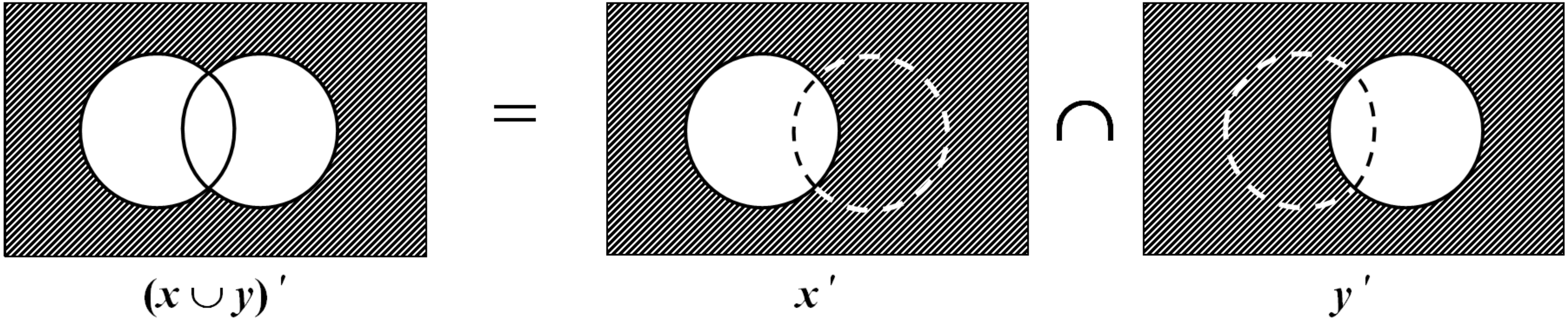
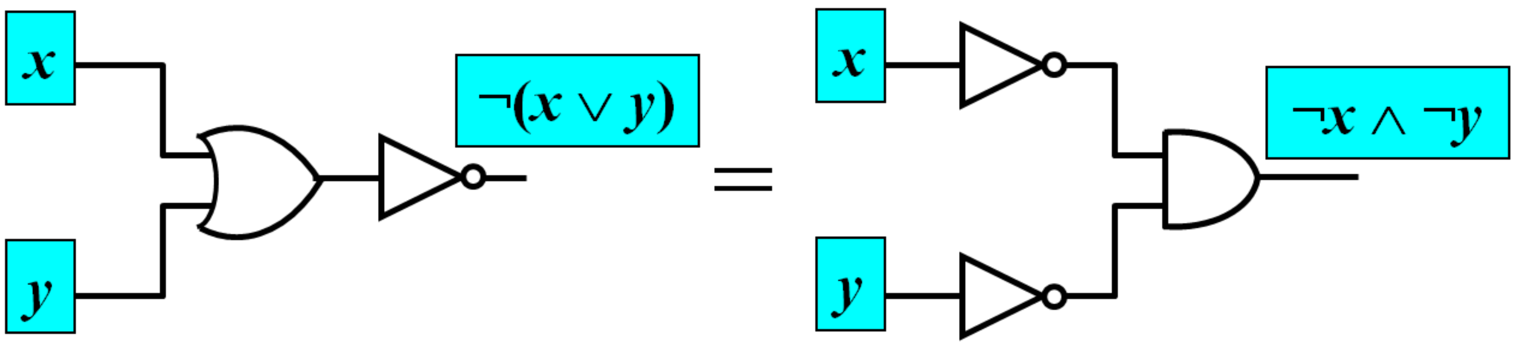
THEOREM OF THE DAY

De Morgan's Laws *If B, a set containing at least two elements, and equipped with the operations +, × and ′ (complement), is a Boolean algebra, then, for any x and y in B,*

$$(x+y)' = x' \times y', \text{ and } (x \times y)' = x' + y'.$$





Truth table verification:

x	y	$\neg$	$(x \vee y)$	$\neg x$	$\wedge$	$\neg y$
0	0	1	0	1	1	1
0	1	0	1	1	0	0
1	0	0	1	0	0	1
1	1	0	1	0	0	0

and $\neg(x \wedge y) = \neg x \vee \neg x$ similarly.

De Morgan's laws are readily derived from the axioms of Boolean algebra and indeed are themselves sometimes treated as axiomatic. They merit special status because of their role in translating between + and ×, which means, for example, that Boolean algebra can be defined entirely in terms of one or the other. This property, entirely absent in the arithmetic of numbers, would seem to mark Boolean algebras as highly specialised creatures, but they are found everywhere from computer circuitry to the sigma-algebras of probability theory. The illustration here shows De Morgan's laws in their set-theoretic, logic circuit guises, and truth table guises.

These laws are named after Augustus De Morgan (1806–1871) as is the building in which resides the London Mathematical Society, whose first president he was.

Web link: www.mathcs.org/analysis/reals/logic/notation.html

Further reading: *Boolean Algebra and Its Applications* by J. Eldon Whitesitt, Dover Publications Inc., 1995.