Past Tripos and Diploma questions

Questions on Discrete Mathematics have in the past been set in Part IA (for both Mathematics and Computer Science) and in the Diploma and Part II (General).

Questions on Regular Languages were set on a course given both to Part IB and to the Diploma and Part II (General) until the year 1994–95. From the year 1995–96 that course was moved to CST IA (the 50% option): the present course covers similar material from a more algebraic viewpoint, the intention being to illustrate discrete mathematical structures and the associated proof techniques.

Many past examples are suitable for the present Diploma and Part II (General) course.

Part A

Relations on sets (including equivalence relations and transitive closure):

- CST Part II (G) 1989 Paper 11 Question 7.
- CST Part II (G) 1990 Paper 12 Question 4.
- CST Part II (G) 1993 Paper 10 Question 11.
- CST Part II (G) 1998 Paper 10 Question 10.
- CST Part II (G) 1999 Paper 10 Question 10.

Partially Ordered sets:

- CST Part II (G) 1991 Paper 10 Question 8.
- CST Part II (G) 2000 Paper 10 Question 10.

Well-founded relations and Induction:

- CST Part II (G) 1989 Paper 12 Question 8.
- CST Part II (G) 1992 Paper 11 Question 8.
- CST Part II (G) 1997 Paper 10 Question 9.
- CST Part II (G) 2001 Paper 10 Question 11.
Part B

Regular Languages and Finite Automata:

CST Part IA 1995 Paper 2 Question 27.
CST Part II (G) 1989 Paper 13 Question 11.
CST Part II (G) 1991 Paper 11 Question 6.
CST Part II (G) 1993 Paper 13 Question 12.
CST Part II (G) 1996 Paper 10 Question 9.
CST Part II (G) 1996 Paper 11 Question 8.
CST Part II (G) 1997 Paper 11 Question 8.
CST Part II (G) 1998 Paper 10 Question 10.
CST Part II (G) 1998 Paper 11 Question 8.
CST Part II (G) 1999 Paper 11 Question 8.
CST Part II (G) 2000 Paper 11 Question 8.
CST Part II (G) 2001 Paper 11 Question 7.

Questions from the Mathematical Tripos Part IA 1988 Paper 6:

9 State the principle of mathematical induction. Prove your statement, assuming that every non-empty subset of the natural numbers has a least element.

The Master of Regent’s College and her husband invite $n$ Fellows and their spouses to a party. After the party the Master asks everyone (including her own husband) how many people they shook hands with, and receives $2n+1$ different answers. Of course, no woman shook hands with her own husband. Show that the person who shook the most hands was not the Master’s husband. How many hands did the Master shake?

10 Let $R$ be a relation on a set $X$. Define the reflexive, symmetric and transitive closures $r(R)$, $s(R)$ and $t(R)$ of $R$. Let $\Delta$ be the relation $\{(x, x) \mid x \in X\}$. Prove that:

(a) $R \circ \Delta = R$,

(b) $(R \cup \Delta)^n = \Delta \cup \bigcup \{R^i \mid 1 \leq i \leq n\}$, for all $n \geq 1$,

(c) $tr(R) = rt(R)$.

Show also that $st(R) \subseteq ts(R)$. If $X = \mathbb{N}$ and

$R = \Delta \cup \{(x, y) \mid x, y \in \mathbb{N} \text{ s.t. } y = p.x \text{ for some prime } p \in \mathbb{N}\}$, describe $st(R)$ and $ts(R)$.

[ Notation. In this question $rt(R)$ stands for $r(t(R))$, and so on. ]