1. Show that a language \( L \) is in \( \text{co-NP} \) if, and only if, there is a nondeterministic Turing machine \( M \) and a polynomial \( p \) such that \( M \) halts in time \( p(n) \) for all inputs of length \( n \), and \( L \) is exactly the set of strings \( x \) such that all computations of \( M \) on input \( x \) end in an accepting state.

2. Define a strong nondeterministic Turing machine as one where each computation has three possible outcomes: accept, reject or maybe. If \( M \) is such a machine, we say that it accepts \( L \), if for every \( x \in L \), every computation path of \( M \) on \( x \) ends in either accept or maybe, with at least one accept and for \( x \notin L \), every computation path of \( M \) on \( x \) ends in reject or maybe, with at least one reject.

Show that if \( L \) is decided by a strong nondeterministic Turing machine running in polynomial time, then \( L \in \text{NP} \cap \text{co-NP} \).

3. Consider the algorithm presented in the lecture which establishes that \text{Reachability} is in \( \text{SPACE}((\log n)^2) \). What is the time complexity of this algorithm?

Can you generalise the time bound to the entire complexity class? That is, give a class of functions \( F \), such that

\[
\text{SPACE}((\log n)^2) \subseteq \bigcup_{f \in F} \text{TIME}(f)
\]

4. Show that, for every nondeterministic machine \( M \) which uses \( O(\log n) \) work space, there is a machine \( R \) with three tapes (input, work and output) which works as follows. On input \( x \), \( R \) produces on its output tape a description of the configuration graph for \( M, x \), and \( R \) uses \( O(\log |x|) \) space on its work tape.

Explain why this means that if \text{Reachability} is in \( L \), then \( L = \text{NL} \).

5. Consider the language \( L \) in the alphabet \( \{a,b\} \) given by \( L = \{a^n b^n \mid n \in \mathbb{N}\} \). \( L \notin \text{SPACE}(c) \) for any constant \( c \). Why?