Circuits

A circuit is a graph $G = (V, E)$, with $V = \{1, \ldots, n\}$ together with a labeling: $l : V \to \{\mathtt{true}, \mathtt{false}, \wedge, \vee, \neg\}$, satisfying:

- If $E(i, j)$, then $i < j$;
- Every node in V has *indegree* at most 2.
- A node v has
  indegree 0 iff $l(v) \in \{\mathtt{true}, \mathtt{false}\}$;
  indegree 1 iff $l(v) = \neg$;
  indegree 2 iff $l(v) \in \{\mathtt{true}, \mathtt{false}\}$

A circuit is a more compact way of representing a Boolean expression.

The value of the expression is given by the value at node n.

CVP

CVP - the *circuit value problem* is, given a circuit, determine the value of the result node n.

CVP is solvable in polynomial time, by the algorithm which examines the nodes in increasing order, assigning a value `true` or `false` to each node.

CVP is complete for P under L reductions.

That is, for every language A in P,

$$A \leq_L \mathsf{CVP}$$