1. In the lecture, a proof of the Time Hierarchy Theorem was sketched. Give a similar argument for the following Space Hierarchy Theorem:

**Space Hierarchy.** For every constructible function $f$, there is a language in $\text{SPACE}(f(n) \cdot \log f(n))$ that is not in $\text{SPACE}(f(n))$.

Could you replace the factor of $\log f(n)$ in this statement with something even smaller?

2. Consider the algorithm presented in the lecture which establishes that **Reachability** is in $\text{SPACE}((\log n)^2)$. What is the time complexity of this algorithm? Can you generalise the time bound to the entire complexity class? That is, give a class of functions $F$, such that

$$\text{SPACE}((\log n)^2) \subseteq \bigcup_{f \in F} \text{TIME}(f)$$

3. Show that, if $\text{SPACE}((\log n)^2) \subseteq \text{P}$, then $L \neq \text{P}$. (Hint: use the Space Hierarchy Theorem from Exercise 1.)

4. Show that, for every nondeterministic machine $M$ which uses $O(\log n)$ work space, there is a machine $R$ with three tapes (input, work and output) which works as follows. On input $x$, $R$ produces on its output tape a description of the configuration graph for $M, x$, and $R$ uses $O(\log |x|)$ space on its work tape. Explain why this means that if **Reachability** is in $L$, then $L = \text{NL}$.

5. Show that a language $L$ is in $\text{co-NP}$ if, and only if, there is a nondeterministic Turing machine $M$ and a polynomial $p$ such that $M$ halts in time $p(n)$ for all inputs of length $x$, and $L$ is exactly the set of strings $x$ such that all computations of $M$ on input $x$ end in an accepting state.