Introduction to Functional Programming
Lent 2004
Suggested Exercises 2

1. **Binary Trees.** Write a function `reverse` which creates the mirror image of a binary tree. That is, if \( T \) is a binary tree, than `reverse(T)` is a binary tree in which, at every node, left and right branches are interchanged.

A binary tree is said to be *balanced* if for each node \( Br(x, t_1, t_2) \) the sizes of \( t_1 \) and \( t_2 \) differ by at most one. Write a function `balanced` of type `'a tree -> bool` which determines whether a tree is balanced. One obvious solution involves checking the size of every subtree, but this is inefficient because it repeats a lot of computation. Can you do this more efficiently?

2. **Arrays** Write a function that takes an array in binary tree form and returns a list of the elements of the array, in order. Can you do this efficiently, i.e. without extracting each element by looking up the subscript?

3. **Merge Sort** Write a generic version of `mergesort`, which takes a comparison function as argument.

4. **Minimum** Write a functional to compute the minimum value \( \min_{i=0}^{n-1} f(i) \) of a function \( f \). Use the functional to express the two dimensional minimum \( \min_{i=0}^{n-1} \min_{j=0}^{n-1} g(i, j) \) of a function \( g \) of two arguments.