



# THEOREM OF THE DAY

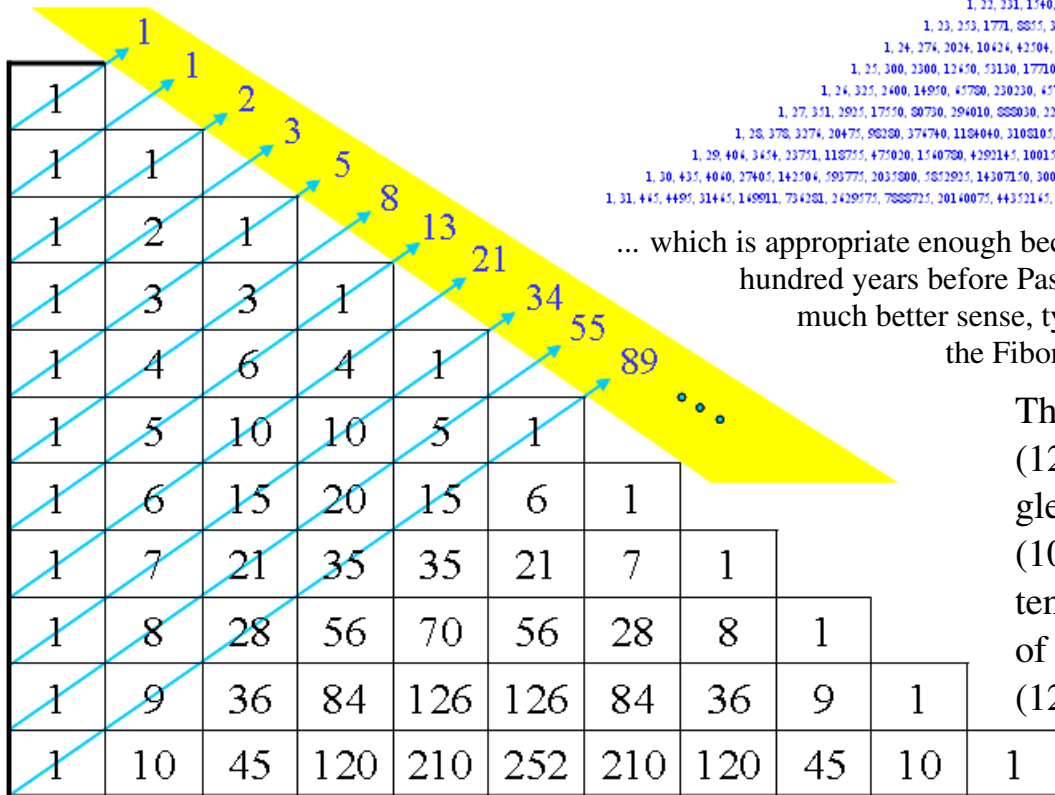
Pascal's Rule For any positive integers  $n$  and  $k$ ,

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

1  
 1 1  
 1 2 1  
 1 3 3 1  
 1 4 6 4 1  
 1 5 10 10 5 1  
 1 6 15 20 15 6 1  
 1 7 21 35 35 21 7 1  
 1 8 28 56 70 56 28 8 1  
 1 9 36 84 126 126 84 36 9 1  
 1 10 45 120 210 252 210 120 45 10 1  
 1 11 55 165 330 462 462 330 165 55 11 1  
 1 12 66 220 495 792 924 792 495 220 66 12 1  
 1 13 78 284 715 1287 1714 1714 1287 715 284 78 13 1  
 1 14 91 344 1001 2002 3003 3432 3003 2002 1001 344 91 14 1  
 1 15 105 455 1345 3003 5005 6435 6435 5005 3003 1345 455 105 15 1  
 1 16 120 560 1820 4348 8008 11440 12870 11440 8008 4348 1820 560 120 16 1  
 1 17 136 680 2380 5985 10948 16320 16320 10948 5985 2380 680 136 17 1  
 1 18 153 816 3042 8124 15848 24310 24310 15848 8124 3042 816 153 18 1  
 1 19 171 969 3674 11428 27132 50388 75582 90378 90378 75582 50388 27132 11428 3674 969 171 19 1  
 1 20 190 1140 4845 15504 38740 77520 125970 147940 147940 125970 77520 38740 15504 4845 1140 190 20 1  
 1 21 210 1330 5985 20349 54284 114280 203490 289890 352714 352714 289890 203490 54284 20349 5985 1330 210 21 1  
 1 22 231 1540 7315 24334 74124 170544 319770 497420 644444 704324 644444 497420 319770 170544 74124 24334 7315 1540 22 21 1  
 1 23 253 1771 8855 33449 100947 241577 490314 817190 1144044 1352078 1352078 1144044 817190 490314 241577 100947 33449 8855 1771 253 23 1  
 1 24 276 2024 10424 42504 134594 344104 735471 1307504 1941254 2494144 2704154 2494144 1941254 1307504 735471 344104 134594 42504 10424 2024 276 24 1  
 1 25 300 2300 12450 53130 177100 480700 1081575 2042975 3248740 4457400 5200300 5200300 4457400 3248740 2042975 1081575 480700 177100 53130 12450 2300 300 25 1  
 1 26 325 2600 14950 63780 230230 637800 1542275 3124550 5311735 7724140 9457700 10400400 9457700 7724140 5311735 3124550 1542275 637800 230230 63780 2600 325 26 1  
 1 27 351 2925 17550 80730 294010 888030 2320075 4484835 8434285 13037895 17383840 20078300 20078300 17383840 13037895 8434285 4484835 2320075 888030 294010 80730 17550 2925 351 27 1  
 1 28 378 3274 20475 98280 374740 1184040 3103105 6064900 13123310 21474130 30421755 37442140 40114400 37442140 30421755 21474130 13123310 6064900 3103105 1184040 374740 98280 20475 3274 378 28 1  
 1 29 404 3654 23751 118755 475020 1540780 4292145 10015005 20030010 34597290 51895935 67843915 77518740 67843915 51895935 34597290 20030010 10015005 4292145 1540780 475020 118755 23751 3654 404 29 1  
 1 30 435 4040 27405 142504 593775 2035800 5832925 14307150 30045015 54427300 84493225 119759850 145422475 155117520 145422475 119759850 84493225 54427300 30045015 14307150 5832925 2035800 593775 142504 27405 4040 435 30 1  
 1 31 465 4495 31445 149811 542831 1498911 734281 2429575 7888725 20140075 44352145 84472315 141120525 204253075 245182325 204253075 141120525 84472315 44352145 20140075 7888725 2429575 734281 1498911 542831 31445 4495 465 31 1

Rows are numbered from zero; cells in each row are likewise numbered from zero. Row zero consists of  $\binom{0}{0} = 1$ ; the  $n$ -th row starts with  $\binom{n}{0} = 1$ .

In words, this is read as “ $n + 1$  choose  $k = n$  choose  $k + n$  choose  $k - 1$ ”, i.e. the number of choices if you must select  $k$  objects from  $n + 1$  is the same as the number of choices if you are selecting from  $n$  objects and have an initial choice of whether to take  $k$  or  $k - 1$ . The rule defines what is usually called Pascal's triangle, presented as shown on the right. However, this is a misnomer for two reasons. Firstly, it isn't a triangle at all, unless font size decreases exponentially with increasing row number; it is more like a Chinese hat!



... which is appropriate enough because, secondly, this triangle and rule were known to the Chinese scholar Jia Xian, six hundred years before Pascal. Aligning the rows of the triangle on the left (as shown on the left) seems to make much better sense, typographically, computationally and combinatorially. A well-known relationship with the Fibonacci series, for instance, becomes immediately apparent as a series of diagonal sums.

The work of Jia Xian has passed to us through the commentary of Yang Hui (1238-1298) and Pascal's triangle is known in China as 'Yang Hui's triangle'. In Iran, it is known as the 'Khayyám triangle' after Omar Khayyám (1048-1131), although it was known to Persian, and Indian, scholars in the tenth century. Peter Cameron cites Robin Wilson as dating Western study of Pascal's triangle as far back as the Majorcan theologian Ramon Llull (1232–1316).

**Web link:** [ptr1.tripod.com](http://ptr1.tripod.com). See the [wikipedia entry](#) on nomenclature.  
**Further reading:** [Pascal's Arithmetical Triangle](#) by A.W.F. Edwards, Johns Hopkins University Press, 2002. The Cameron citation appears in *Combinatorics: Topics, Techniques, Algorithms*, by Peter J. Cameron, CUP, 1994, section 3.3.

