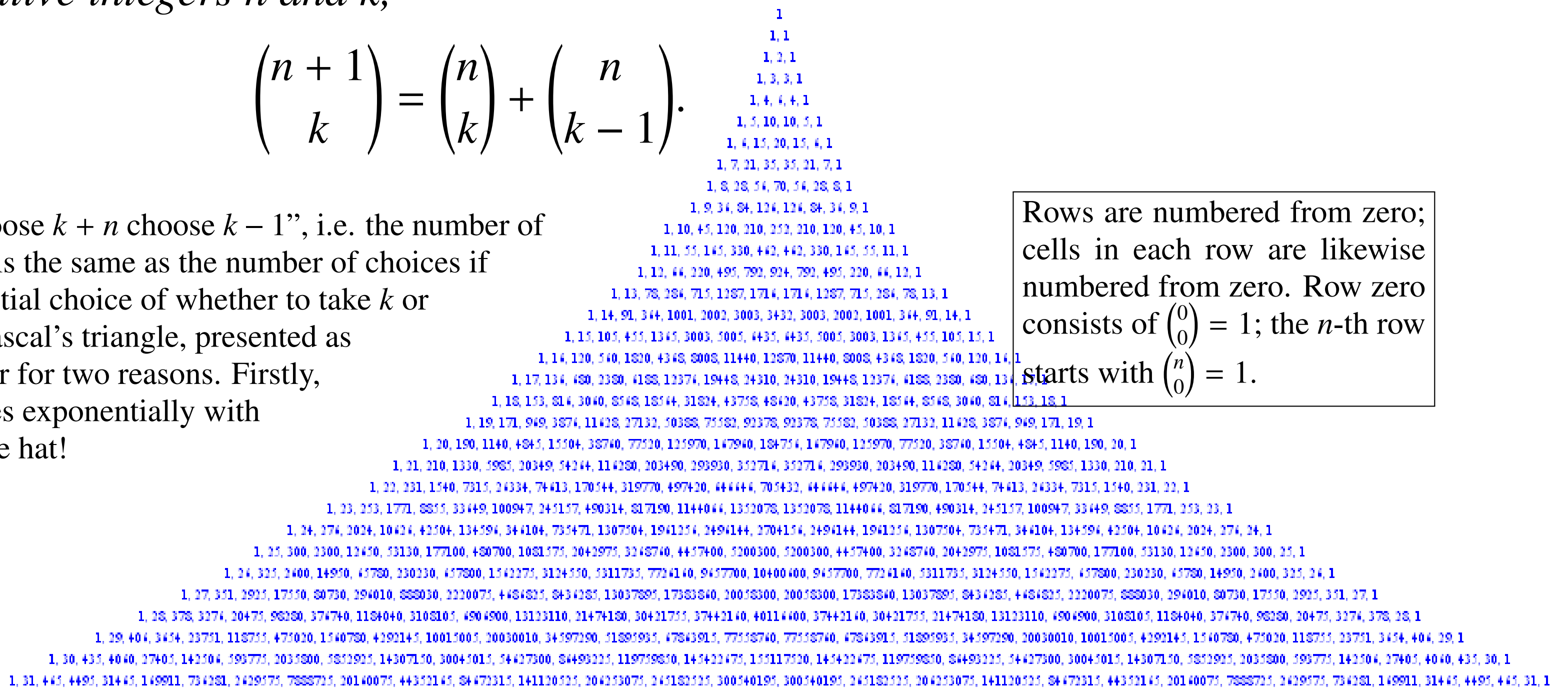
THEOREM OF THE DAY

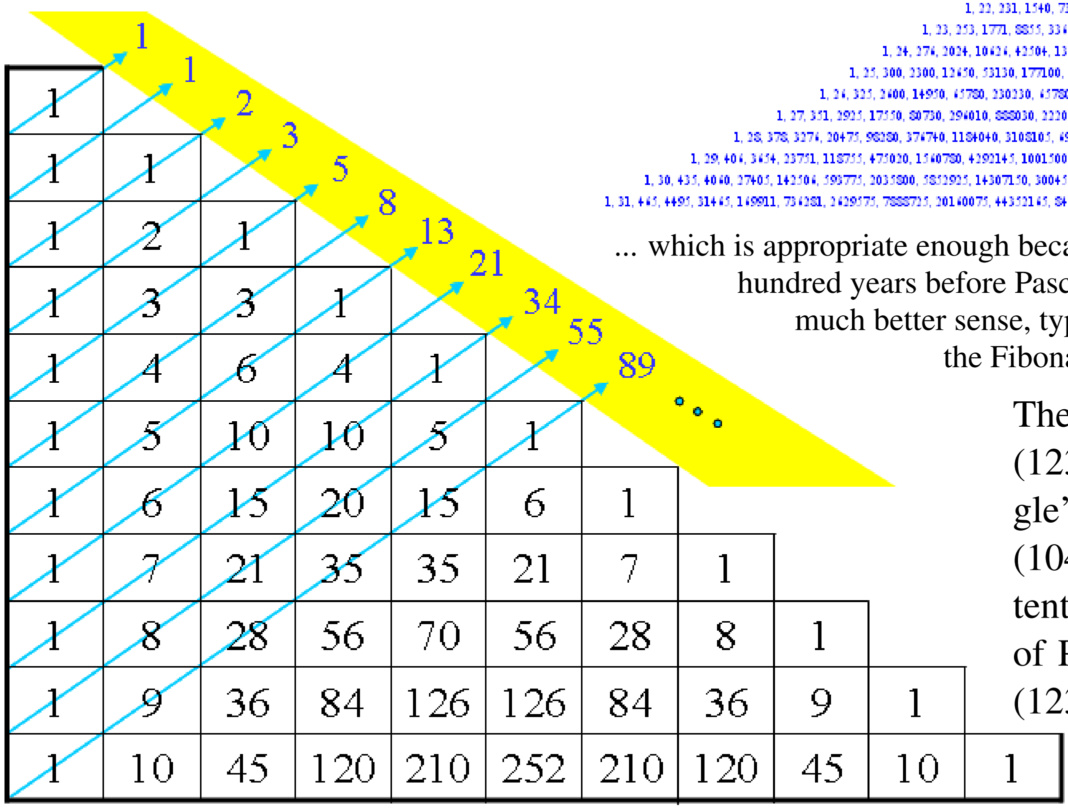
Pascal's Rule *For any positive integers n and k,*

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}.$$



Rows are numbered from zero; cells in each row are likewise numbered from zero. Row zero consists of $\binom{0}{0} = 1$; the n-th row starts with $\binom{n}{0} = 1$.

In words, this is read as "$n+1$ choose $k = n$ choose $k + n$ choose $k-1$", i.e. the number of choices if you must select k objects from $n+1$ is the same as the number of choices if you are selecting from n objects and have an initial choice of whether to take k or $k-1$. The rule defines what is usually called Pascal's triangle, presented as shown on the right. However, this is a misnomer for two reasons. Firstly, it isn't a triangle at all, unless font size decreases exponentially with increasing row number; it is more like a Chinese hat!



... which is appropriate enough because, secondly, this triangle and rule were known to the Chinese scholar Jia Xian, six hundred years before Pascal. Aligning the rows of the triangle on the left (as shown on the left) seems to make much better sense, typographically, computationally and combinatorially. A well-known relationship with the Fibonacci series, for instance, becomes immediately apparent as a series of diagonal sums.

The work of Jia Xian has passed to us through the commentary of Yang Hui (1238-1298) and Pascal's triangle is known in China as 'Yang Hui's triangle'. In Iran, it is known as the 'Khayyám triangle' after Omar Khayyám (1048-1131), although it was known to Persian, and Indian, scholars in the tenth century. Peter Cameron cites Robin Wilson as dating Western study of Pascal's triangle as far back as the Majorcan theologian Ramon Llull (1232–1316).

Web link: ptri1.tripod.com. See the wikipedia entry on nomenclature.
Further reading: Pascal's Arithmetical Triangle by A.W.F. Edwards, Johns Hopkins University Press, 2002. The Cameron citation appears in *Combinatorics: Topics, Techniques, Algorithms,* by Peter J. Cameron, CUP, 1994, section 3.3.