What are Computer Graphics & Image Processing?

- Scene description
- Computer graphics
- Digital image
- Image analysis & computer vision
- Image processing
- Image capture
- Image display

Why bother with CG & IP?

- All visual computer output depends on Computer Graphics
  - Printed output
  - Monitor (CRT/LCD/whatever)
  - All visual computer output consists of real images generated by the computer from some internal digital image
What are CG & IP used for?

- **2D computer graphics**
  - graphical user interfaces: Mac, Windows, X, ...
  - graphic design: posters, cereal packets, ...
  - typesetting: book publishing, report writing, ...
- **Image processing**
  - photograph retouching: publishing, posters, ...
  - photocollaging: satellite imagery, ...
  - art: new forms of artwork based on digitised images
- **3D computer graphics**
  - visualisation: scientific, medical, architectural, ...
  - Computer Aided Design (CAD)
  - entertainment: special effect, games, movies, ...

Course Structure

- **Background [3L]**
  - images, human vision, displays
- **2D computer graphics [4L]**
  - lines, curves, clipping, polygon filling, transformations
- **3D computer graphics [6L]**
  - projection (3D -> 2D), surfaces, clipping, transformations, lighting, filling, ray tracing, texture mapping
- **Image processing [3L]**
  - filtering, compositing, half-toning, dithering, encoding, compression

Course books

- **Computer Graphics: Principles & Practice**
  - Foley, van Dam, Feiner & Hughes, Addison-Wesley, 1990
  - Older version: Fundamentals of Interactive Computer Graphics
    - Foley & van Dam, Addison-Wesley, 1982
- **Computer Graphics & Virtual Environments**
  - Slater, Steed, & Chrysanthou, Addison-Wesley, 2002
- **Digital Image Processing**
  - Gonzalez & Woods, Addison-Wesley, 1992
  - Alternatives:
    - Digital Image Processing, Gonzalez & Wintz
    - Digital Picture Processing, Rosenfeld & Kak

Past exam questions

- Dr Dodgson has been lecturing the course since 1996
  - the course changed considerably between 1996 and 1997
  - all questions from 1997 onwards are good examples of his question setting style
  - do not worry about the last 5 marks of 97/5/2
    - this is now part of Advanced Graphics syllabus
- do not attempt exam questions from 1994 or earlier
  - the course was so different back then that they are not helpful
Background

- what is a digital image?
  - what are the constraints on digital images?
- how does human vision work?
  - what are the limits of human vision?
  - what can we get away with given these constraints & limits?
- how do displays & printers work?
  - how do we fool the human eye into seeing what we want it to see?

What is a digital image?

- a contradiction in terms
  - if you can see it, it’s not digital
  - if it’s digital, it’s just a collection of numbers
- a sampled and quantised version of a real image
- a rectangular array of intensity or colour values

What is an image?

- two dimensional function
- value at any point is an intensity or colour
- not digital!

Image capture

- a variety of devices can be used
  - scanners
    - line CCD in a flatbed scanner
    - spot detector in a drum scanner
  - cameras
    - area CCD
Image capture example

Image display example

Image display

Different ways of displaying the same digital image

A real image

A digital image

The image data

Displayed on a CRT

+ a digital image is an array of integers, how do you display it?
+ reconstruct a real image on some sort of display device
  - CRT - computer monitor, TV set
  - LCD - portable computer
  - printer - dot matrix, laser printer, dye sublimation
**Sampling**

- A digital image is a rectangular array of intensity values.
- Each value is called a *pixel*.
- “picture element”
- Sampling resolution is normally measured in pixels per inch (ppi) or dots per inch (dpi).
  - Computers monitors have a resolution around 100 ppi.
  - Laser printers have resolutions between 300 and 1200 ppi.

**Quantisation**

- Each intensity value is a number.
- For digital storage, the intensity values must be quantised.
  - Limits the number of different intensities that can be stored.
  - Limits the brightest intensity that can be stored.
- How many intensity levels are needed for human consumption?
  - 8 bits usually sufficient.
  - Some applications use 10 or 12 bits.

**Sampling resolution**

<table>
<thead>
<tr>
<th>256×256</th>
<th>128×128</th>
<th>64×64</th>
<th>32×32</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="256x256.png" alt="Image" /></td>
<td><img src="128x128.png" alt="Image" /></td>
<td><img src="64x64.png" alt="Image" /></td>
<td><img src="32x32.png" alt="Image" /></td>
</tr>
<tr>
<td>2×2</td>
<td>4×4</td>
<td>8×8</td>
<td>16×16</td>
</tr>
</tbody>
</table>

**Quantisation levels**

<table>
<thead>
<tr>
<th>8 bits</th>
<th>7 bits</th>
<th>6 bits</th>
<th>5 bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>(256 levels)</td>
<td>(128 levels)</td>
<td>(64 levels)</td>
<td>(32 levels)</td>
</tr>
<tr>
<td><img src="8bits.png" alt="Image" /></td>
<td><img src="7bits.png" alt="Image" /></td>
<td><img src="6bits.png" alt="Image" /></td>
<td><img src="5bits.png" alt="Image" /></td>
</tr>
<tr>
<td>1 bit</td>
<td>2 bits</td>
<td>3 bits</td>
<td>4 bits</td>
</tr>
<tr>
<td>(2 levels)</td>
<td>(4 levels)</td>
<td>(8 levels)</td>
<td>(16 levels)</td>
</tr>
</tbody>
</table>
The workings of the human visual system

To understand the requirements of displays (resolution, quantisation and colour) we need to know how the human eye works...

The lens of the eye forms an image of the world on the retina: the back surface of the eye.

The retina consists of ~150 million light receptors. Retina outputs information to the brain along the optic nerve.

- There are ~1 million nerve fibres in the optic nerve.
- The retina performs significant pre-processing to reduce the number of signals from 150M to 1M.
- Pre-processing includes:
  - Averaging multiple inputs together
  - Colour signal processing
  - Edge detection

Some of the processing in the eye

- Discrimination
  - Discriminates between different intensities and colours

- Adaptation
  - Adapts to changes in illumination level and colour.
  - Can see about 1:100 contrast at any given time.
  - But can adapt to see light over a range of $10^{10}$.

- Persistence
  - Integrates light over a period of about 1/30 second.

- Edge detection and edge enhancement
  - Visible in e.g. Mach banding effects.

Simultaneous contrast

As well as responding to changes in overall light, the eye responds to local changes.

The centre square is the same intensity in all four cases.
**Mach bands**

- show the effect of edge enhancement in the retina’s pre-processing

Each of the nine rectangles is a constant colour

---

**Light detectors in the retina**

- two classes
  - rods
  - cones

- cones come in three types
  - sensitive to short, medium and long wavelengths

- the fovea is a densely packed region in the centre of the retina
  - contains the highest density of cones
  - provides the highest resolution vision

---

**Ghost squares**

- another effect caused by retinal pre-processing

---

**Foveal vision**

- 150,000 cones per square millimetre in the fovea
  - high resolution
  - colour

- outside fovea: mostly rods
  - lower resolution
  - principally monochromatic

- provides peripheral vision
  - allows you to keep the high resolution region in context
  - allows you to avoid being hit by passing branches

GW Fig 2.1, 2.2
Summary of what human eyes do...

- sample the image that is projected onto the retina
- adapt to changing conditions
- perform non-linear processing
  - makes it very hard to model and predict behaviour
- pass information to the visual cortex
  - which performs extremely complex processing
  - discussed in the Computer Vision course

What is required for vision?

- illumination
  - some source of light
- objects
  - which reflect (or transmit) the light
- eyes
  - to capture the light as an image

Light: wavelengths & spectra

- light is electromagnetic radiation
  - visible light is a tiny part of the electromagnetic spectrum
  - visible light ranges in wavelength from 700nm (red end of spectrum) to 400nm (violet end)
- every light has a spectrum of wavelengths that it emits
- every object has a spectrum of wavelengths that it reflects (or transmits)
- the combination of the two gives the spectrum of wavelengths that arrive at the eye

Classifying colours

- we want some way of classifying colours and, preferably, quantifying them
- we will discuss:
  - Munsell's artists' scheme
    - which classifies colours on a perceptual basis
  - the mechanism of colour vision
    - how colour perception works
  - various colour spaces
    - which quantify colour based on either physical or perceptual models of colour
Munsell’s colour classification system

- three axes
  - hue > the dominant colour
  - lightness > bright colours/dark colours
  - saturation > vivid colours/dull colours
- can represent this as a 3D graph
- any two adjacent colours are a standard “perceptual” distance apart
  - worked out by testing it on people
- but how does the eye actually see colour?

Invented by A. H. Munsell, an American artist, in 1905 in an attempt to systematically classify colours.

Colour signals sent to the brain

- the signal that is sent to the brain is pre-processed by the retina

- this theory explains:
  - colour-blindness effects
  - why red, yellow, green and blue are perceptually important
  - why you can see e.g. a yellowish red but not a greenish red

Colour vision

- three types of cone
  - each responds to a different spectrum
    - very roughly long, medium, and short wavelengths
    - each has a response function \( l(\lambda), m(\lambda), s(\lambda) \)
  - different numbers of the different types
    - far fewer of the short wavelength receptors
    - so cannot see fine detail in blue
  - overall intensity response of the eye can be calculated
    - \( y(\lambda) = l(\lambda) + m(\lambda) + s(\lambda) \)
    - \( y = k \int P(\lambda) y(\lambda) d\lambda \) is the perceived luminance

Chromatic metamerism

- many different spectra will induce the same response in our cones
  - the values of the three perceived values can be calculated as:
    - \( l = k \int P(\lambda) l(\lambda) d\lambda \)
    - \( m = k \int P(\lambda) m(\lambda) d\lambda \)
    - \( s = k \int P(\lambda) s(\lambda) d\lambda \)
  - \( k \) is some constant, \( P(\lambda) \) is the spectrum of the light incident on the retina
  - two different spectra (e.g. \( P_1(\lambda) \) and \( P_2(\lambda) \)) can give the same values of \( l, m, s \)
  - we can thus fool the eye into seeing (almost) any colour by mixing correct proportions of some small number of lights
Mixing coloured lights

- by mixing different amounts of red, green, and blue lights we can generate a wide range of responses in the human eye

\[ \text{red} \quad \text{green} \quad \text{blue} \]

\[ \text{light} \quad \text{off} \quad \text{fully on} \]

XYZ colour space

- not every wavelength can be represented as a mix of red, green, and blue
- but matching & defining coloured light with a mixture of three fixed primaries is desirable
- CIE define three standard primaries: \( X, Y, Z \)
  - \( Y \) matches the human eye’s response to light of a constant intensity at each wavelength (luminous-efficiency function of the eye)
  - \( X, Y, \) and \( Z \) are not themselves colours, they are used for defining colours – you cannot make a light that emits one of these primaries

\[ x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \quad z = \frac{Z}{X+Y+Z} \quad : \quad x + y + z = 1 \]

CIE chromaticity diagram

- chromaticity values are defined in terms of \( x, y, z \)

**RGB in XYZ space**

- CRTs and LCDs mix red, green, and blue to make all other colours
- the red, green, and blue primaries each map to a point in XYZ space
- any colour within the resulting triangle can be displayed
  - any colour outside the triangle cannot be displayed
  - for example: CRTs cannot display very saturated purples, blues, or greens
Colour spaces

- **CIE XYZ, Yxy**
- **Pragmatic**
  - used because they relate directly to the way that the hardware works
  - **RGB, CMY, CMYK**
- **Munsell-like**
  - considered by many to be easier for people to use than the pragmatic colour spaces
  - **HSV, HLS**
- **Uniform**
  - equal steps in any direction make equal perceptual differences
  - **L^*a^*b^*, L^*u^*v^***

Summary of colour spaces

- the eye has three types of colour receptor
- therefore we can validly use a three-dimensional co-ordinate system to represent colour
- **XYZ** is one such co-ordinate system
  - **Y** is the eye’s response to intensity (luminance)
  - **X** and **Z** are, therefore, the colour co-ordinates
    - same **Y**, change **X** or **Z** ⇒ same intensity, different colour
    - same **X** and **Z**, change **Y** ⇒ same colour, different intensity
- **some other systems use three colour co-ordinates**
  - luminance can then be derived as some function of the three
  - e.g. in **RGB**: \( Y = 0.299R + 0.587G + 0.114B \)

Implications of vision on resolution

- in theory you can see about 600dpi, 30cm from your eye
- in practice, opticians say that the acuity of the eye is measured as the ability to see a white gap, 1 minute wide, between two black lines
  - about 300dpi at 30cm
- resolution decreases as contrast decreases
- colour resolution is much worse than intensity resolution
  - this is exploited in TV broadcast

Implications of vision on quantisation

- humans can distinguish, at best, about a 2% change in intensity
- not so good at distinguishing colour differences

- for TV ⇒ 10 bits of intensity information
  - 8 bits is usually sufficient
    - why use only 8 bits? why is it usually acceptable?
  - for movie film ⇒ 14 bits of intensity information

- for TV the brightest white is about 25x as bright as the darkest black
- movie film has about 10x the contrast ratio of TV

©2003 Neil A. Dodgson
Storing images in memory

+ 8 bits has become a de facto standard for greyscale images
  + 8 bits = 1 byte
  + an image of size $W \times H$ can therefore be stored in a block of $W \times H$ bytes
  + one way to do this is to store $\text{pixel}[x][y]$ at memory location $\text{base} + x + W \times y$
    - memory is 1D, images are 2D

![Image diagram]

Colour images

+ tend to be 24 bits per pixel
  + 3 bytes: one red, one green, one blue
  + can be stored as a contiguous block of memory
    + of size $W \times H \times 3$
  + more common to store each colour in a separate “plane”
    + each plane contains just $W \times H$ values
  + the idea of planes can be extended to other attributes associated with each pixel
    + alpha plane (transparency), z-buffer (depth value), A-buffer (pointer to a data structure containing depth and coverage information), overlay planes (e.g. for displaying pop-up menus)

The frame buffer

+ most computers have a special piece of memory reserved for storage of the current image being displayed
  + the frame buffer normally consists of dual-ported Dynamic RAM (DRAM)
  - sometimes referred to as Video RAM (VRAM)

![Diagram of frame buffer]

Double buffering

+ if we allow the currently displayed image to be updated then we may see bits of the image being displayed halfway through the update
  - this can be visually disturbing, especially if we want the illusion of smooth animation
+ double buffering solves this problem: we draw into one frame buffer and display from the other
  - when drawing is complete we flip buffers

![Diagram of double buffering]
**Image display**

- A handful of technologies cover over 99% of all display devices
  - Active displays
    - Cathode ray tube: most common, declining use
    - Liquid crystal display: rapidly increasing use
    - Plasma displays: still rare, but increasing use
    - Special displays: e.g., LEDs for special applications
  - Printers (passive displays)
    - Laser printers
    - Ink jet printers
    - Several other technologies

**Cathode ray tubes**

- Focus an electron gun on a phosphor screen
  - Produces a bright spot
- Scan the spot back and forth, up and down to cover the whole screen
- Vary the intensity of the electron beam to change the intensity of the spot
- Repeat this fast enough and humans see a continuous picture
  - Displaying pictures sequentially at > 20Hz gives illusion of continuous motion
  - But humans are sensitive to flicker at frequencies higher than this...

**Liquid crystal display**

- Liquid crystal can twist the polarisation of light
- Control is by the voltage that is applied across the liquid crystal
  - Either on or off: transparent or opaque
- Greyscale can be achieved in some liquid crystals by varying the voltage
- Colour is achieved with colour filters
- Low power consumption but image quality not as good as cathode ray tubes

**How fast do CRTs need to be?**

- Speed at which entire screen is updated is called the “refresh rate”
- 50Hz (PAL TV, used in most of Europe)
  - Many people can see a slight flicker
- 60Hz (NTSC TV, used in USA and Japan)
  - Better
- 80-90Hz
  - 99% of viewers see no flicker, even on very bright displays
- 100Hz (newer “flicker-free” PAL TV sets)
  - Practically no-one can see the image flickering
Colour CRTs: shadow masks
- use three electron guns & colour phosphors
- electrons have no colour
  - use shadow mask to direct electrons from each gun onto the appropriate phosphor
- the electron beams’ spots are bigger than the shadow mask pitch
  - can get spot size down to 7/4 of the pitch
  - pitch can get down to 0.25mm with delta arrangement of phosphor dots
  - with a flat tension shadow mask can reduce this to 0.15mm

Printer resolution
- laser printer
  - up to 1200dpi, generally 600dpi
- ink jet
  - used to be lower resolution & quality than laser printers but now have comparable resolution
- phototypesetter
  - up to about 3000dpi
- bi-level devices: each pixel is either black or white

Printers
- many types of printer
  - ink jet
    - sprays ink onto paper
  - dot matrix
    - pushes pins against an ink ribbon and onto the paper
  - laser printer
    - uses a laser to lay down a pattern of charge on a drum; this picks up charged toner which is then pressed onto the paper
- all make marks on paper
  - essentially binary devices: mark/no mark

What about greyscale?
- achieved by halftoning
  - divide image into cells, in each cell draw a spot of the appropriate size for the intensity of that cell
  - on a printer each cell is $m\times m$ pixels, allowing $m^2+1$ different intensity levels
  - e.g. 300dpi with $4\times 4$ cells ⇒ 75 cells per inch, 17 intensity levels
  - phototypesetters can make 256 intensity levels in cells so small you can only just see them
- an alternative method is dithering
  - dithering photocopies badly, halftoning photocopies well

will discuss halftoning and dithering in Image Processing section of course
Dye sublimation printers: true greyscale

- dye sublimation gives true greyscale
- dye sublimes off dye sheet and onto paper in proportion to the heat level
- colour is achieved by using four different coloured dye sheets in sequence — the heat mixes them

What about colour?

- generally use cyan, magenta, yellow, and black inks (CMYK)
- inks absorb colour
  - c.f. lights which emit colour
  - CMY is the inverse of RGB
- why is black (K) necessary?
  - inks are not perfect absorbers
  - mixing C + M + Y gives a muddy grey, not black
  - lots of text is printed in black: trying to align C, M and Y perfectly for black text would be a nightmare

How do you produce halftoned colour?

- print four halftone screens, one in each colour
- carefully angle the screens to prevent interference (moiré) patterns

- Standard angles
  - Magenta 45°
  - Cyan 15°
  - Yellow 90°
  - Black 75°

- Standard rulings (in lines per inch)
  - 65 lpi
  - 85 lpi newsprint
  - 100 lpi
  - 120 lpi uncoated offset paper
  - 133 lpi uncoated offset paper
  - 150 lpi matt coated offset paper or art paper
  - 200 lpi very smooth, expensive paper

2D Computer Graphics

- lines
  - how do I draw a straight line?
- curves
  - how do I specify curved lines?
- clipping
  - what about lines that go off the edge of the screen?
- filled areas
- transformations
  - scaling, rotation, translation, shearing
- applications
Drawing a straight line

- A straight line can be defined by:
  \[ y = mx + c \]
  
- A mathematical line is “length without breadth”
- A computer graphics line is a set of pixels
- Which pixels do we need to turn on to draw a given line?

Which pixels do we use?

- There are two reasonably sensible alternatives:
  - Every pixel through which the line passes
    (can have either one or two pixels in each column)
  - The “closest” pixel to the line in each column
    (always have just one pixel in every column)

A line drawing algorithm - preparation 1

- Pixel \((x, y)\) has its centre at real co-ordinate \((x, y)\)
- It thus stretches from \((x - \frac{1}{2}, y - \frac{1}{2})\) to \((x + \frac{1}{2}, y + \frac{1}{2})\)

A line drawing algorithm - preparation 2

- The line goes from \((x_0, y_0)\) to \((x_1, y_1)\)
- The line lies in the first octant \((0 \leq m \leq 1)\)
- \(x_0 < x_1\)
Bresenham’s line drawing algorithm 1

Initialisation

\[ d = \frac{(y_1 - y_0)}{(x_1 - x_0)} \]
\[ x = x_0 \]
\[ y_1 = y_0 \]
\[ y = y_0 \]
\[ \text{DRAW}(x,y) \]

Iteration

\[ \text{WHILE } x < x_1 \text{ DO} \]
\[ x = x + 1 \]
\[ y_1 = y_1 + d \]
\[ y = \text{ROUND}(y) \]
\[ \text{DRAW}(x,y) \]
\[ \text{END WHILE} \]


Bresenham’s line drawing algorithm 2

naive algorithm involves floating point arithmetic & rounding inside the loop ⇒ slow

Speed up A:
- separate integer and fractional parts of \( y_i \) (into \( y \) and \( y_f \))
- replace rounding by an IF
  - removes need to do rounding

\[ d = \frac{(y_1 - y_0)}{(x_1 - x_0)} \]
\[ x = x_0 \]
\[ y_f = 0 \]
\[ y = y_0 \]
\[ \text{DRAW}(x,y) \]
\[ \text{WHILE } x < x_1 \text{ DO} \]
\[ x = x + 1 \]
\[ y_f = y_f + d \]
\[ \text{IF } (y_f > \frac{1}{2}) \text{ THEN} \]
\[ y = y + 1 \]
\[ y_f = y_f - 1 \]
\[ \text{END IF} \]
\[ \text{DRAW}(x,y) \]
\[ \text{END WHILE} \]

Bresenham’s line drawing algorithm 3

Speed up B:
- multiply all operations involving \( y_f \) by \( 2(x_1 - x_0) \)
  - \( y_f = y_f + Dy/dx \) → \( y_f = y_f + 2dy \)
  - \( y_f > \frac{1}{2} \) → \( y_f > dx \)
  - \( y_f = y_f - 1 \) → \( y_f = y_f - 2dx \)
- removes need to do floating point arithmetic if end-points have integer co-ordinates

\[ d = \frac{(y_1 - y_0)}{(x_1 - x_0)} \]
\[ dx = (x_1 - x_0) \]
\[ x = x_0 \]
\[ y_f = 0 \]
\[ y = y_0 \]
\[ \text{DRAW}(x,y) \]
\[ \text{WHILE } x < x_1 \text{ DO} \]
\[ x = x + 1 \]
\[ y_f = y_f + d \]
\[ \text{IF } (y_f > dx) \text{ THEN} \]
\[ y = y + 1 \]
\[ y_f = y_f - 2dx \]
\[ \text{END IF} \]
\[ \text{DRAW}(x,y) \]
\[ \text{END WHILE} \]

Bresenham’s algorithm for floating point end points

\[ d = \frac{(y_1 - y_0)}{(x_1 - x_0)} \]
\[ x = \text{ROUND}(x_0) \]
\[ y_f = y_0 + d \ству (x_0,x_0) \]
\[ y = \text{ROUND}(y) \]
\[ y_f = y_f + d \]
\[ \text{WHILE } x < (x_1 + \frac{1}{2}) \text{ DO} \]
\[ x = x + 1 \]
\[ y_f = y_f + d \]
\[ \text{IF } (y_f > \frac{1}{2}) \text{ THEN} \]
\[ y = y + 1 \]
\[ y_f = y_f - 1 \]
\[ \text{END IF} \]
\[ \text{DRAW}(x,y) \]
\[ \text{END WHILE} \]
Bresenham’s algorithm — more details
+ we assumed that the line is in the first octant
  • can do fifth octant by swapping end points
  • therefore need four versions of the algorithm

Exercise: work out what changes need to be made to the algorithm for it to work in each of the other three octants

A second line drawing algorithm
+ a line can be specified using an equation of the form:
  \[ k = ax + by + c \]
+ this divides the plane into three regions:
  • above the line \( k < 0 \)
  • below the line \( k > 0 \)
  • on the line \( k = 0 \)

Midpoint line drawing algorithm 1
+ given that a particular pixel is on the line, the next pixel must be either immediately to the right (E) or to the right and up one (NE)
+ use a decision variable (based on \( k \)) to determine which way to go

Midpoint line drawing algorithm 2
+ decision variable needs to make a decision at point \((x+1, y+\frac{1}{2})\)
  \[ d = a(x+1) + b(y + \frac{1}{2}) + c \]
+ if go E then the new decision variable is at \((x+2, y+\frac{1}{2})\)
  \[ d' = a(x+2) + b(y + \frac{1}{2}) + c = d + a \]
+ if go NE then the new decision variable is at \((x+2, y+1\frac{1}{2})\)
  \[ d' = a(x+2) + b(y + 1\frac{1}{2}) + c = d + a + b \]
Midpoint line drawing algorithm 3

**Initialisation**

\[
\begin{align*}
a &= (y_1 - y_0) \\
b &= -(x_1 - x_0) \\
c &= x_1y_0 - x_0y_1 \\
x &= \text{ROUND}(x_0) \\
y &= \text{ROUND}(y_0 - (x - x_0)(a/b)) \\
d &= a(x+1) + b(y+1) + c \\
\text{DRAW}(x,y)
\end{align*}
\]

**Iteration**

WHILE \( x < (x_1 - 1/2) \) DO

IF \( d < 0 \) THEN

\[ d = d + a \]

ELSE

\[ d = d + a + b \]

\[ y = y + 1 \]

END IF

\[ x = x + 1 \]

\[ \text{DRAW}(x,y) \]

END WHILE

If end-points have integer co-ordinates then all operations can be in integer arithmetic.

Midpoint - comments

- this version only works for lines in the first octant
- extend to other octants as for Bresenham
- Sproull has proven that Bresenham and Midpoint give identical results
- Midpoint algorithm can be generalised to draw arbitrary circles & ellipses
  - Bresenham can only be generalised to draw circles with integer radii

Midpoint circle algorithm 1

- equation of a circle is \( x^2 + y^2 = r^2 \)
  - centred at the origin

- decision variable can be \( d = x^2 + y^2 - r^2 \)
  - \( d < 0 \) on the circle, \( d > 0 \) outside, \( d < 0 \) inside

- divide circle into eight octants
  - on the next slide we consider only the second octant, the others are similar

Curves

- circles & ellipses
- Bezier cubics
  - Pierre Bézier, worked in CAD for Renault
  - widely used in Graphic Design
- Overhauser cubics
  - Overhauser, worked in CAD for Ford
- NURBS
  - Non-Uniform Rational B-Splines
  - more powerful than Bezier & now more widely used
  - consider these in Part II

Midpoint circle algorithm 1

- equation of a circle is \( x^2 + y^2 = r^2 \)
  - centred at the origin

- decision variable can be \( d = x^2 + y^2 - r^2 \)
  - \( d < 0 \) on the circle, \( d > 0 \) outside, \( d < 0 \) inside

- divide circle into eight octants
  - on the next slide we consider only the second octant, the others are similar
**Midpoint circle algorithm 2**

- Decision variable needs to make a decision at point \((x+1, y-\frac{1}{2})\)
  
  \[ d = (x+1)^2 + (y - \frac{1}{2})^2 - r^2 \]

- If go E then the new decision variable is at \((x+2, y-\frac{1}{2})\)
  
  \[ d' = (x+2)^2 + (y - \frac{1}{2})^2 - r^2 \]
  
  \[ = d + 2x + 3 \]

- If go SE then the new decision variable is at \((x+2, y-1\frac{1}{2})\)
  
  \[ d'' = (x+2)^2 + (y - 1\frac{1}{2})^2 - r^2 \]
  
  \[ = d + 2x - 2y + 5 \]

**Exercise:** Complete the circle algorithm for the second octant.

---

**Are circles & ellipses enough?**

- Simple drawing packages use ellipses & segments of ellipses
- For graphic design & CAD need something with more flexibility
  - Use cubic polynomials

---

**Why cubics?**

- Lower orders cannot:
  - Have a point of inflection
  - Match both position and slope at both ends of a segment
  - Be non-planar in 3D
- Higher orders:
  - Can wiggle too much
  - Take longer to compute

**Taking circles further**

- The algorithm can be easily extended to circles not centred at the origin
- A similar method can be derived for ovals
  - But: cannot naively use octants
    - Use points of 45° slope to divide oval into eight sections
  - And: ovals must be axis-aligned
    - There is a more complex algorithm which can be used for non-axis aligned ovals

---

©2003 Neil A. Dodgson
**Hermite cubic**

- the Hermite form of the cubic is defined by its two end-points and by the tangent vectors at these end-points:

\[
P(t) = (2t^3 - 3t^2 + 1)P_0^t + (-2t^3 + 3t^2)P_1^t + (t^3 - 2t^2 + t)T_0^t + (t^3 - t^2)T_1^t
\]

- two Hermite cubics can be smoothly joined by matching both position and tangent at an end point of each cubic

Charles Hermite, mathematician, 1822–1901

**Bezier cubic**

- difficult to think in terms of tangent vectors
- Bezier defined by two end points and two other control points

\[
P(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1^P + 3t^2 (1 - t) P_2 + t^3 P_3
\]

where: \(P_i = (x_i, y_i)\)

Pierre Bézier worked for Citroën in the 1960s

**Bezier properties**

- Bezier is equivalent to Hermite

\[
T_0 = 3(P_1 - P_0) \quad T_1 = 3(P_3 - P_2)
\]

- Weighting functions are Bernstein polynomials

\[
b_i(t) = (1-t)^3 \quad b_i(t) = 3t(1-t)^2 \quad b_i(t) = 3t^2(1-t) \quad b_i(t) = t^3
\]

- Weighting functions sum to one

\[
\sum_{i=0}^{3} b_i(t) = 1
\]

- Bezier curve lies within convex hull of its control points

**Types of curve join**

- each curve is smooth within itself

joins at endpoints can be:

- \(C_1\) – continuous in both position and tangent vector
  - smooth join
- \(C_0\) – continuous in position
  - “corner”
- discontinuous in position

\(C_n\) = continuous in all derivatives up to the \(n\)th derivative
Computer Graphics & Image Processing

Drawing a Bezier cubic – naïve method
- draw as a set of short line segments equispaced in parameter space, \( t \)

\[
(x_0, y_0) = \text{Bezier}(0) \\
\text{FOR } t = 0.05 \text{ TO } 1 \text{ STEP } 0.05 \text{ DO} \\
(x_1, y_1) = \text{Bezier}(t) \\
\text{DrawLine( } (x_0,y_0), (x_1,y_1) ) \\
(x_0, y_0) = (x_1,y_1) \\
\text{END FOR}
\]

- problems:
  - cannot fix a number of segments that is appropriate for all possible Beziers: too many or too few segments
  - distance in real space, \((x,y)\), is not linearly related to distance in parameter space, \( t \)

Drawing a Bezier cubic (continued)
- adaptive subdivision
  - check if a straight line between \( P_1 \) and \( P_3 \) is an adequate approximation to the Bezier
  - if so: draw the straight line
  - if not: divide the Bezier into two halves, each a Bezier, and repeat for the two new Beziers
  - need to specify some tolerance for when a straight line is an adequate approximation
    - when the Bezier lies within half a pixel width of the straight line along its entire length

Subdividing a Bezier cubic into two halves
- a Bezier cubic can be easily subdivided into two smaller Bezier cubics

\[
\begin{align*}
Q_0 &= P_0 \\
Q_1 &= \frac{1}{4} P_0 + \frac{1}{2} P_1 + \frac{1}{4} P_2 \\
Q_2 &= \frac{1}{4} P_2 + \frac{1}{2} P_3 + \frac{1}{4} P_1 \\
Q_3 &= \frac{1}{4} P_3 + \frac{1}{2} P_1 + \frac{1}{4} P_2
\end{align*}
\]

\[
\begin{align*}
R_0 &= \frac{1}{4} P_0 + \frac{1}{2} P_1 + \frac{1}{4} P_2 + \frac{1}{4} P_3 \\
R_1 &= \frac{1}{4} P_1 + \frac{1}{2} P_2 + \frac{1}{4} P_3 \\
R_2 &= \frac{1}{4} P_3 + \frac{1}{2} P_2 + \frac{1}{4} P_1 \\
R_3 &= P_3
\end{align*}
\]

Exercise: prove that the Bezier cubic curves defined by \( Q_0, Q_1, Q_2, Q_3 \) and \( R_0, R_1, R_2, R_3 \) match the Bezier cubic curve defined by \( P_0, P_1, P_2, P_3 \) over the ranges \( t \in [0, \frac{1}{2}] \) and \( t \in [\frac{1}{2}, 1] \) respectively.
What if we have no tangent vectors?
- base each cubic piece on the four surrounding data points
- at each data point the curve must depend solely on the three surrounding data points
  - define the tangent at each point as the direction from the preceding point to the succeeding point
    - tangent at \( P_1 \) is \( \frac{1}{4}(P_2 - P_0) \), at \( P_2 \) is \( \frac{1}{4}(P_3 - P_1) \)
- this is the basis of Overhauser's cubic

Overhauser's cubic
- method
  - calculate the appropriate Bezier or Hermite values from the given points
  - e.g. given points \( A, B, C, D \), the Bezier control points are:
    - \( P_0 = B \)
    - \( P_1 = B + (C - A)/6 \)
    - \( P_2 = C \)
    - \( P_3 = C - (D - B)/6 \)
- (potential) problem
  - moving a single point modifies the surrounding four curve segments (c.f. Bezier where moving a single point modifies just the two segments connected to that point)
  - good for control of movement in animation

Simplifying line chains
- the problem: you are given a chain of line segments at a very high resolution, how can you reduce the number of line segments without compromising the quality of the line
  - e.g. given the coastline of Britain defined as a chain of line segments at 10m resolution, draw the entire outline on a 1280×1024 pixel screen
- the solution: Douglas & Pücker's line chain simplification algorithm

This can also be applied to chains of Bezier curves at high resolution: most of the curves will each be approximated (by the previous algorithm) as a single line segment. Douglas & Pücker's algorithm can then be used to further simplify the line chain

Douglas & Pücker's algorithm
- find point, \( C \), at greatest distance from line \( AB \)
- if distance from \( C \) to \( AB \) is more than some specified tolerance then subdivide into \( AC \) and \( CB \), repeat for each of the two subdivisions
- otherwise approximate entire chain from \( A \) to \( B \) by the single line segment \( AB \)

Exercises: (1) How do you calculate the distance from \( C \) to \( AB \)?
(2) What special cases need to be considered? How should they be handled?

Overhauser worked for the Ford motor company in the 1960s.

Douglas & Pücker, Canadian Cartographer, 10(2), 1973
Clipping

- What about lines that go off the edge of the screen?

- Need to clip them so that we only draw the part of the line that is actually on the screen.

Clipping against a rectangle

- Need to check four inequalities:

\[
\begin{align*}
&x \geq x_L \\
&x \leq x_R \\
y \geq y_L \\
y \leq y_T
\end{align*}
\]

Cohen-Sutherland clipper 1

- Make a four bit code, one bit for each inequality:

\[
A \equiv x < x_L, \quad B \equiv x > x_R, \quad C \equiv y < y_L, \quad D \equiv y > y_T
\]

- Evaluate this for both endpoints of the line.

\[
Q_1 = A\bar{B}\bar{C}D_1, \quad Q_2 = A\bar{B}C\bar{D}_2
\]

Cohen-Sutherland clipper 2

- If \(Q_1 = Q_2 = 0\), both ends in rectangle ACCEPT

- If \(Q_1 \land Q_2 \neq 0\), both ends outside and in same half-plane REJECT

- Otherwise

- Need to intersect line with one of the edges and start again.

The 1 bits tell you which edge to clip against.

Example

\[
\begin{align*}
x_L' &= x_L \\
y_L' &= y_L + \frac{(y_L - y_T)(x_L - x_L)}{x_R - x_L} \\
x_R' &= x_R \\
y_R' &= y_R + \frac{(y_T - y_R)(x_R - x_L)}{x_R - x_L}
\end{align*}
\]
Cohen-Sutherland clipper 3

- if code has more than a single 1 then you cannot tell which is the best: simply select one and loop again
- horizontal and vertical lines are not a problem
- need a line drawing algorithm that can cope with floating-point endpoint co-ordinates

Exercise: what happens in each of the cases at left? [Assume that, where there is a choice, the algorithm always tries to intersect with \( x_L \) or \( x_R \) before \( y_B \) or \( y_T \).]

Try some other cases of your own devising.

Polygon filling

- which pixels do we turn on?
  - those whose centres lie inside the polygon
    - this is a naïve assumption, but is sufficient for now

Scanline polygon fill algorithm

1. take all polygon edges and place in an edge list (EL), sorted on lowest \( y \) value
2. start with the first scanline that intersects the polygon, get all edges which intersect that scan line and move them to an active edge list (AEL)
3. for each edge in the AEL: find the intersection point with the current scanline; sort these into ascending order on the \( x \) value
4. fill between pairs of intersection points
5. move to the next scanline (increment \( y \)); remove edges from the AEL if endpoint < \( y \); move new edges from EL to AEL if start point \( \leq y \); if any edges remain in the AEL go back to step 1

Scanline polygon fill example
Scanline polygon fill details
- how do we efficiently calculate the intersection points?
  - use a line drawing algorithm to do incremental calculation
- what if endpoints exactly intersect scanlines?
  - need to cope with this, e.g. add a tiny amount to the y coordinate to ensure that they don't exactly match
- what about horizontal edges?
  - throw them out of the edge list, they contribute nothing

Sutherland-Hodgman polygon clipping 1
- clips an arbitrary polygon against an arbitrary convex polygon
  - basic algorithm clips an arbitrary polygon against a single infinite clip edge
  - the polygon is clipped against one edge at a time, passing the result on to the next stage

Sutherland & Hodgman, “Reentrant Polygon Clipping,” Comm. ACM, 17(1), 1974

Clipping polygons

Sutherland-Hodgman polygon clipping 2
- the algorithm progresses around the polygon checking if each edge crosses the clipping line and outputting the appropriate points

Exercise: the Sutherland-Hodgman algorithm may introduce new edges along the edge of the clipping polygon — when does this happen and why?
2D transformations

+ scale
  - about origin
  - by factor $m$

+ rotate
  - about origin
  - by angle $\theta$

+ translate
  - along vector $(x_0, y_0)$

+ (shear)
  - parallel to $x$ axis
  - by factor $a$

why?

• it is extremely useful to be able to transform predefined objects to an arbitrary location, orientation, and size
• any reasonable graphics package will include transforms
  - 2D $\rightarrow$ Postscript
  - 3D $\rightarrow$ OpenGL

Basic 2D transformations

- scale
  - about origin
  - by factor $m$

- rotate
  - about origin
  - by angle $\theta$

- translate
  - along vector $(x_0, y_0)$

- shear
  - parallel to $x$ axis
  - by factor $a$

Matrix representation of transformations

+ scale
  - about origin, factor $m$

+ rotate
  - about origin, angle $\theta$

+ do nothing
  - identity

+ shear
  - parallel to $x$ axis, factor $a$

Homogeneous 2D co-ordinates

- translations cannot be represented using simple $2D$ matrix multiplication on $2D$ vectors, so we switch to homogeneous co-ordinates

- an infinite number of homogeneous co-ordinates map to every $2D$ point

- $w=0$ represents a point at infinity

- usually take the inverse transform to be:

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} m \\ 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$
Concatenating transformations

- Often necessary to perform more than one transformation on the same object.
- Can concatenate transformations by multiplying their matrices.
- E.g. a shear followed by a scaling:

\[
\begin{pmatrix}
    x' \\
    y' \\
    w'
\end{pmatrix} = \begin{pmatrix}
    \cos \theta & -\sin \theta & 0 \\
    \sin \theta & \cos \theta & 0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    x \\
    y \\
    w
\end{pmatrix}
\]

In homogeneous coordinates:

\[
\begin{pmatrix}
    x' \\
    y' \\
    w'
\end{pmatrix} = \begin{pmatrix}
    1 & 0 & x_0 \\
    0 & 1 & y_0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    x \\
    y \\
    w
\end{pmatrix}
\]

In conventional coordinates:

\[
\begin{align*}
x' &= x + w x_0 \\
y' &= y + w y_0 \\
w' &= w
\end{align*}
\]

Translation by matrix algebra

\[
\begin{pmatrix}
    x' \\
    y' \\
    w'
\end{pmatrix} = \begin{pmatrix}
    1 & 0 & x_0 \\
    0 & 1 & y_0 \\
    0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
    x \\
    y \\
    w
\end{pmatrix}
\]

Concatenation is not commutative

- Be careful of the order in which you concatenate transformations.
Scaling about an arbitrary point

1. scale by a factor \( m \) about point \((x_0, y_0)\)
2. translate point \((x, y)\) to the origin
3. scale by a factor \( m \) about the origin
4. translate the origin to \((x_0, y_0)\)

\[
\begin{bmatrix}
x'' \\
y'' \\
w''
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & -x_0 \\
0 & 1 & -y_0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x' \\
y' \\
w'
\end{bmatrix} =
\begin{bmatrix}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x'' \\
y'' \\
w''
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & x_0 \\
0 & 1 & y_0 \\
0 & 0 & 1
\end{bmatrix}
\]

Exercise: show how to perform rotation about an arbitrary point

Clipping with bounding boxes

1. do a quick accept/reject/unsure test to the bounding box then apply clipping to only the unsure objects

\[
BB_l > x_l \lor BB_x < x_L \lor BB_y > y_l \lor BB_t < y_0 \Rightarrow REJECT
\]
\[
BB_r \geq x_L \land BB_x \leq x_x \land BB_y \geq y_L \land BB_t \leq y_7 \Rightarrow ACCEPT
\]
on otherwise ⇒ clip at next higher level of detail

Bounding boxes

1. when working with complex objects, bounding boxes can be used to speed up some operations

Object inclusion with bounding boxes

1. including one object (e.g. a graphics) file inside another can be easily done if bounding boxes are known and used

use the eight values to translate and scale the original to the appropriate position in the destination document
**Bit block transfer (BitBlT)**

- It is sometimes preferable to predraw something and then copy the image to the correct position on the screen as and when required.
  - E.g. icons
  - E.g. games
- Copying an image from place to place is essentially a memory operation.
  - Can be made very fast.
  - E.g. 32×32 pixel icon can be copied, say, 8 adjacent pixels at a time, if there is an appropriate memory copy operation.

**XOR drawing**

- Generally we draw objects in the appropriate colours, overwriting what was already there.
- Sometimes, usually in HCI, we want to draw something temporarily, with the intention of wiping it out (almost) immediately e.g. when drawing a rubber-band line.
- If we bitwise XOR the object’s colour with the colour already in the frame buffer we will draw an object of the correct shape (but wrong colour).
- If we do this twice we will restore the original frame buffer.
- Saves drawing the whole screen twice.

**Application 1: user interface**

- Tend to use objects that are quick to draw.
  - Straight lines
  - Filled rectangles
- Complicated bits done using predrawn icons.
- Typefaces also tend to be predrawn.

**Application 2: typography**

- Typeface: a family of letters designed to look good together.
  - Usually has upright (roman/regular), italic (oblique), bold and bold-italic members.
- Two forms of typeface used in computer graphics.
  - Pre-rendered bitmaps:
    - Single resolution (don’t scale well).
    - Use BitBlT to put into frame buffer.
  - Outline definitions:
    - Multi-resolution (can scale).
    - Need to render (fill) to put into frame buffer.
### Application 3: Postscript

* industry standard rendering language for printers
* developed by Adobe Systems
* stack-based interpreted language

**Basic features**
- object outlines made up of lines, arcs & Bezier curves
- objects can be filled or stroked
- whole range of 2D transformations can be applied to objects
- typeface handling built in
- halftoning
- can define your own functions in the language

### 3D ⇒ 2D projection

* to make a picture
  - 3D world is projected to a 2D image
    - like a camera taking a photograph
    - the three dimensional world is projected onto a plane

The 3D world is described as a set of (mathematical) objects

- e.g. sphere radius (3.4) centre (0.2,9.2)
- e.g. box size (2.4,3) centre (7.2,9) orientation (27°, 156°)

### 3D Computer Graphics

* 3D ⇒ 2D projection
* 3D versions of 2D operations
  - clipping, transforms, matrices, curves & surfaces
* 3D scan conversion
  - depth-sort, BSP tree, z-Buffer, A-buffer
* sampling
* lighting
* ray tracing

### Types of projection

* parallel
  - e.g. \((x, y, z) \rightarrow (x, y)\)
  - useful in CAD, architecture, etc
  - looks unrealistic
* perspective
  - e.g. \((x, y, z) \rightarrow (\frac{x}{z}, \frac{y}{z})\)
  - things get smaller as they get farther away
  - looks realistic
  - this is how cameras work!
Viewing volume

- the rectangular pyramid is the viewing volume
- everything within the viewing volume is projected onto the viewing plane

Geometry of perspective projection

\[ \begin{align*}
  x' &= x \frac{d}{z} \\
  y' &= y \frac{d}{z}
\end{align*} \]

Perspective projection with an arbitrary camera

- we have assumed that:
  - screen centre at \((0,0,d)\)
  - screen parallel to \(xy\)-plane
  - \(z\)-axis into screen
  - \(y\)-axis up and \(x\)-axis to the right
  - eye (camera) at origin \((0,0,0)\)

- for an arbitrary camera we can either:
  - work out equations for projecting objects about an arbitrary point onto an arbitrary plane
  - transform all objects into our standard coordinate system (viewing coordinates) and use the above assumptions

3D transformations

- 3D homogeneous co-ordinates
  \((x, y, z, w) \rightarrow (\frac{x}{w}, \frac{y}{w}, \frac{z}{w})\)
- 3D transformation matrices

<table>
<thead>
<tr>
<th>Translation</th>
<th>Identity</th>
<th>Rotation about x-axis</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 \\
\end{bmatrix}
\] |

<table>
<thead>
<tr>
<th>Scale</th>
<th>Rotation about y-axis</th>
<th>Rotation about z-axis</th>
</tr>
</thead>
</table>
| \[
\begin{bmatrix}
  s_x & 0 & 0 & 0 \\
  0 & s_y & 0 & 0 \\
  0 & 0 & s_z & 0 \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
  \cos \theta & -\sin \theta & 0 & 0 \\
  \sin \theta & \cos \theta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\] | \[
\begin{bmatrix}
  \cos \phi & 0 & \sin \phi & 0 \\
  0 & 1 & 0 & 0 \\
 -\sin \phi & 0 & \cos \phi & 0 \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}
\] |
3D transformations are not commutative

- 90° rotation about z-axis
- 90° rotation about x-axis
- opposite faces

Viewing transform 2

- translate eye point, \((e_x,e_y,e_z)\), to origin, \((0,0,0)\)

\[
T = \begin{bmatrix}
1 & 0 & 0 & -e_z \\
0 & 1 & 0 & -e_x \\
0 & 0 & 1 & -e_y \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

- scale so that eye point to look point distance, \(\|\vec{e}l\|\), is distance from origin to screen centre, \(d\)

\[
\|\vec{e}l\| = \sqrt{(l_x - e_x)^2 + (l_y - e_y)^2 + (l_z - e_z)^2}
\]

\[
S = \begin{bmatrix}
\frac{d}{\|\vec{e}l\|} & 0 & 0 & 0 \\
0 & \frac{d}{\|\vec{e}l\|} & 0 & 0 \\
0 & 0 & \frac{d}{\|\vec{e}l\|} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Viewing transform 3

- need to align line \(\vec{e}l\) with \(z\)-axis

1. first transform \(e\) and \(l\) into new coordinate system \(e'' = S \times T \times e = 0\)
   \(l'' = S \times T \times l = 0\)
2. then rotate \(e''l''\) into \(yz\)-plane, rotating about \(y\)-axis

\[
R_y = \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

\[
\theta = \arccos \frac{\vec{e''} \cdot \vec{l''}}{\|\vec{e''}\| \cdot \|\vec{l''}\|}
\]

\[
\begin{aligned}
& (0, l''_y, 0, l''_z) \\
& (e'', l'')
\end{aligned}
\]
Viewing transform 4

- Having rotated the viewing vector onto the $yz$ plane, rotate it about the $x$-axis so that it aligns with the $z$-axis.

$$I'' = R_x \times I'$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \phi & \sin \phi & 0 \\ 0 & -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \arccos \frac{m''_x}{\sqrt{m''_x^2 + m''_y^2}}$$

Viewing transform 5

- The final step is to ensure that the up vector actually points up, i.e. along the positive $y$-axis.
- Actually need to rotate the up vector about the $z$-axis so that it lies in the positive $y$ half of the $yz$ plane.

$$u'''' = R_z \times R_y \times u$$

$$R_z = \begin{bmatrix} \cos \psi & \sin \psi & 0 & 0 \\ -\sin \psi & \cos \psi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\psi = \arccos \frac{u''''_z}{\sqrt{u''''_x^2 + u''''_y^2}}$$

Another transformation example

- A well known graphics package (Open Inventor) defines a cylinder to be:
  - Centre at the origin, $(0,0,0)$
  - Radius 1 unit
  - Height 2 units, aligned along the $y$-axis
- This is the only cylinder that can be drawn, but the package has a complete set of 3D transformations.

- We want to draw a cylinder of:
  - Radius 2 units
  - The centres of its two ends located at $(1,2,3)$ and $(2,4,5)$
  - Its length is thus 3 units.

- What transforms are required?
  - And in what order should they be applied?
A variety of transformations

- The modelling transform and viewing transform can be multiplied together to produce a single matrix taking an object directly from object co-ordinates into viewing co-ordinates.
- Either or both of the modelling transform and viewing transform matrices can be the identity matrix.
- E.g., objects can be specified directly in viewing co-ordinates, or directly in world co-ordinates.
- This is a useful set of transforms, not a hard and fast model of how things should be done.

What about clipping in z?

- Need to at least check for \( z < 0 \) to stop things behind the camera from projecting onto the screen.
- Can also have front and back clipping planes: \( z > z_f \) and \( z < z_b \).
- Resulting clipping volume is called the viewing frustum.

Clipping in 3D

- Clipping against a volume in viewing co-ordinates
  - A point \((x, y, z)\) can be clipped against the pyramid by checking it against four planes:
    - \( x > -\frac{a}{d} \) \( x < \frac{a}{d} \)
    - \( y > -\frac{b}{d} \) \( y < \frac{b}{d} \)
    - \( z > -\frac{d}{a} \) \( z < \frac{d}{a} \)

Clipping in 3D — two methods

- Clipping against the viewing frustum
  - Need to clip against six planes:
    - \( x = -\frac{a}{d} \) \( x = \frac{a}{d} \)
    - \( y = -\frac{b}{d} \) \( y = \frac{b}{d} \)
    - \( z = z_f \) \( z = z_b \)

- Project to 2D (retaining z) and clip against the axis-aligned cuboid
  - Still need to clip against six planes:
    - \( x = -a \) \( x = a \)
    - \( y = -b \) \( y = b \)
    - \( z = z_f \) \( z = z_b \)
  - These are simpler planes against which to clip.
  - This is equivalent to clipping in 2D with two extra clips for z.
Bounding volumes & clipping
- can be very useful for reducing the amount of work involved in clipping
- what kind of bounding volume?
  - axis aligned box
  - sphere
- can have multiple levels of bounding volume

Curves in 3D
- same as curves in 2D, with an extra co-ordinate for each point
- e.g. Bezier cubic in 3D:
  \[ P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3 \]
  where: \( P_i = (x_i, y_i, z_i) \)

Surfaces in 3D: polygons
- lines generalise to planar polygons
  - 3 vertices (triangle) must be planar
  - > 3 vertices, not necessarily planar

Splitting polygons into triangles
- some graphics processors accept only triangles
- an arbitrary polygon with more than three vertices isn’t guaranteed to be planar; a triangle is
  - which is preferable?
**Surfaces in 3D: patches**

- curves generalise to patches
  - a Bezier patch has a Bezier curve running along each of its four edges and four extra internal control points

**Bezier patch definition**

- the Bezier patch defined by the sixteen control points, $P_{0,0}, P_{0,1}, \ldots, P_{3,3}$, is:
  \[
P(s, t) = \sum_{i=0}^{3} \sum_{j=0}^{3} b_i(s) b_j(t) P_{ij}
\]
  where: $b_i(t) = (1-t)^i$, $b_i(t) = 3t(1-t)^i$, $b_i(t) = 3t^i(1-t)$, $b_i(t) = t^3$

- compare this with the 2D version:
  \[
P(t) = \sum_{i=0}^{3} b_i(t) P_i
\]

**Continuity between Bezier patches**

- each patch is smooth within itself
- ensuring continuity in 3D:
  - $C_0$ – continuous in position
    - the four edge control points must match
  - $C_1$ – continuous in both position and tangent vector
    - the four edge control points must match
    - the two control points on either side of each of the four edge control points must be co-linear with both the edge point and each another and be equidistant from the edge point

**Drawing Bezier patches**

- in a similar fashion to Bezier curves, Bezier patches can be drawn by approximating them with planar polygons
- method:
  - check if the Bezier patch is sufficiently well approximated by a quadrilateral, if so use that quadrilateral
  - if not then subdivide it into two smaller Bezier patches and repeat on each
    - subdivide in different dimensions on alternate calls to the subdivision function
  - having approximated the whole Bezier patch as a set of (non-planar) quadrilaterals, further subdivide these into (planar) triangles
    - be careful to not leave any gaps in the resulting surface!
**Subdividing a Bezier patch - example**

1. Initial Bezier patch
2. Subdividing the patch
3. Further subdivisions
4. Final subdivided patch

**Triangulating the subdivided patch**

- Final quadrilateral mesh
- Naive triangulation
- More intelligent triangulation

**3D scan conversion**

- Lines
- Polygons
  - Depth sort
  - Binary Space-Partitioning tree
  - Z-buffer
  - A-buffer
- Ray tracing

**3D line drawing**

- Given a list of 3D lines we draw them by:
  - Projecting end points onto the 2D screen
  - Using a line drawing algorithm on the resulting 2D lines
- This produces a wireframe version of whatever objects are represented by the lines
Hidden line removal

- by careful use of cunning algorithms, lines that are hidden by surfaces can be carefully removed from the projected version of the objects
  - still just a line drawing
  - will not be covered further in this course

3D polygon drawing

- given a list of 3D polygons we draw them by:
  - projecting vertices onto the 2D screen
    - but also keep the \( z \) information
  - using a 2D polygon scan conversion algorithm on the resulting 2D polygons
- in what order do we draw the polygons?
  - some sort of order on \( z \):
    - depth sort
    - Binary Space-Partitioning tree
- is there a method in which order does not matter?
  - \( z \)-buffer

Depth sort algorithm

- transform all polygon vertices into viewing co-ordinates and project these into 2D, keeping \( z \) information
- calculate a depth ordering for polygons, based on the most distant \( z \) co-ordinate in each polygon
- resolve any ambiguities caused by polygons overlapping in \( z \)
- draw the polygons in depth order from back to front
  - "painter's algorithm": later polygons draw on top of earlier polygons
- steps 1 and 2 are simple, step 3 is 2D polygon scan conversion, step 4 requires more thought

Resolving ambiguities in depth sort

- may need to split polygons into smaller polygons to make a coherent depth ordering
Resolving ambiguities: algorithm

1. for the rearmost polygon, $P$, in the list, need to compare each polygon, $Q$, which overlaps $P$ in $z$
   - the question is: can I draw $P$ before $Q$?
     a. do the polygons $P$ extents not overlap?
     b. do the polygons $Q$ extents not overlap?
     c. is $P$ entirely on the opposite side of $Q$'s plane from the viewpoint?
     d. is $Q$ entirely on the same side of $P$'s plane as the viewpoint?
     e. do the projections of the two polygons into the $xy$ plane not overlap?

2. If all 5 tests fail, repeat $P$ and $Q$ with $P$ and $Q$ swapped (i.e. can I draw $Q$ before $P$?), if true swap $P$ and $Q$

3. Otherwise split either $P$ or $Q$ by the plane of the other, throw away the original polygon and insert the two pieces into the list

4. draw rearmost polygon once it has been completely checked

Depth sort: comments

1. the depth sort algorithm produces a list of polygons which can be scan-converted in 2D, backmost to frontmost, to produce the correct image
2. reasonably cheap for small number of polygons, becomes expensive for large numbers of polygons
3. the ordering is only valid from one particular viewpoint

Back face culling: a time-saving trick

1. if a polygon is a face of a closed polyhedron and faces backwards with respect to the viewpoint then it need not be drawn at all because front facing faces would later obscure it anyway
   a. saves drawing time at the the cost of one extra test per polygon
   b. assumes that we know which way a polygon is oriented
2. back face culling can be used in combination with any 3D scan-conversion algorithm

Binary Space-Partitioning trees

1. BSP trees provide a way of quickly calculating the correct depth order:
   a. for a collection of static polygons
   b. from an arbitrary viewpoint
2. the BSP tree trades off an initial time- and space-intensive pre-processing step against a linear display algorithm ($O(N)$) which is executed whenever a new viewpoint is specified
3. the BSP tree allows you to easily determine the correct order in which to draw polygons by traversing the tree in a simple way
**BSP tree: basic idea**
- A given polygon will be correctly scan-converted if:
  - All polygons on the far side of it from the viewer are scan-converted first
  - Then it is scan-converted
  - Then all the polygons on the near side of it are scan-converted

**Making a BSP tree**
- Given a set of polygons
  - Select an arbitrary polygon as the root of the tree
  - Divide all remaining polygons into two subsets:
    - Those in front of the selected polygon’s plane
    - Those behind the selected polygon’s plane
  - Any polygons through which the plane passes are split into two polygons and the two parts put into the appropriate subsets
  - Make two BSP trees, one from each of the two subsets
    - These become the front and back subtrees of the root

**Drawing a BSP tree**
- If the viewpoint is in front of the root’s polygon’s plane then:
  - Draw the BSP tree for the back child of the root
  - Draw the root’s polygon
  - Draw the BSP tree for the front child of the root
- Otherwise:
  - Draw the BSP tree for the front child of the root
  - Draw the root’s polygon
  - Draw the BSP tree for the back child of the root

**Scan-line algorithms**
- Instead of drawing one polygon at a time:
  - Modify the 2D polygon scan-conversion algorithm to handle all of the polygons at once
  - The algorithm keeps a list of the active edges in all polygons and proceeds one scan-line at a time
    - There is thus one large active edge list and one (even larger) edge list
    - Enormous memory requirements
  - Still fill in pixels between adjacent pairs of edges on the scan-line but:
    - Need to be intelligent about which polygon is in front and therefore what colours to put in the pixels
    - Every edge is used in two pairs:
      - One to the left and one to the right of it
z-buffer polygon scan conversion

- depth sort & BSP-tree methods involve clever sorting algorithms followed by the invocation of the standard 2D polygon scan conversion algorithm
- by modifying the 2D scan conversion algorithm we can remove the need to sort the polygons
  - makes hardware implementation easier

z-buffer basics

- store both colour and depth at each pixel
- when scan converting a polygon:
  - calculate the polygon's depth at each pixel
  - if the polygon is closer than the current depth stored at that pixel
    - then store both the polygon's colour and depth at that pixel
    - otherwise do nothing

z-buffer example

This is essentially the 2D polygon scan conversion algorithm with depth calculation and depth comparison added.
Interpolating depth values 1

- just as we incrementally interpolate $x$ as we move down the edges of the polygon, we can incrementally interpolate $z$:
  - as we move down the edges of the polygon
  - as we move across the polygon's projection

Interpolating depth values 2

- we thus have 2D vertices, with added depth information $[(x'_i, y'_i), z_i]$
- we can interpolate $x$ and $y$ in 2D
  
  \[
  x' = (1-t)x'_1 + t(x'_2) \\
  y' = (1-t)y'_1 + t(y'_2)
  \]
- but $z$ must be interpolated in 3D
  
  \[
  z = (1-t)\frac{1}{z_1} + t(1)\frac{1}{z_2}
  \]

Comparison of methods

- BSP is only useful for scenes which do not change
- as number of polygons increases, average size of polygon decreases, so time to draw a single polygon decreases
- z-buffer easy to implement in hardware: simply give it polygons in any order you like
- other algorithms need to know about all the polygons before drawing a single one, so that they can sort them into order

Putting it all together - a summary

- a 3D polygon scan conversion algorithm needs to include:
  - a 2D polygon scan conversion algorithm
  - 2D or 3D polygon clipping
  - projection from 3D to 2D
  - some method of ordering the polygons so that they are drawn in the correct order
**Sampling**
- all of the methods so far take a single sample for each pixel at the precise centre of the pixel
  - i.e. the value for each pixel is the colour of the polygon which happens to lie exactly under the centre of the pixel
- this leads to:
  - stair step (jagged) edges to polygons
  - small polygons being missed completely
  - thin polygons being missed completely or split into small pieces

**Anti-aliasing**
- these artefacts (and others) are jointly known as aliasing
- methods of ameliorating the effects of aliasing are known as anti-aliasing
  - in signal processing aliasing is a precisely defined technical term for a particular kind of artefact
  - in computer graphics its meaning has expanded to include most undesirable effects that can occur in the image
  - this is because the same anti-aliasing techniques which ameliorate true aliasing artefacts also ameliorate most of the other artefacts

**Anti-aliasing method 1: area averaging**
- average the contributions of all polygons to each pixel
  - e.g. assume pixels are square and we just want the average colour in the square
  - Ed Catmull developed an algorithm which does this:
    - works a scan-line at a time
    - clips all polygons to the scan-line
    - determines the fragment of each polygon which projects to each pixel
    - determines the amount of the pixel covered by the visible part of each fragment
    - pixel's colour is a weighted sum of the visible parts
  - expensive algorithm!

**Anti-aliasing method 2: super-sampling**
- sample on a finer grid, then average the samples in each pixel to produce the final colour
  - for an \( n \times n \) sub-pixel grid, the algorithm would take roughly \( n^2 \) times as long as just taking one sample per pixel
  - can simply average all of the sub-pixels in a pixel or can do some sort of weighted average
The A-buffer

- A significant modification of the z-buffer, which allows for sub-pixel sampling without as high an overhead as straightforward super-sampling
- Basic observation:
  - A given polygon will cover a pixel:
    - totally
    - partially
    - not at all
  - Sub-pixel sampling is only required in the case of pixels which are partially covered by the polygon


A-buffer: details

- For each pixel, a list of masks is stored
- Each mask shows how much of a polygon covers the pixel
- The masks are sorted in depth order
- A mask is a 4x8 array of bits:
  - 1 = polygon covers this sub-pixel
  - 0 = polygon doesn’t cover this sub-pixel
  - Sampling is done at the centre of each of the sub-pixels

A-buffer: example

- To get the final colour of the pixel you need to average together all visible bits of polygons

Making the A-buffer more efficient

- If a polygon totally covers a pixel then:
  - Do not need to calculate a mask, because the mask is all 1s
  - All masks currently in the list which are behind this polygon can be discarded
  - Any subsequent polygons which are behind this polygon can be immediately discounted (without calculating a mask)
  - In most scenes, therefore, the majority of pixels will have only a single entry in their list of masks
  - The polygon scan-conversion algorithm can be structured so that it is immediately obvious whether a pixel is totally or partially within a polygon
A-buffer: calculating masks

- clip polygon to pixel
- calculate the mask for each edge bounded by the right hand side of the pixel
  - there are few enough of these that they can be stored in a look-up table
- XOR all masks together

A-buffer: comments

- the A-buffer algorithm essentially adds anti-aliasing to the z-buffer algorithm in an efficient way
- most operations on masks are AND, OR, NOT, XOR
  - very efficient boolean operations
- why 4x8?
  - algorithm originally implemented on a machine with 32-bit registers (VAX 11/780)
  - on a 64-bit register machine, 8x8 seems more sensible
- what does the A stand for in A-buffer?
  - anti-aliased, area averaged, accumulator

A-buffer: extensions

- as presented the algorithm assumes that a mask has a constant depth (z value)
  - can modify the algorithm and perform approximate intersection between polygons
- can save memory by combining fragments which start life in the same primitive
  - e.g. two triangles that are part of the decomposition of a Bezier patch
- can extend to allow transparent objects

Illumination & shading

- until now we have assumed that each polygon is a uniform colour and have not thought about how that colour is determined
- things look more realistic if there is some sort of illumination in the scene
- we therefore need a mechanism of determining the colour of a polygon based on its surface properties and the positions of the lights
- we will, as a consequence, need to find ways to shade polygons which do not have a uniform colour
Illumination & shading (continued)

- in the real world every light source emits millions of photons every second
- these photons bounce off objects, pass through objects, and are absorbed by objects
- a tiny proportion of these photons enter your eyes allowing you to see the objects

- tracing the paths of all these photons is not an efficient way of calculating the shading on the polygons in your scene

Comments on reflection

- the surface can absorb some wavelengths of light
  - e.g. shiny gold or shiny copper
- specular reflection has “interesting” properties at glancing angles owing to occlusion of micro-facets by one another
- plastics are good examples of surfaces with:
  - specular reflection in the light’s colour
  - diffuse reflection in the plastic’s colour

How do surfaces reflect light?

- perfect reflection (mirror)
- specular reflection
- diffuse reflection (Lambertian reflection)

Calculating the shading of a polygon

- gross assumptions:
  - there is only diffuse (Lambertian) reflection
  - all light falling on a polygon comes directly from a light source
  - there is no interaction between polygons
  - no polygon casts shadows on any other
  - so can treat each polygon as if it were the only polygon in the scene
  - light sources are considered to be infinitely distant from the polygon
  - the vector to the light is the same across the whole polygon

- observation:
  - the colour of a flat polygon will be uniform across its surface, dependent only on the colour & position of the polygon and the colour & position of the light sources
**Diffuse shading calculation**

\[ L \text{ is a normalised vector pointing in the direction of the light source} \]
\[ N \text{ is the normal to the polygon} \]
\[ I_s \text{ is the intensity of the light source} \]
\[ k_d \text{ is the proportion of light which is diffusely reflected by the surface} \]
\[ I \text{ is the intensity of the light reflected by the surface} \]

\[ I = I_s k_d \cos \theta \]
\[ = I_s k_d (N \cdot L) \]

use this equation to set the colour of the whole polygon and draw the polygon using a standard polygon scan-conversion routine.

**Diffuse shading: comments**

- can have different \( I_s \) and different \( k_d \) for different wavelengths (colours)
- watch out for \( \cos \theta < 0 \)
  - implies that the light is behind the polygon and so it cannot illuminate this side of the polygon
- do you use one-sided or two-sided polygons?
  - one sided: only the side in the direction of the normal vector can be illuminated
    - if \( \cos \theta < 0 \) then both sides are black
  - two sided: the sign of \( \cos \theta \) determines which side of the polygon is illuminated
    - need to invert the sign of the intensity for the back side

**Gouraud shading**

- for a polygonal model, calculate the diffuse illumination at each vertex rather than for each polygon
  - calculate the normal at the vertex, and use this to calculate the diffuse illumination at that point
  - normal can be calculated directly if the polygonal model was derived from a curved surface
- interpolate the colour across the polygon, in a similar manner to that used to interpolate:
  - surface will look smoothly curved
  - rather than looking like a set of polygons
  - surface outline will still look polygonal


**Specular reflection**

- Phong developed an easy-to-calculate approximation to specular reflection

\[ I = I_s k_s \cos \alpha \]
\[ = I_s k_s (R \cdot V)^n \]

\( L \text{ is a normalised vector pointing in the direction of the light source} \)
\( R \text{ is the vector of perfect reflection} \)
\( N \text{ is the normal to the polygon} \)
\( V \text{ is a normalised vector pointing at the viewer} \)
\( I_s \text{ is the intensity of the light source} \)
\( k_s \text{ is the proportion of light which is specularly reflected by the surface} \)
\( n \text{ is Phong’s ad hoc “roughness” coefficient} \)
\( I \text{ is the intensity of the specularly reflected light} \)

Phong Bui-Tuong, “Illumination for computer generated pictures”, CACM, 18(6), 1975, 311–7
Phong shading

- similar to Gouraud shading, but calculate the specular component in addition to the diffuse component
- therefore need to interpolate the normal across the polygon in order to be able to calculate the reflection vector

N.B. Phong’s approximation to specular reflection ignores (amongst other things) the effects of glancing incidence

The gross assumptions revisited

- only diffuse reflection
  - now have a method of approximating specular reflection
- no shadows
  - need to do ray tracing to get shadows
- lights at infinity
  - can add local lights at the expense of more calculation
- need to interpolate the L vector
- no interaction between surfaces
  - cheat!
  - assume that all light reflected off all other surfaces onto a given polygon can be amalgamated into a single constant term: “ambient illumination”, add this onto the diffuse and specular illumination

Shading: overall equation

- the overall shading equation can thus be considered to be the ambient illumination plus the diffuse and specular reflections from each light source

\[ I = I_a k_a + \sum_i I_i k_d mass(L_i \cdot N) + \sum_i I_i k_s (R_i \cdot V)^s \]

- the more lights there are in the scene, the longer this calculation will take

Illumination & shading: comments

- how good is this shading equation?
  - gives reasonable results but most objects tend to look as if they are made out of plastic
  - Cook & Torrance have developed a more realistic (and more expensive) shading model which takes into account:
    - micro-facet geometry (which models, amongst other things, the roughness of the surface)
    - Fresnel’s formula for reflectance off a surface
    - there are other, even more complex, models
- is there a better way to handle inter-object interaction?
  - “ambient illumination” is, frankly, a gross approximation
  - distributed ray tracing can handle specular inter-reflection
  - radiosity can handle diffuse inter-reflection
Ray tracing

- a powerful alternative to polygon scan-conversion techniques
- given a set of 3D objects, shoot a ray from the eye through the centre of every pixel and see what it hits

Ray tracing algorithm

1. select an eye point and a screen plane
2. FOR every pixel in the screen plane
   - determine the ray from the eye through the pixel’s centre
   - FOR each object in the scene
     - IF the object is intersected by the ray
       - IF the intersection is the closest (so far) to the eye
         - record intersection point and object
     - END IF;
   - END IF;
3. END FOR;
4. set pixel’s colour to that of the object at the closest intersection point
5. END FOR;

Intersection of a ray with an object 1

- plane

  ![Diagram of a ray and a plane]

  ray: \( P = O + sD, s \geq 0 \)
  plane: \( P \cdot N + d = 0 \)
  \[ s = \frac{d + N \cdot O}{N \cdot D} \]

- box, polygon, polyhedron
  - defined as a set of bounded planes

Intersection of a ray with an object 2

- sphere

  ![Diagram of a circle and a sphere]

  ray: \( P = O + sD, s \geq 0 \)
  circle: \( (P - C) \cdot (P - C) - r^2 = 0 \)
  \[ d = \sqrt{b^2 - 4ac} \]
  \( d = \text{real} \)
  \( d = \text{imaginary} \)
  \[ s_1 = \frac{-b + d}{2a} \]
  \[ s_2 = \frac{-b - d}{2a} \]

- cylinder, cone, torus
  - all similar to sphere
Ray tracing: shading

- Once you have the intersection of a ray with the nearest object, you can also:
  - Calculate the normal to the object at that intersection point.
  - Shoot rays from that point to all of the light sources, and calculate the diffuse and specular reflections off the object at that point.
  - This (plus ambient illumination) gives the colour of the object (at that point).

Ray tracing: reflection

- If a surface is totally or partially reflective, new rays can be spawned to find the contribution to the pixel's colour given by the reflection.
  - This is perfect (mirror) reflection.

Ray tracing: shadows

- Because you are tracing rays from the intersection point to the light, you can check whether another object is between the intersection and the light and is hence casting a shadow.
  - Also need to watch for self-shadowing.

Ray tracing: transparency & refraction

- Objects can be totally or partially transparent.
  - This allows objects behind the current one to be seen through it.
- Transparent objects can have refractive indices.
  - Bending the rays as they pass through the objects.
- Transparency + reflection means that a ray can split into two parts.
Sampling in ray tracing

- **single point**
  - shoot a single ray through the pixel's centre

- **super-sampling for anti-aliasing**
  - shoot multiple rays through the pixel and average the result
  - regular grid, random, jittered, Poisson disc

- **adaptive super-sampling**
  - shoot a few rays through the pixel, check the variance of the resulting values, if similar enough stop, otherwise shoot some more rays

Types of super-sampling 1

- **regular grid**
  - divide the pixel into a number of sub-pixels and shoot a ray through the centre of each
  - problem: can still lead to noticeable aliasing unless a very high resolution sub-pixel grid is used

- **random**
  - shoot $N$ rays at random points in the pixel
  - replaces aliasing artefacts with noise artefacts
    - the eye is far less sensitive to noise than to aliasing

Types of super-sampling 2

- **Poisson disc**
  - shoot $N$ rays at random points in the pixel with the proviso that no two rays shall pass through the pixel closer than $\epsilon$ to one another
  - for $N$ rays this produces a better looking image than pure random sampling
  - very hard to implement properly

Types of super-sampling 3

- **jittered**
  - divide pixel into $N$ sub-pixels and shoot one ray at a random point in each sub-pixel
  - an approximation to Poisson disc sampling
  - for $N$ rays it is better than pure random sampling
  - easy to implement
More reasons for wanting to take multiple samples per pixel

- Super-sampling is only one reason why we might want to take multiple samples per pixel.
- Many effects can be achieved by distributing the multiple samples over some range.
  - Called distributed ray tracing.
  - N.B. Distributed means distributed over a range of values.
- Can work in two ways.
  - Each of the multiple rays shot through a pixel is allocated a random value from the relevant distribution(s).
  - All effects can be achieved this way with sufficient rays per pixel.
  - Each ray spawns multiple rays when it hits an object.
    - This alternative can be used, for example, for area lights.

Examples of distributed ray tracing

- Distribute the samples for a pixel over the pixel area.
  - Get random (or jittered) super-sampling.
  - Used for anti-aliasing.
- Distribute the rays going to a light source over some area.
  - Allows area light sources in addition to point and directional light sources.
  - Produces soft shadows with penumbras.
- Distribute the camera position over some area.
  - Allows simulation of a camera with a finite aperture lens.
  - Produces depth of field effects.
- Distribute the samples in time.
  - Produces motion blur effects on any moving objects.

Distributed ray tracing for specular reflection

- Previously we could only calculate the effect of perfect reflection.
- We can now distribute the reflected rays over the range of directions from which specularly reflected light could come.
- Provides a method of handling some of the inter-reflections between objects in the scene.
- Requires a very large number of ray per pixel.

Handling direct illumination

- Diffuse reflection.
  - Handled by ray tracing and polygon scan conversion.
  - Assumes the object is a perfect Lambertian reflector.
- Specular reflection.
  - Also handled by ray tracing and polygon scan conversion.
  - Use Phong's approximation to true specular reflection.
Handing indirect illumination:

1. **diffuse to specular**
   - handled by distributed ray tracing

2. **specular to specular**
   - also handled by distributed ray tracing

Multiple inter-reflection

- light may reflect off many surfaces on its way from the light to the camera
- standard ray tracing and polygon scan conversion can handle a single diffuse or specular bounce
- distributed ray tracing can handle multiple specular bounces
- radiosity can handle multiple diffuse bounces
- the general case cannot be handled by any efficient algorithm

(diffuse | specular)*

Hybrid algorithms

- **polygon scan conversion** and **ray tracing** are the two principal 3D rendering mechanisms
  - each has its advantages
    - polygon scan conversion is faster
    - ray tracing produces more realistic looking results
  - hybrid algorithms exist
  - these generally use the speed of polygon scan conversion for most of the work and use ray tracing only to achieve particular special effects
Surface detail

- So far we have assumed perfectly smooth, uniformly coloured surfaces.
- Real life isn't like that:
  - Multicoloured surfaces (e.g., a painting, a food can, a page in a book)
  - Bumpy surfaces (e.g., almost any surface! Very few things are perfectly smooth)
  - Textured surfaces (e.g., wood, marble)

Basic texture mapping

- A texture is simply an image, with a 2D coordinate system \((u,v)\).
- Each 3D object is parameterised in \((u,v)\) space.
- Each pixel maps to some part of the surface.
- That part of the surface maps to part of the texture.

Texture mapping

- Without:
  - All surfaces are smooth and of uniform colour.

- With:
  - Most surfaces are textured with 2D texture maps.
  - The pillars are textured with a solid texture.

Parameterising a primitive

- Polygon: Give \((u,v)\) coordinates for three vertices, or treat as part of a plane.
- Plane: Give \(u\)-axis and \(v\)-axis directions in the plane.
- Cylinder: One axis goes up the cylinder, the other around the cylinder.
Sampling texture space

Find \((u, v)\) coordinate of the sample point on the object and map this into texture space as shown.

Sampling texture space: finding the value

- nearest neighbour: the sample value is the nearest pixel value to the sample point
- bilinear reconstruction: the sample value is the weighted mean of pixels around the sample point

Sampling texture space: interpolation methods

- nearest neighbour: fast with many artefacts
- bilinear: reasonably fast, blurry
- can we get better results?
  - bicubic gives better results
    - uses 16 values \((4 \times 4)\) around the sample location
    - but runs at one quarter the speed of bilinear
  - biquadratic
    - use 9 values \((3 \times 3)\) around the sample location
    - faster than bicubic, slower than linear, results seem to be nearly as good as bicubic

Texture mapping examples

- nearest-neighbour
- bilinear
**Down-sampling**

If the pixel covers quite a large area of the texture, then it will be necessary to average the texture across that area, not just take a sample in the middle of the area.

**Multi-resolution texture**

Rather than down-sampling every time you need to, have multiple versions of the texture at different resolutions and pick the appropriate resolution to sample from...

You can use tri-linear interpolation to get an even better result: that is, use bi-linear interpolation in the two nearest levels and then linearly interpolate between the two interpolated values.

**The MIP map**

An efficient memory arrangement for a multi-resolution colour image. A pixel \((x,y)\) is a bottom level pixel location (level 0); for an image of size \((m,n)\), it is stored at these locations in level \(k\):

Red: \[
\begin{pmatrix}
\frac{m+x}{2^k} \\
\frac{y}{2^k}
\end{pmatrix}
\]

Blue: \[
\begin{pmatrix}
\frac{x}{2^k} \\
\frac{m+y}{2^k}
\end{pmatrix}
\]

Green: \[
\begin{pmatrix}
\frac{m+x}{2^k} \\
\frac{m+y}{2^k}
\end{pmatrix}
\]

**Solid textures**

Texture mapping applies a 2D texture to a surface \(colour = f(u,v)\).

Solid textures have colour defined for every point in space \(colour = f(x,y,z)\). This permits the modelling of objects which appear to be carved out of a material.
What can a texture map modify?

- any (or all) of the colour components
  - ambient, diffuse, specular
- transparency
  - “transparency mapping”
- reflectiveness
- but also the surface normal
  - “bump mapping”

Bump mapping

- the surface normal is used in calculating both diffuse and specular reflection
- bump mapping modifies the direction of the surface normal so that the surface appears more or less bumpy
- rather than using a texture map, a 2D function can be used which varies the surface normal smoothly across the plane
- but bump mapping doesn’t change the object’s outline

Filtering

- move a filter over the image, calculating a new value for every pixel

Image Processing

- filtering
  - convolution
  - nonlinear filtering
- point processing
  - intensity/colour correction
- compositing
- halftoning & dithering
- compression
  - various coding schemes
Filters - discrete convolution

Convolve a discrete filter with the image to produce a new image

- in one dimension:
  \[ f'(x) = \sum_{i=-\infty}^{\infty} h(i) \times f(x-i) \]
  where \( h(i) \) is the filter

- in two dimensions:
  \[ f'(x, y) = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h(i, j) \times f(x-i, y-j) \]

Example filters - averaging/blurring

- Basic 3x3 blurring filter
  \[ \frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \]

- Gaussian 3x3 blurring filter
  \[ \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 2 \\ 1 & 2 & 1 \end{bmatrix} \]

- Gaussian 5x5 blurring filter
  \[ \frac{1}{12} \times \begin{bmatrix} 1 & 2 & 4 & 2 & 1 \\ 2 & 6 & 9 & 6 & 2 \\ 4 & 9 & 16 & 9 & 4 \\ 2 & 6 & 9 & 6 & 2 \\ 1 & 2 & 4 & 2 & 1 \end{bmatrix} \]

Example filters - edge detection

- Horizontal edge detection filter
  \[ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \]

- Vertical edge detection filter
  \[ \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \]

- Diagonal edge detection filter
  \[ \begin{bmatrix} 1 & 0 & -1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \]

Example filter - horizontal edge detection

- Image Result
  \[ \begin{bmatrix} 300 & 300 & 300 & 300 & 300 \\ 300 & 300 & 300 & 300 & 300 \\ 300 & 300 & 300 & 300 & 300 \\ 300 & 300 & 300 & 300 & 300 \\ 300 & 300 & 300 & 300 & 300 \end{bmatrix} \times \begin{bmatrix} 1/12 \times \begin{bmatrix} 1 & 2 & 4 & 2 & 1 \\ 2 & 6 & 9 & 6 & 2 \\ 4 & 9 & 16 & 9 & 4 \\ 2 & 6 & 9 & 6 & 2 \\ 1 & 2 & 4 & 2 & 1 \end{bmatrix} \end{bmatrix} \]

©2003 Neil A. Dodgson
**Example filter - horizontal edge detection**

- **Original image**
- **After use of a 3x3 Prewitt horizontal edge detection filter**
  
  *Mid-grey = no edge, black or white = strong edge*

---

**Median filtering**

- **Not a convolution method**
- The new value of a pixel is the median of the values of all the pixels in its neighbourhood

**Median filter - example**

- Original image
- Add shot noise
- Median filter
- Gaussian blur

**Median filter - limitations**

- Copes well with shot (impulse) noise
- Not so good at other types of noise

**In this example,**

- Median filter reduces noise but does not eliminate it
- Gaussian filter eliminates noise at the expense of excessive blurring
Point processing

- each pixel's value is modified
- the modification function only takes that pixel's value into account

\[ p'(i, j) = f\{p(i, j)\} \]

- where \( p(i, j) \) is the value of the pixel and \( p'(i, j) \) is the modified value
- the modification function, \( f(p) \), can perform any operation that maps one intensity value to another

Point processing

inverting an image

Point processing

improving an image's contrast

Point processing

modifying the output of a filter
**Point processing: gamma correction**

- The intensity displayed on a CRT is related to the voltage on the electron gun by:
  \[ i \propto V^\gamma \]
- The voltage is directly related to the pixel value:
  \[ V \propto p \]
- Gamma correction modifies pixel values in the inverse manner:
  \[ p' = p^{1/\gamma} \]
- Thus generating the appropriate intensity on the CRT:
  \[ i \propto V^\gamma \propto p^{1/\gamma} \propto p \]
- CRTs generally have gamma values around 2.0

**Simple compositing**

- Copy pixels from one image to another
- Only copying the pixels you want
- Use a mask to specify the desired pixels
  ![Masked images](mask.png)
  - The mask determines which image is used for each pixel.

**Image compositing**

- Merging two or more images together
  ![Image compositing](images.png)
  - What does this operator do?

**Alpha blending for compositing**

- Instead of a simple boolean mask, use an alpha mask
- Value of alpha mask determines how much of each image to blend together to produce final pixel
  \[ d = ma + (1 - m)b \]
  - The mask determines how to blend the two source pixel values.
**Arithmetic operations**

+ images can be manipulated arithmetically
  - simply apply the operation to each pixel location in turn
+ multiplication
  - used in masking
+ subtraction (difference)
  - used to compare images
  - e.g. comparing two x-ray images before and after injection of a dye into the bloodstream

**Difference example**

The two images are taken from slightly different viewpoints

\[ d = 1 - |a - b| \]

where 1 = white and 0 = black

black = large difference
white = no difference

**Halftoning & dithering**

+ mainly used to convert greyscale to binary
  - e.g. printing greyscale pictures on a laser printer
  - 8-bit to 1-bit
+ is also used in colour printing, normally with four colours:
  - cyan, magenta, yellow, black

**Halftoning**

+ each greyscale pixel maps to a square of binary pixels
  - e.g. five intensity levels can be approximated by a 2x2 pixel square
  - 1-to-4 pixel mapping

<table>
<thead>
<tr>
<th>0-51</th>
<th>52-102</th>
<th>103-153</th>
<th>154-204</th>
<th>205-255</th>
</tr>
</thead>
</table>

8-bit values that map to each of the five possibilities
Halftoning dither matrix

- One possible set of patterns for the $3 \times 3$ case is:

  
  \[
  \begin{array}{ccc}
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \end{array}
  \quad
  \begin{array}{ccc}
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \end{array}
  \]

- These patterns can be represented by the dither matrix:

  \[
  \begin{array}{ccc}
  7 & 9 & 5 \\
  2 & 1 & 4 \\
  6 & 3 & 8 \\
  \end{array}
  \]

  1-to-9 pixel mapping

Rules for halftone pattern design

- Mustn’t introduce visual artefacts in areas of constant intensity
  - E.g. this won’t work very well:

  
  \[
  \begin{array}{ccc}
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \cdot & \cdot & \cdot \\
  \end{array}
  \]

- Every on pixel in intensity level $j$ must also be on in levels $> j$
  - I.e. on pixels form a growth sequence

- Pattern must grow outward from the centre
  - Simulates a dot getting bigger

- All on pixels must be connected to one another
  - This is essential for printing, as isolated on pixels will not print very well (if at all)

Ordered dither

- Halftone prints and photocopies well, at the expense of large dots
- An ordered dither matrix produces a nicer visual result than a halftone dither matrix

1-to-1 pixel mapping

- A simple modification of the ordered dither method can be used
  - Turn a pixel on if its intensity is greater than (or equal to) the value of the corresponding cell in the dither matrix

  \[
  b_{ij} = \left( q_{ij} \geq \frac{d_{ij}}{15} \right)
  \]

  Quantise 8 bit pixel value

  \[
  q_{ij} = \frac{p_{ij}}{15}
  \]

  Find binary value

  \[
  d_{ij} = \frac{d_{ij}}{15}
  \]
Error diffusion

+ Error diffusion gives a more pleasing visual result than ordered dither
+ Method:
  - Work left to right, top to bottom
  - Map each pixel to the closest quantised value
  - Pass the quantisation error on to the pixels to the right and below, and add in the errors before quantising these pixels

Error diffusion - example (1)

+ Map 8-bit pixels to 1-bit pixels
+ Quantise and calculate new error values

<table>
<thead>
<tr>
<th>8-bit value $f_{i,j}$</th>
<th>1-bit value $b_{i,j}$</th>
<th>Error $e_{i,j}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-127</td>
<td>0</td>
<td>$f_{i,j}$</td>
</tr>
<tr>
<td>128-255</td>
<td>1</td>
<td>$f_{i,j} - 255$</td>
</tr>
</tbody>
</table>

- Each 8-bit value is calculated from pixel and error values:
  $$f_{i,j} = p_{i,j} + \frac{1}{2} e_{i-1,j} + \frac{1}{2} e_{i,j-1}$$
  In this example the errors from the pixels to the left and above are taken into account

Error diffusion - example (2)

Floyd & Steinberg developed the error diffusion method in 1975
- Often called the “Floyd-Steinberg algorithm”
- Their original method diffused the errors in the following proportions:

{\begin{align*}
\frac{7}{16} &\quad \frac{1}{16} &\quad \frac{1}{16} &\quad \frac{3}{16} \\
\end{align*}}

Pixels still to be processed

Current pixel

Pixels that have been processed
**Halftoning & dithering — examples**

- **ordered dither**
  - halftoning (4x4 cells)
  - the regular dither pattern is clearly visible
  - therefore looks more attractive to the human eye

- **error diffused**
  - halftoning (5x5 cells)
  - more random than ordered dither

**Halftoning & dithering — examples**

- **original**
  - halftoned with a very fine screen
  - thresholding
  - <128 ⇒ black
  - ≥128 ⇒ white

- **halftoning**
  - the larger the cell size, the more intensity levels available
  - the smaller the cell, the less noticeable the halftone dots

**Encoding & compression**

- introduction
- various coding schemes
  - difference, predictive, run-length, quadtree
- transform coding
  - Fourier, cosine, wavelets, JPEG

**What you should note about image data**

- there's lots of it!
  - an A4 page scanned at 300 ppi produces:
    - 24MB of data in 24 bit per pixel colour
    - 1MB of data at 1 bit per pixel
    - the Encyclopaedia Britannica would require 25GB at 300 ppi, 1 bit per pixel
- adjacent pixels tend to be very similar
- compression is therefore both feasible and necessary
Encoding - overview

- mapper
  - maps pixel values to some other set of values
  - designed to reduce inter-pixel redundancies

- quantiser
  - reduces the accuracy of the mapper’s output
  - designed to reduce psychovisual redundancies

- symbol encoder
  - encodes the quantiser’s output
  - designed to reduce symbol redundancies

Lossless vs lossy compression

- lossless
  - allows you to exactly reconstruct the pixel values from the encoded data
  - implies no quantisation stage and no losses in either of the other stages

- lossy
  - loses some data, you cannot exactly reconstruct the original pixel values

Raw image data

- can be stored simply as a sequence of pixel values
  - no mapping, quantisation, or encoding

Symbol encoding on raw data

- pixels are encoded by variable length symbols
  - the length of the symbol is determined by the frequency of the pixel value’s occurrence

<table>
<thead>
<tr>
<th>p</th>
<th>P(p)</th>
<th>Code 1</th>
<th>Code 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.19</td>
<td>0.19</td>
<td>000</td>
<td>11</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25</td>
<td>001</td>
<td>01</td>
</tr>
<tr>
<td>0.21</td>
<td>0.21</td>
<td>010</td>
<td>10</td>
</tr>
<tr>
<td>0.16</td>
<td>0.16</td>
<td>011</td>
<td>001</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
<td>100</td>
<td>0001</td>
</tr>
<tr>
<td>0.06</td>
<td>0.06</td>
<td>101</td>
<td>00001</td>
</tr>
<tr>
<td>0.03</td>
<td>0.03</td>
<td>110</td>
<td>00000</td>
</tr>
<tr>
<td>0.02</td>
<td>0.02</td>
<td>111</td>
<td>000000</td>
</tr>
</tbody>
</table>

With Code 1 each pixel requires 3 bits
With Code 2 each pixel requires 2.7 bits
Code 2 thus encodes the data in 90% of the space of Code 1
Quantisation as a compression method

(quantisation, on its own, is not normally used for compression because of the visual degradation of the resulting image)

however, an 8-bit to 4-bit quantisation using error diffusion would compress an image to 50% of the space

Difference mapping

(every pixel in an image will be very similar to those either side of it)

(a simple mapping is to store the first pixel value and, for every other pixel, the difference between it and the previous pixel)

this distribution of values will work well with a variable length code
Predictive mapping  
(an example of mapping)

- when transmitting an image left-to-right top-to-bottom, we already know the values above and to the left of the current pixel
- predictive mapping uses those known pixel values to predict the current pixel value, and maps each pixel value to the difference between its actual value and the prediction

\[ \hat{p}_{i,j} = \frac{1}{2} p_{i-1,j} + \frac{1}{2} p_{i,j-1} \]

\[ d_{i,j} = p_{i,j} - \hat{p}_{i,j} \]

Run-length encoding  
(an example of symbol encoding)

- based on the idea that images often contain runs of identical pixel values
- method:
  - encode runs of identical pixels as run length and pixel value
  - encode runs of non-identical pixels as run length and pixel values

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- based on the idea that images often contain runs of identical pixel values
- method:
  - encode runs of identical pixels as run length and pixel value
  - encode runs of non-identical pixels as run length and pixel values

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images

Run-length encoding - example (1)

- run length is encoded as an 8-bit value:
  - first bit determines type of run
    - 0 = identical pixels, 1 = non-identical pixels
  - other seven bits code length of run
    - binary value of run length - 1 (run length ∈ \{1, ..., 128\})
- pixels are encoded as 8-bit values
  - best case: all runs of 128 identical pixels
    - compression of 2/128 = 1.56%
  - worst case: no runs of identical pixels
    - compression of 129/128=100.78% (expansion!)

Run-length encoding - example (2)

- works well for computer generated imagery
- not so good for real-life imagery
- especially bad for noisy images
**CCITT fax encoding**

- fax images are binary
- 1D CCITT group 3
  - binary image is stored as a series of run lengths
  - don’t need to store pixel values!
- 2D CCITT group 3 & 4
  - predict this line’s runs based on previous line’s runs
  - encode differences

**Transform coding**

- transform $N$ pixel values into coefficients of a set of $N$ basis functions
- the basis functions should be chosen so as to squash as much information into as few coefficients as possible
- quantise and encode the coefficients

**Mathematical foundations**

- each of the $N$ pixels, $f(x)$, is represented as a weighted sum of coefficients, $F(u)$

$$f(x) = \sum_{u=0}^{N-1} F(u)H(u,x)$$

e.g., $H(u,x)$

<table>
<thead>
<tr>
<th>$u$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

- 0: +1 +1 +1 +1 +1 +1 +1 +1
- 1: +1 +1 +1 +1 -1 -1 -1 -1
- 2: +1 +1 -1 -1 +1 +1 -1 -1
- 3: +1 +1 -1 -1 -1 -1 +1 +1
- 4: +1 -1 +1 -1 +1 -1 +1 -1
- 5: +1 -1 +1 -1 -1 +1 -1 +1
- 6: +1 -1 -1 +1 +1 -1 -1 +1
- 7: +1 -1 -1 +1 -1 +1 +1 -1

$H(u,x)$ is the array of weights

**Calculating the coefficients**

- the coefficients can be calculated from the pixel values using this equation:

$$F(u) = \sum_{x=0}^{N-1} f(x)h(x,u)$$

- compare this with the equation for a pixel value, from the previous slide:

$$f(x) = \sum_{u=0}^{N-1} F(u)H(u,x)$$
Walsh-Hadamard transform

“square wave” transform

\[ h(x,u) = \frac{1}{N} H(u,x) \]

invented by Walsh (1923) and Hadamard (1893) - the two variants give the same results for \( N \) a power of 2

the first sixteen Walsh basis functions (Hadamard basis functions are the same, but numbered differently)

2D transforms

the two-dimensional versions of the transforms are an extension of the one-dimensional cases

\[
\begin{align*}
F(u) &= \sum_{x=0}^{N-1} f(x)h(x,u) \\
F(u,v) &= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y)h(x,y,u,v)
\end{align*}
\]

forward transform

\[
\begin{align*}
f(x) &= \sum_{u=0}^{N-1} F(u)H(u,x) \\
f(x,y) &= \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u,v)H(u,v,x,y)
\end{align*}
\]

inverse transform

2D Walsh basis functions

these are the Walsh basis functions for \( N=4 \)
in general, there are \( N^2 \) basis functions operating on an \( N \times N \) portion of an image

Discrete Fourier transform (DFT)

forward transform:

\[
F(u) = \sum_{x=0}^{N-1} f(x)e^{\frac{2\pi iux}{N}}
\]

inverse transform:

\[
f(x) = \frac{1}{N} \sum_{u=0}^{N-1} F(u)e^{\frac{2\pi iux}{N}}
\]

thus:

\[
\begin{align*}
h(x,u) &= \frac{1}{N} e^{\frac{2\pi iux}{N}} \\
H(u,x) &= e^{\frac{2\pi iux}{N}}
\end{align*}
\]
DFT - alternative interpretation

- the DFT uses complex coefficients to represent real pixel values
- it can be reinterpreted as:

\[ f(x) = \sum_{u=0}^{N-1} A(u) \cos(2\pi xu + \theta(u)) \]

where \( A(u) \) and \( \theta(u) \) are real values
- a sum of weighted & offset sinusoids

Discrete cosine transform (DCT)

- forward transform:

\[ F(u) = \sum_{x=0}^{N-1} f(x) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \]

- inverse transform:

\[ f(x) = \sum_{u=0}^{N-1} F(u) \alpha(u) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \]

where:

\[ \alpha(u) = \begin{cases} \sqrt{\frac{N}{2}} & u = 0 \\ \sqrt{\frac{1}{N}} & u \in \{1,2,\ldots,N-1\} \end{cases} \]

Haar transform: wavelets

- “square wave” transform, similar to Walsh-Hadamard
- Haar basis functions get progressively more local
  - c.f. Walsh-Hadamard, where all basis functions are global
- simplest wavelet transform

DCT basis functions

the first eight DCT basis functions showing the values of \( h_{u,x} \) for \( N=8 \)

DCT basis functions

the first eight DCT basis functions showing the values of \( h_{u,x} \) for \( N=8 \)
Haar basis functions

the first sixteen Haar basis functions

Karhunen-Loève transform (KLT)
"eigenvector", "principal component", "Hotelling" transform

+ based on statistical properties of the image source
+ theoretically best transform encoding method
+ but different basis functions for every different image source

JPEG: a practical example

+ compression standard
  JPEG = Joint Photographic Expert Group
+ three different coding schemes:
  + baseline coding scheme
    + based on DCT, lossy
    + adequate for most compression applications
  + extended coding scheme
    + for applications requiring greater compression or higher precision or progressive reconstruction
  + independent coding scheme
    + lossless, doesn't use DCT

JPEG sequential baseline scheme

+ input and output pixel data limited to 8 bits
+ DCT coefficients restricted to 11 bits
+ three step method

the following slides describe the steps involved in the JPEG compression of an 8 bit/pixel image
JPEG example: DCT transform

- Subtract 128 from each (8-bit) pixel value
- Subdivide the image into 8x8 pixel blocks
- Process the blocks left-to-right, top-to-bottom
- Calculate the 2D DCT for each block

JPEG example: quantisation

- Quantise each coefficient, $F(u,v)$, using the values in the quantisation matrix and the formula:

$$
\hat{F}(u,v) = \text{round}\left[ \frac{F(u,v)}{Z(u,v)} \right]
$$

- Reorder the quantised values in a zigzag manner to put the most important coefficients first

JPEG example: symbol encoding

- The DC coefficient (mean intensity) is coded relative to the DC coefficient of the previous 8x8 block
- Each non-zero AC coefficient is encoded by a variable length code representing both the coefficient's value and the number of preceding zeroes in the sequence
- This is to take advantage of the fact that the sequence of 63 AC coefficients will normally contain long runs of zeroes