Desirable features in CAD
+ Need to handle any surface
+ Need guaranteed continuity
  - Continuity of slope (C1)
    - Smooth surfaces
  - Continuity of curvature (C2)
    - Smoothly reflecting surfaces
    - Required for some aerodynamics
+ Need to allow for discontinuities
  - Edges, creases and holes
+ Needs to be easy to use

Traditional tools
+ Bezier patches
+ B-spline patches
+ NURBS patches

Bezier & B-spline patches
+ A rectangular array of control points
+ A mathematical function determines where the surface goes based on those points
+ Move a control point to change the surface

The first problem
+ Very few objects are made up of a single rectangular patch, so we need to join patches together

The mathematics of joins
+ We want to preserve certain types of mathematical continuity across joins
  - C0: continuity of position
    - Prevents holes at the join
  - C1: continuity of slope
    - Prevents a sharp edge at the join
  - C2: continuity of curvature
    - Strongly related to aesthetics
    - Most often visible in reflections
    - Prevents sharp edges in reflected lines
  - These are continuity of the zeroth, first and second derivatives

Joining two Bezier patches
+ C0 but not C1
  - Four edge points are the same
+ C0 and C1
  - Four edge points are the same
  - Next four points out in either direction are constrained
Part II
Advanced Graphics 2004
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Slide 13
An example: the car’s roof
+ The car
+ Curvature plot of its roof

Slide 14
Definition of the car’s roof
+ 5x2 grid of biquintic Bezier patches
  - 36 control points per patch
  - 286 control points overall
  - Moving one point also moves several others to maintain C2 continuity

Slide 15
B-spline patches
+ A rectangular array of points define a rectangular array of automatically joined patches
+ Example
  - The black points control the central patch
  - All points together define a surface of many joined patches

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The second problem
+ What do we do at special points where other than four patches meet?
  - Either we cannot get C2, which means that curvature is not continuous
  - Or we get C2 by forcing curvature to be zero, which produces a flat spot
  - Or we get C2 using very high degree patches, which are very hard for a designer to control

Slide 17
Subdivision surfaces
- Developed in the 1970s, adopted in computer animation in 1990s
- Replace the patch-based representation of B-splines and Beziers
- Base a curve or surface solely on its control points and their connectivity
- A simple mechanism produces a larger, more refined set of control points from the current set
- Iterate refinement until the appropriate level of detail is achieved

Slide 18
Subdivision
+ Advantages
  - Reproduces everything which can be done by B-splines
  - Handles extraordinary points much more easily
+ Disadvantages
  - Cannot get C2 unless you produce a flat spot
  - Generates other visual artefacts, not seen in B-spline surfaces
+ Commercial position
  - Subdivision is replacing B-splines in computer animation
  - Subdivision is not replacing B-splines in CAD