1. In the lecture, a proof was sketched showing a $\Omega(n \log n)$ lower bound on the complexity of the sorting problem. It was also stated that a similar analysis could be used to establish the same bound for the Travelling Salesman Problem. Give a detailed sketch of such an argument. Can you think of a way to improve the lower bound?

2. On page 10 of the notes, a number of functions are listed as being constructible. Show that this is the case by giving, for each one, a description of an appropriate Turing machine.

Prove that if $f$ and $g$ are constructible functions and $f(n) \geq n$, then so are $f(g)$, $f + g$, $f \cdot g$ and $2^f$.

3. For any constructible function $f$, and any language $L \in \text{NTIME}(f(n))$, there is a nondeterministic machine $M$ that accepts $L$ and all of whose computations terminate in time $O(f(n))$ for all inputs of length $n$. Give a detailed argument for this statement, describing how $M$ might be obtained from a machine accepting $L$ in time $f(n)$.

4. Consider the language $L$ in the alphabet $\{a, b\}$ given by $L = \{a^n b^n \mid n \in \mathbb{N}\}$. $L \not\in \text{SPACE}(c)$ for any constant $c$. Why?