1. Prove the statements of Lecture X by structural induction.

2. Let
\[
\text{fun foldl } f \ e \ [] = e \\
| \text{foldl } f \ e \ (h::t) = \text{foldl } f \ (f(x,e)) \ t
\]
(a) For all \( f : \alpha \rightarrow \beta, b : \beta, \) and \( \ell_0, \ell_1 : \alpha \text{ list}, \) show that
\[
\text{foldl } f \ b \ (\ell_0 @ \ell_1) = \text{foldl } f \ (\text{foldl } f \ b \ \ell_0) \ \ell_1 : \beta
\]
(b) For \( \oplus : \beta \rightarrow \beta \) an associative function show that, for all \( b_0, b_1 : \beta \) and \( \ell : \alpha \text{ list}, \)
\[
\text{foldl } \oplus \ (b_1 \oplus b_0) \ \ell = (\text{foldl } \oplus \ b_1 \ \ell) \oplus b_0 : \beta
\]

3. Let
\[
\text{fun foldr } f \ e \ [] = e \\
| \text{foldr } f \ e \ (h::t) = f(h, \text{foldr } f \ e \ t)
\]
(a) For all \( \ell_0, \ell_1 : \alpha \text{ list}, \) show that
\[
\text{foldr } (\text{op}::) \ \ell_0 \ \ell_1 = \ell_1 @ \ell_0 : \alpha \text{ list}
\]
(b) For \( \otimes : \beta \rightarrow \beta \) an associative function and \( e : \beta \) such that \( \otimes(e, x) = x \) for all \( x : \beta, \) show that
\[
(foldr \ \otimes \ e \ \ell) \otimes b = foldr \ \otimes \ b \ \ell
\]
and
\[
\text{foldr } (\text{fn}(l,b) \Rightarrow foldr \ \otimes \ b \ l) \ e \ \ell = foldr \ \otimes \ e \ (\text{map } (\text{foldr } \ \otimes \ e) \ \ell) : \beta
\]
for all \( b : \beta \) and \( \ell : \beta \text{ list}. \)