1. In the lecture, a proof of the Time Hierarchy Theorem was sketched. Give a similar argument for the following Space Hierarchy Theorem:

**Space Hierarchy.** For every constructible function \( f \), there is a language in \( \text{SPACE}(f(n) \cdot \log f(n)) \) that is not in \( \text{SPACE}(f(n)) \).

Could you replace the factor of \( \log f(n) \) in this statement with something even smaller?

2. Consider the algorithm presented in the lecture which establishes that Reachability is in \( \text{SPACE}((\log n)^2) \). What is the time complexity of this algorithm? Can you generalise the time bound to the entire complexity class? That is, give a class of functions \( F \), such that

\[
\text{SPACE}((\log n)^2) \subseteq \bigcup_{f \in F} \text{TIME}(f)
\]

3. Show that, if \( \text{SPACE}((\log n)^2) \subseteq \text{P} \), then \( \text{L} = \text{NP} \). (Hint: use the Space Hierarchy Theorem from Exercise 1.)

4. Show that, for every nondeterministic machine \( M \) which uses \( O(\log n) \) work space, there is a machine \( R \) with three tapes (input, work and output) which works as follows. On input \( x \), \( R \) produces on its output tape a description of the configuration graph for \( M, x \), and \( R \) uses \( O(\log |x|) \) space on its work tape. Explain why this means that if Reachability is in \( \text{L} \), then \( \text{L} = \text{NL} \).

5. Show that a language \( L \) is in \( \text{co} - \text{NP} \) if, and only if, there is a nondeterministic Turing machine \( M \) and a polynomial \( p \) such that \( M \) halts in time \( p(n) \) for all inputs of length \( x \), and \( L \) is exactly the set of strings \( x \) such that all computations of \( M \) on input \( x \) end in an accepting state.