PROBABILITY

These notes accompany the twelve lectures of the course on Probability given to Part IA Computer Scientists. An outline schedule of the lectures is presented overleaf.

Prerequisites
It is assumed throughout this document that the reader studied Mathematics to A-level or equivalent and followed the Michælmas Term course in Mathematics given to Part IA Natural Scientists.
No familiarity with Probability is assumed but a little account is taken of those with previous experience of Probability, perhaps having studied the topic at A-level. Even in the earlier lectures material is presented in a way which is likely to be different from the approach used at A-level and many of the exercises are quite demanding.

Recommended Books
There are numerous books on Probability. In the preparation of this course the book which was most extensively consulted was Probability: An Introduction by G. Grimmet and D. Welsh. An important standard text is An Introduction to Probability Theory, Volume I by W. Feller.

Appendix
This is not a course on statistics but, in response to requests from previous first-year Computer Scientists, there is an appendix at the end of this document which addresses a particular question: Why is it that the formula for population variance is $\Sigma(x - \bar{x})^2/n$ but the formula for sample variance is $\Sigma(x - \bar{x})^2/(n - 1)$?

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F.H. King
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4. **MEANS AND VARIANCES.** Use of derived random variables and generalised expectation. Variance and standard deviation. Geometric distribution. Poisson distribution. Revision of summation (double-sigma sign). Mean and variance when there are two or more random variables. Covariance.

5. **CORRELATION.** Mean and variance of the Binomial distribution. Correlation coefficient. Complete positive and complete negative correlation. $P(X+Y=t)$. A polynomial with probabilities as coefficients.


7. **DIFFERENCE EQUATIONS.** Introduction to linear, second-order difference equations with constant coefficients. How these equations are found in Probability. Solving homogeneous and inhomogeneous difference equations.


11. **TRANSFORMING DENSITY FUNCTIONS.** Integration by substitution. Application to probability density functions. Transforming a uniform distribution. Transforming a Uniform distribution into a Normal distribution using Excel.

12. **TRANSFORMING BIVARIATE DENSITY FUNCTIONS.** Integration with two independent variables. Jacobians. Application to bivariate probability density functions. The Box–Muller Transformation.