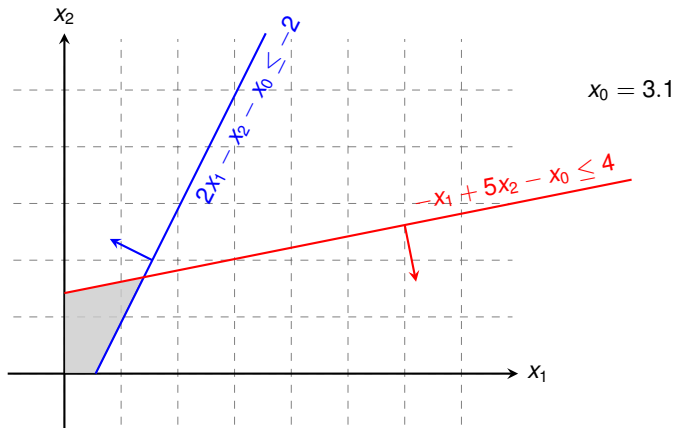
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Geometric Illustration

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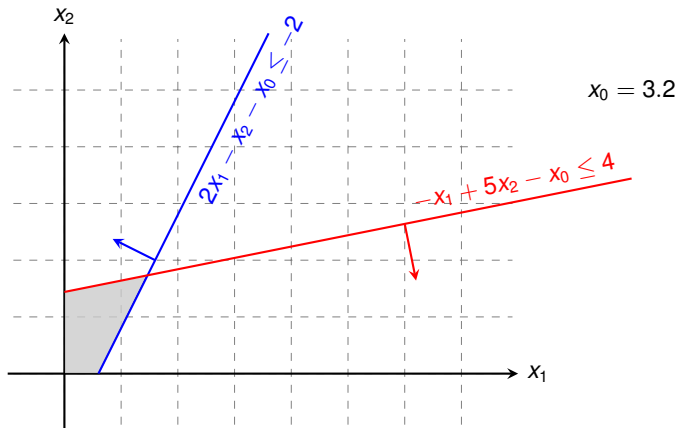
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



Geometric Illustration

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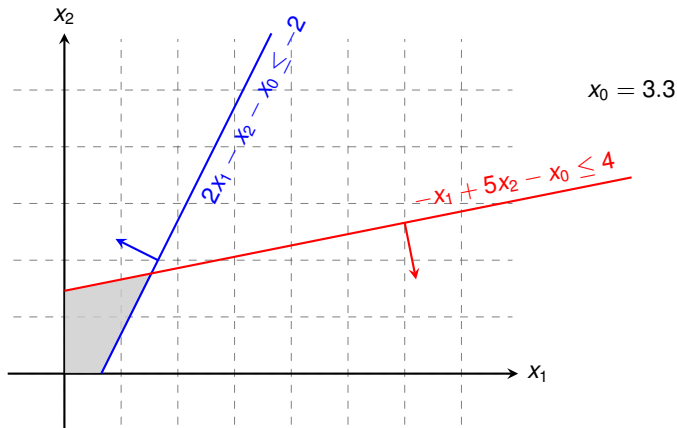
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Geometric Illustration

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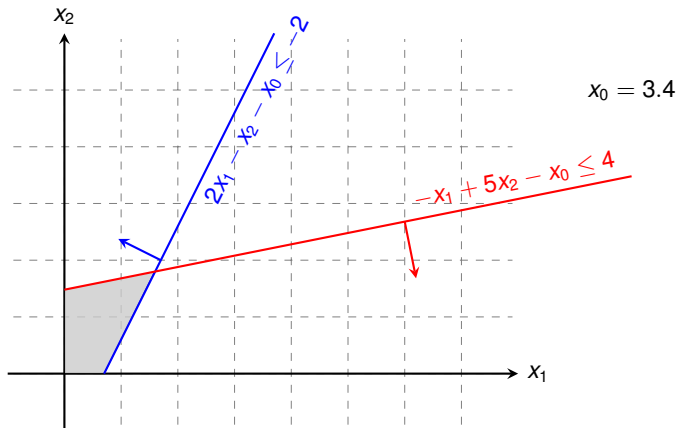
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Geometric Illustration

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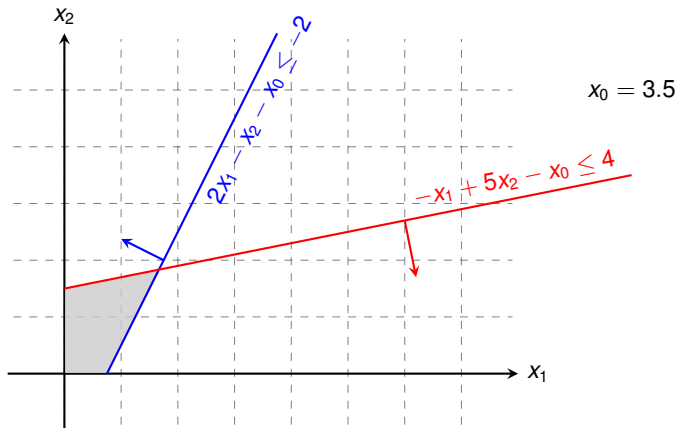
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Geometric Illustration

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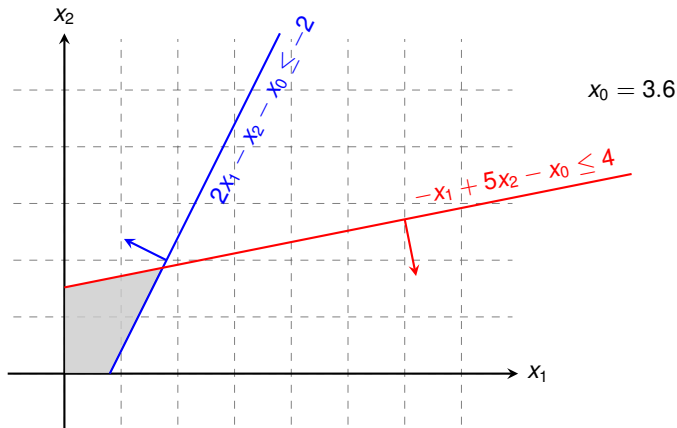
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Geometric Illustration

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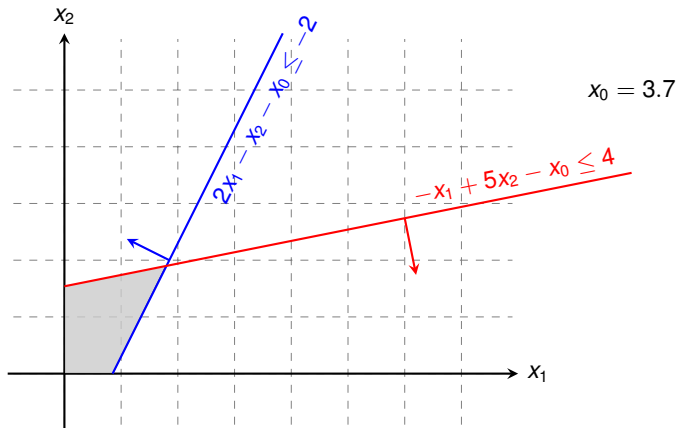
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Geometric Illustration

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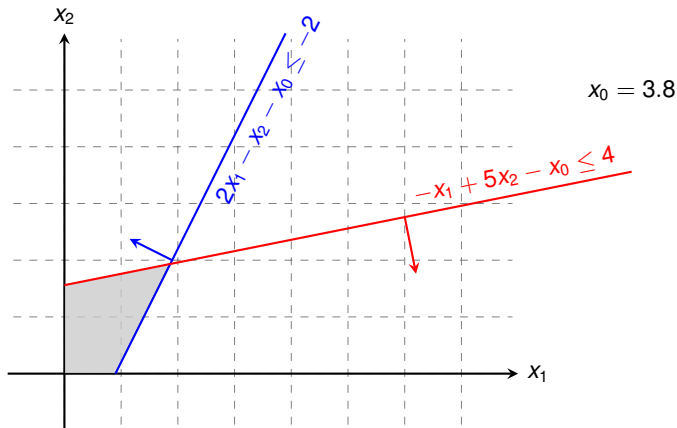
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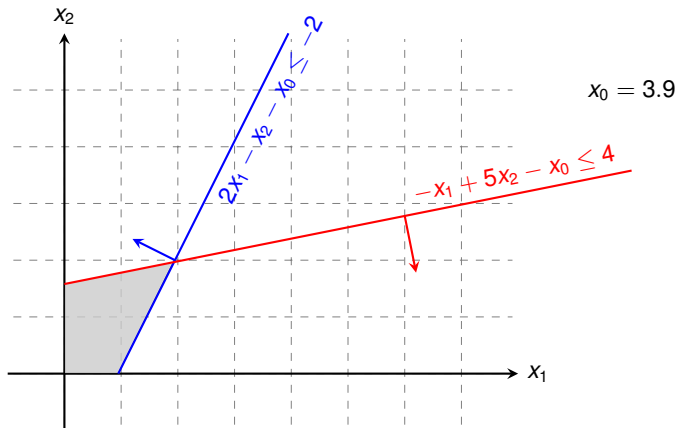
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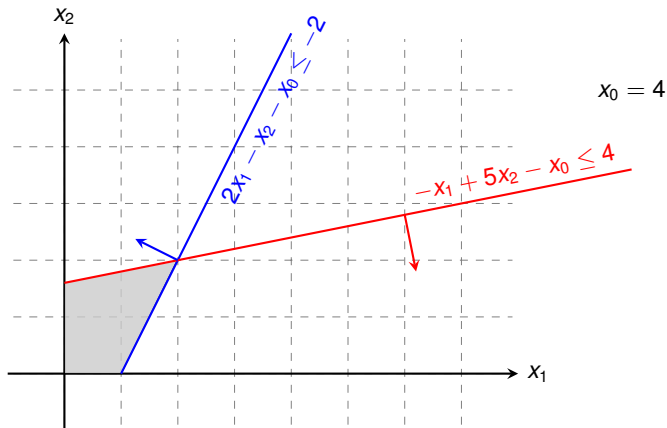
$$\begin{array}{rcccccc} 2x_1 & - & x_2 & - & x_0 & \leq & -2 \\ -x_1 & + & 5x_2 & - & x_0 & \leq & 4 \\ & & & & x_0, x_1, x_2 & \geq & 0 \end{array}$$



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Now the Feasible Region of the Auxiliary LP in 3D



