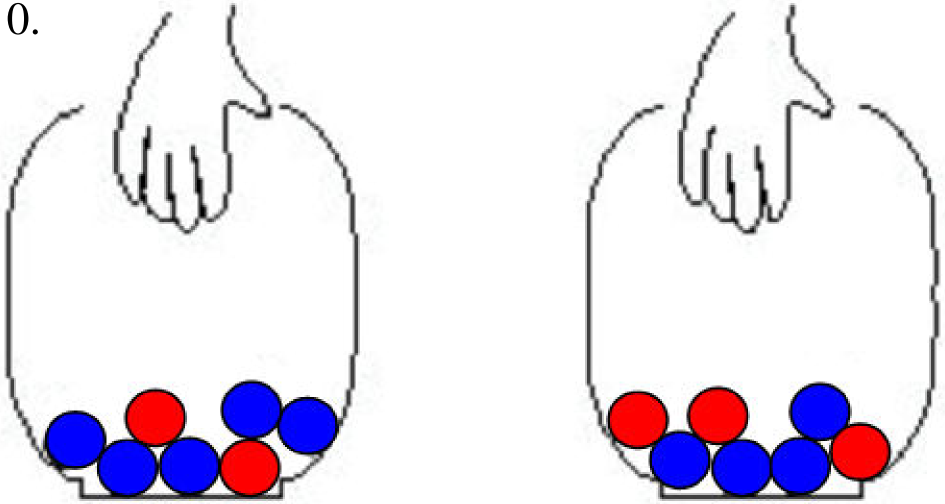
THEOREM OF THE DAY

Fermat's Last Theorem *If x, y, z and n are integers satisfying*

$$x^n + y^n = z^n,$$

then either $n \leq 2$ or $xyz = 0$.



Urn A **Urn B**

It is easy to see that we can assume that all the integers in the theorem are positive. So the following is a legitimate, but totally different, way of asserting the theorem: we take a ball at random from Urn A; then replace it and take a 2nd ball at random. Do the same for Urn B. The probability that both A balls are blue, for the urns shown here, is $\frac{5}{7} \times \frac{5}{7}$. The probability that both B balls are the same colour (both blue or both red) is $(\frac{4}{7})^2 + (\frac{3}{7})^2$. Now the Pythagorean triple $5^2 = 3^2 + 4^2$ tells us that the probabilities are equal: $\frac{25}{49} = \frac{9}{49} + \frac{16}{49}$. What if we choose $n > 2$ balls with replacement? Can we again fill each of the urns with N balls, red and blue, so that taking n with replacement will give equal probabilities? Fermat's Last Theorem says: only in the trivial case where all the balls in Urn A are blue (which includes, vacuously, the possibility that $N = 0$.)

Another, much more profound restatement: if $a^n + b^n$, for $n > 2$ and positive integers a and b, is again an n-th power of an integer then the elliptic curve $y^2 = x(x - a^n)(x + b^n)$, known as the **Frey curve**, cannot be modular (is not a rational map of a modular curve). So it is enough to prove the **Taniyama-Shimura-Weil conjecture:** all rational elliptic curves are modular.

Fermat's innocent statement was famously left unproved when he died in 1665 and was the last of his unproved 'theorems' to be settled true or false, hence the name. The non-modularity of the Frey curve was established in the 1980s by the successive efforts of Gerhard Frey, Jean-Pierre Serre and Ken Ribet. The Taniyama-Shimura-Weil conjecture was at the time thought to be 'inaccessible' but the technical virtuosity (not to mention the courage and stamina) of Andrew Wiles resolved the 'semistable' case, which was enough to settle Fermat's assertion. His work was extended to a full proof of Taniyama-Shimura-Weil during the late 90s by Christophe Breuil, Brian Conrad, Fred Diamond and Richard Taylor.

Web link: math.stanford.edu/~lekheng/flt/kleiner.pdf

Further reading: *Fermat's Last Theorem* by Simon Singh, Fourth Estate Ltd, London, 1997.