1 Example Class (23rd May 2019, 16.15-17.30)

Question 1. We consider the KNAPSACK-problem, where we are given n items each of which comes with an integral weight $w_i > 0$ and integral value $v_i > 0$. The knapsack has capacity C and the goal is to fill the knapsack so as to maximise its total value. Further, we denote by $OPT \leq \max\{C, \sum_{i=1}^{n} v_i\}$ the value obtained by an optimal solution. As a side remark, we may assume that for all items $1 \leq i \leq n$, $w_i \leq C$.

1. Design a simple ("the arguably most natural") greedy algorithm and analyse its approximation ratio.

2. Consider a modified greedy algorithm, which takes the better solution of the algorithm from Part 1 and item with the largest value. Prove that the approximation ratio of this new algorithm is two.
   Hint: One way of establishing this approximation ratio involves the following steps:

   (a) First define a LP relaxation of the knapsack problem.

   (b) Find a mathematical formula for the optimum solution of the LP relaxation.

   (c) Use the result from (b) to argue that the solution of the algorithm is within a factor of two of the optimum LP solution.

3. Consider the dynamic programming technique. Derive two algorithms based on this technique that achieve a runtime of $O(n \cdot C)$ and $O(n \cdot OPT)$, respectively.
   (Question: Why are both of these algorithms *not* polynomial-time?)

4. Design a FPTAS based on the dynamic programming algorithm with runtime $O(n^3/\epsilon)$.
   Hint: Round down all values so that they will lie in a suitable range (depending of course on $\epsilon > 0$!).