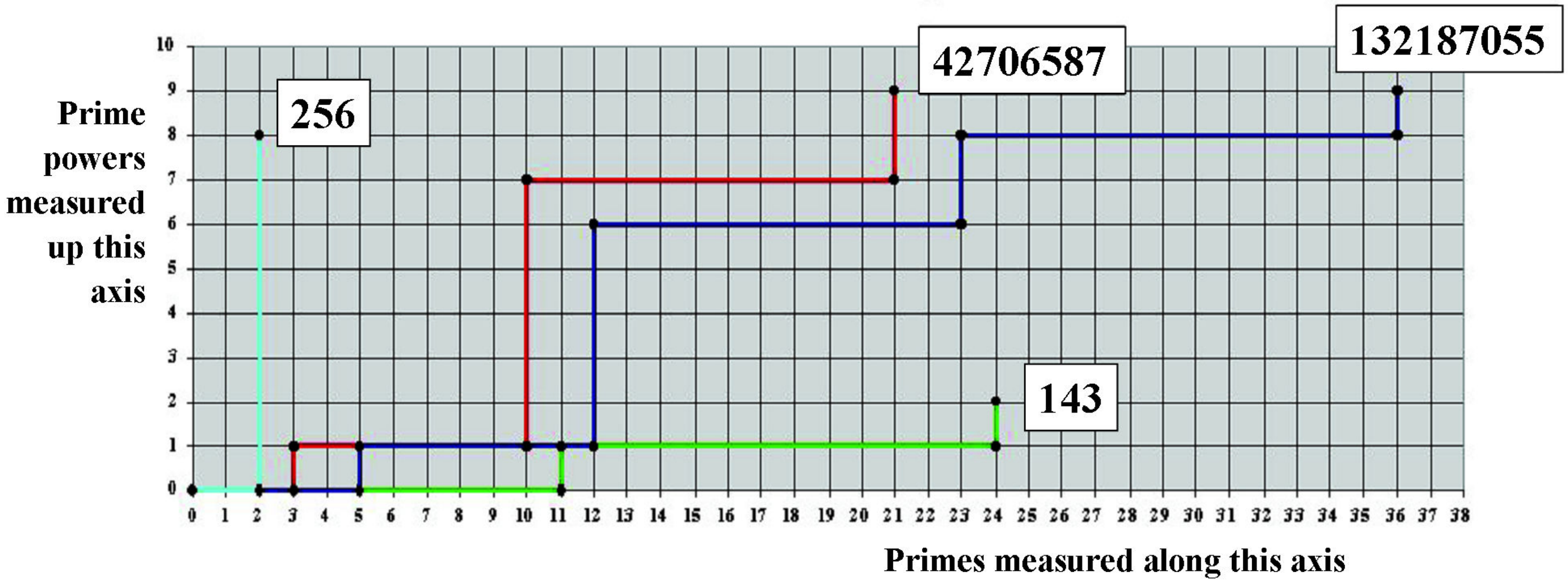
THEOREM OF THE DAY

The Fundamental Theorem of Arithmetic *Every integer greater than one can be expressed uniquely (up to order) as a product of powers of primes.*

Some Fundamental Paths



Every number corresponds to a unique path (which we may call a *fundamental path*) plotted on the xy-plane. Starting at $(0, 0)$ we progress horizontally along the x axis for each prime factor, taking the primes in ascending order. After each prime, we ascend the y axis to represent its power. Thus: $256 = 2^8$ $\quad 143 = 11 \times 13\ (= 11^1.13^1)$ $\quad 42706587 = 3.7^6.11^2$ $\quad 132187055 = 5.7^5.11^2.13.$

The end-points of fundamental paths may be called *fundamental points*. Some well-known conjectures about primes can be expressed in terms of questions about fundamental points: Goldbach's conjecture that every even integer greater than 2 is the sum of two primes could be solved if we knew which points on the line $y = 2$ were fundamental (the line for 143 shows that 24=11+13, for instance.) The 'twin primes conjecture', that there are infinitely many primes separated by 2 is a question about fundamental points on the line $y = 1$ (for example, $(3, 1)$ and $(5, 1)$ are fundamental points.)

Euclid, **Book 7, Proposition 30** of the *Elements*, proves that if a prime divides the product of two numbers then it must divide one or both of these numbers. This provided a key ingredient of the Fundamental Theorem which then had to wait more than two thousand years before it was finally established as the bedrock of modern number theory by Gauss, in 1798, in his *Disquisitiones Arithmeticae*.

Web link: www.dpmms.cam.ac.uk/~wtg10/FTA.html

Further reading: *Elementary Number Theory* by Gareth Jones and Mary Jones, Springer, Berlin, 1998.