Deep Learning for Natural Language Processing

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3. Training and Optimization

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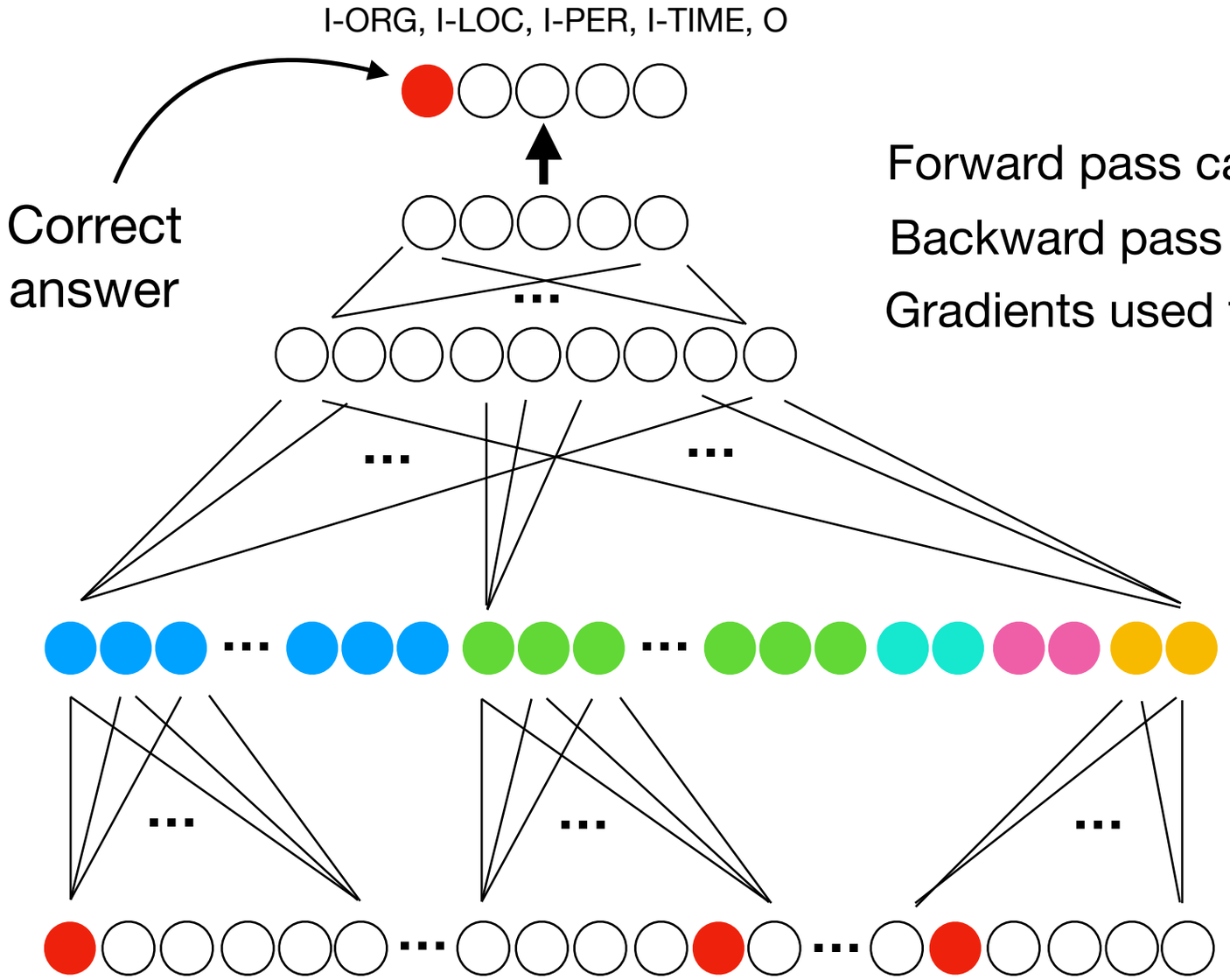
Supervised Training



Forward pass calculates loss

Backward pass (backprop) calculates gradients

Gradients used for parameter updates

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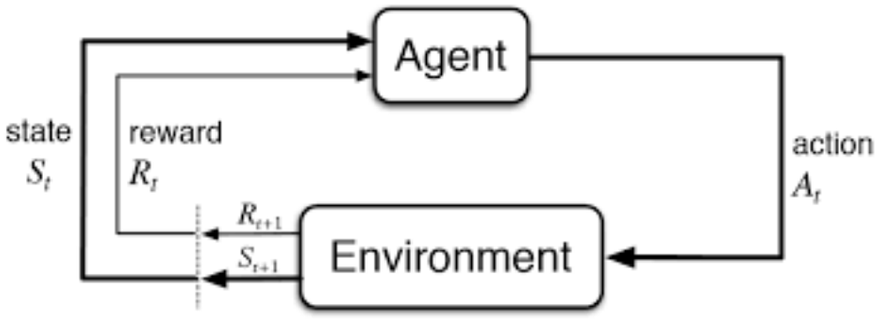
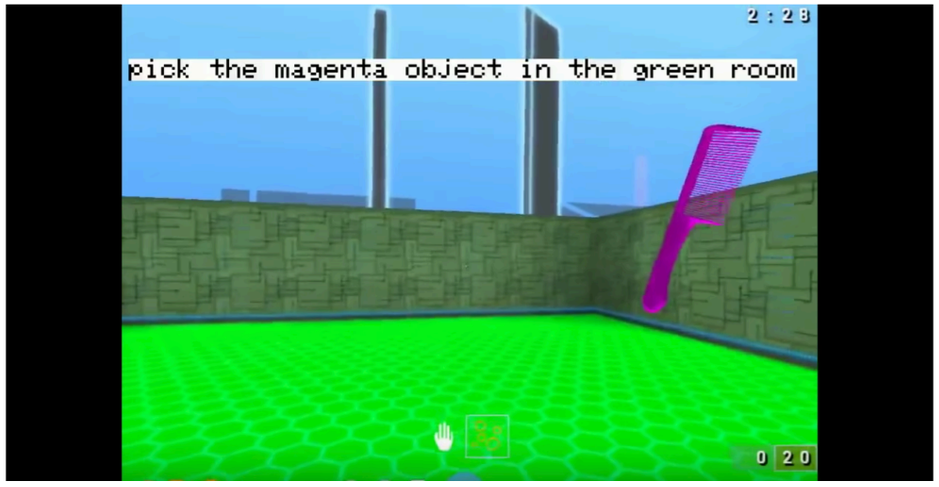
An Aside on Reinforcement Learning



- We're seeing more work on RL in NLP (e.g. dialog)



- RL problems typically have "trial-and-error search and delayed reward"

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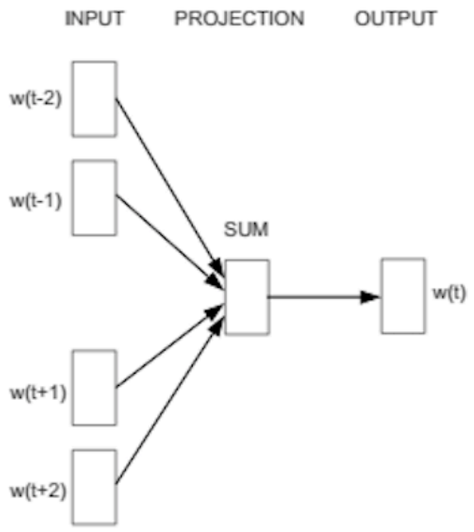
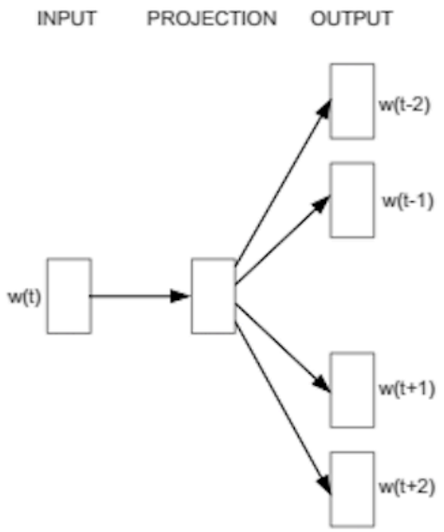
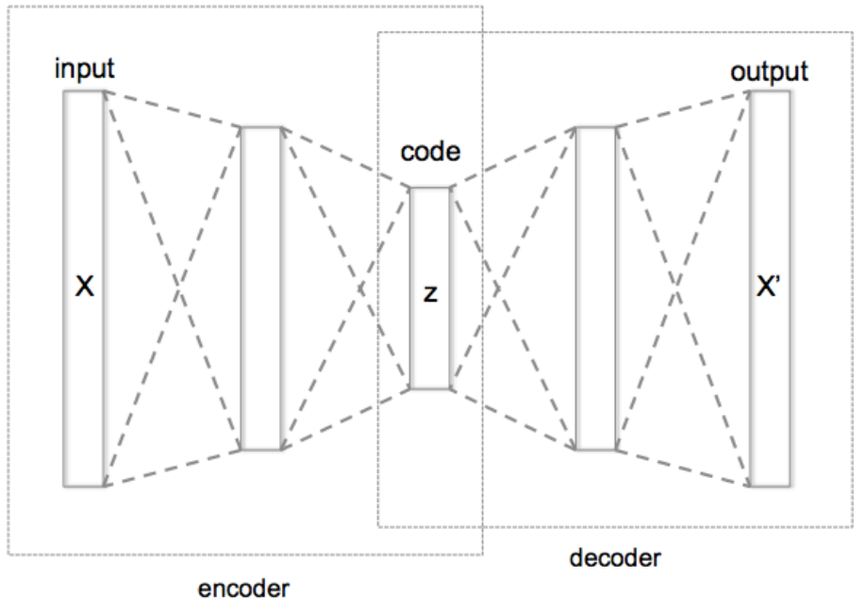
An Aside on Unsupervised Learning



CBOW



Skip-gram



- Important in deriving general-purpose representations

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The Loss

- For language tasks, typically negative log-likelihood

Ian Botham *bowled* for Somerset	**O**
Botham bowled *for* Somerset cricket	**O**
bowled for *Somerset* cricket club	**I-ORG**
for Somerset *cricket* club in	**I-ORG**
Somerset cricket *club* in the	**I-ORG**
cricket club *in* the 1980s	**O**

<SOS> **The**
<SOS> The **dog**
<SOS> The dog **sleeps**
<SOS> The dog sleeps **before**
<SOS> The dog sleeps before **chasing**
<SOS> The dog sleeps before chasing **the**
<SOS> The dog sleeps before chasing the **cat**

$$-\sum_{i=1}^{m} \log P(y^{(i)}|\mathbf{x}^{(i)})$$

$$-\sum_{i=1}^{m} \sum_{j=1}^{|\mathbf{y}^{(i)}|} \log P(\mathbf{y}_j^{(i)}|\mathbf{y}_{<j}^{(i)})$$

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Empirical Risk Minimization

- What we'd ideally like to minimize:

$$J^*(\theta) = \mathbb{E}_{(\mathbf{x},y)\sim p_{\text{data}}} L(f(\mathbf{x};\theta), y)$$

- What we minimize in practice:

$$J(\theta) = \mathbb{E}_{(\mathbf{x},y)\sim \hat{p}_{\text{data}}} L(f(\mathbf{x};\theta), y)$$
$$= \frac{1}{m}\sum_{i=1}^{m} L(f(\mathbf{x}^{(i)};\theta), y^{(i)})$$

θ are the parameters, $\{(\mathbf{x}^{(i)}, y^{(i)})\}_{i=1}^{m}$ the training data, L the loss

Regularization

- *Overfitting* is a major problem with powerful deep models (esp. when data is scarce)

- We'd like our models to perform well on unseen test data (possibly at the expense of training set performance)

- Requiring good *generalization* is what separates machine learning from pure optimization problems

It's All About the Gradients

$$J(\theta) = \mathbb{E}_{\mathbf{x},y\sim\hat{p}_{\text{data}}} L(\mathbf{x}, y, \theta) = \frac{1}{m}\sum_{i=1}^{m} L(\mathbf{x}^{(i)}, y^{(i)}, \theta)$$

$$\text{where } L(\mathbf{x}, y, \theta) = -\log p(y|\mathbf{x};\theta)$$

$$\nabla_\theta J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \nabla_\theta L(\mathbf{x}^{(i)}, y^{(i)}, \theta)$$

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It's All About the Gradients

$$\nabla_\theta J(\theta) = \frac{1}{m}\sum_{i=1}^{m} \nabla_\theta L(\mathbf{x}^{(i)}, y^{(i)}, \theta)$$

$$\nabla_\theta L(\mathbf{x}^{(i)}, y^{(i)}, \theta) = \left\{ \frac{\partial}{\partial \theta_j} L(\mathbf{x}^{(i)}, y^{(i)}, \theta) \right\}_j$$

$$= \left\langle \frac{\partial L}{\partial \theta_1}, \frac{\partial L}{\partial \theta_2}, \frac{\partial L}{\partial \theta_3}, \ldots, \frac{\partial L}{\partial \theta_{|\theta|}} \right\rangle$$

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Gradient-based Optimization

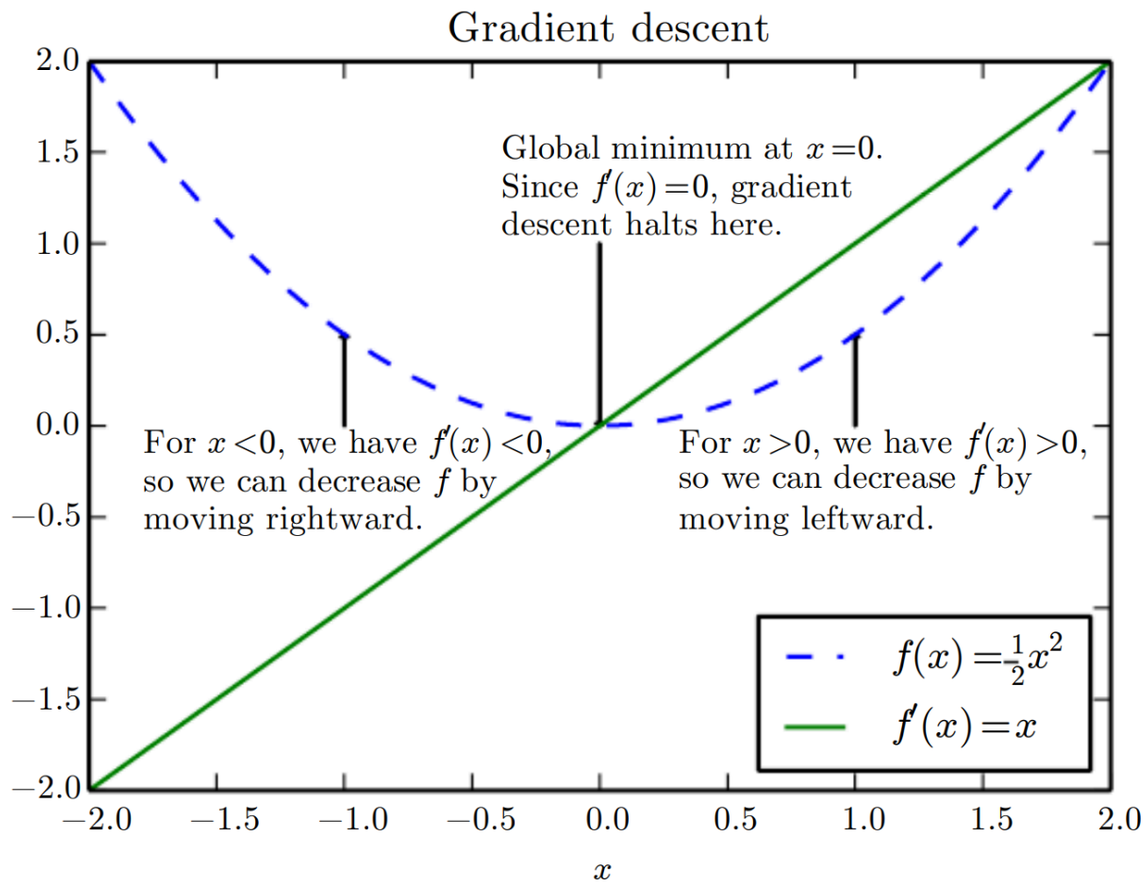

Figure 4.1: An illustration of how the derivatives of a function can be used to follow the function downhill to a minimum. This technique is called *gradient descent*.

p.83 of the Deep Learning textbook

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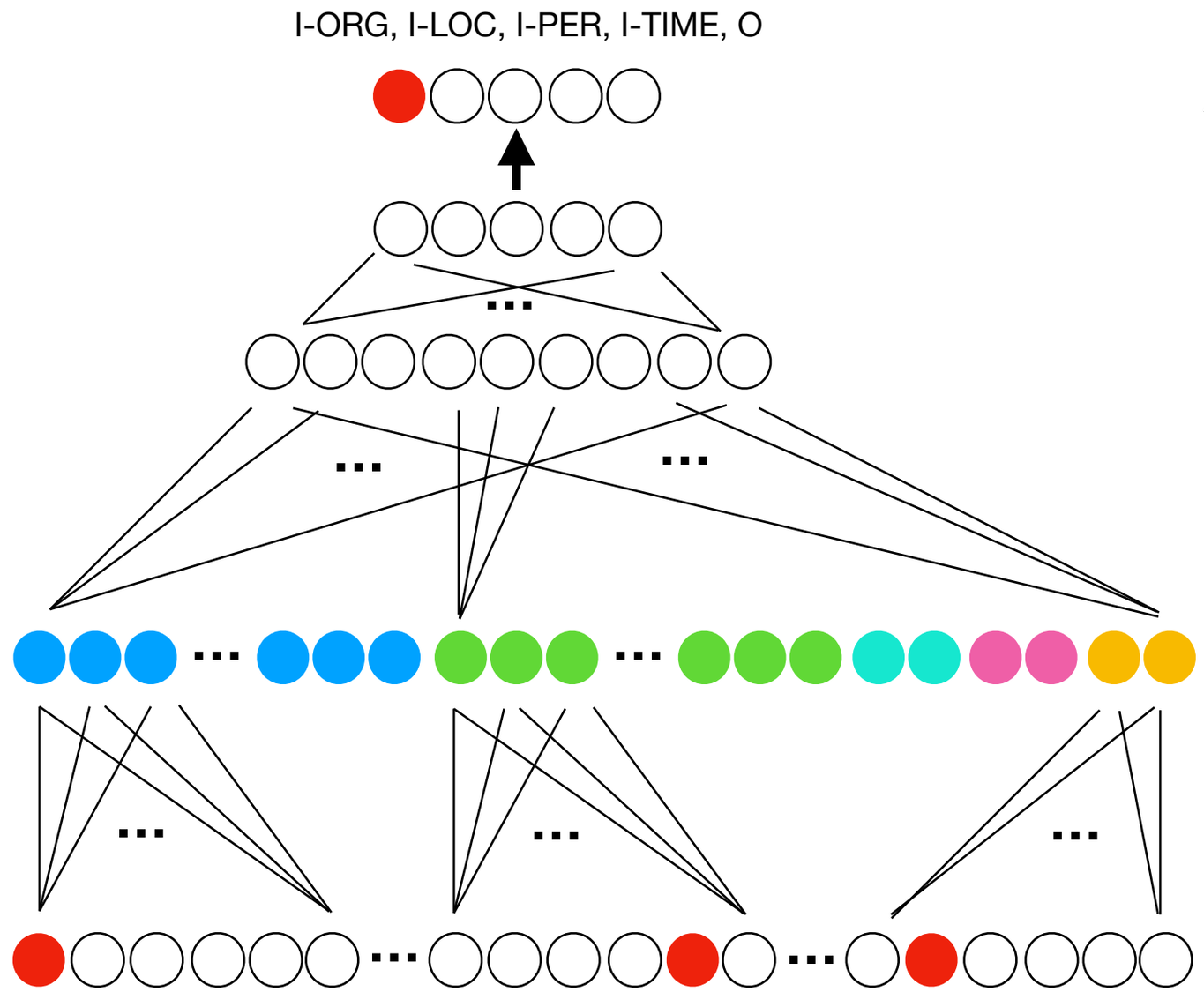
Lots of Parameters



All need to be learned (or updated)

$$\begin{pmatrix} 0.1063 & -0.0214 & -0.4013 & 0.9999 & -0.1109 \\ 0.0211 & 0.0086 & 0.0102 & 0.2311 & 0.0876 \\ & & \dots & & \\ -0.1176 & 1.0012 & 1.2150 & 0.7610 & 1.1211 \end{pmatrix}$$

$$\begin{pmatrix} 0.1043 & -0.0135 & -0.1113 & \dots & 0.1209 \end{pmatrix}$$

$$\begin{pmatrix} -0.1063 & -0.0234 & -0.4013 & \dots & -0.1109 \\ 0.0011 & 0.1986 & -0.0122 & \dots & 0.0876 \\ & & \dots & & \\ -0.9876 & 1.0092 & -1.9650 & \dots & 1.1200 \end{pmatrix}$$

$$\begin{pmatrix} -0.0537 & 0.0263 & -0.0003 & \dots & 0.0099 \\ 0.0000 & 0.0986 & -0.1023 & \dots & 0.0654 \\ & & \dots & & \\ 0.1020 & -0.8750 & 0.0000 & \dots & -0.1200 \end{pmatrix}$$

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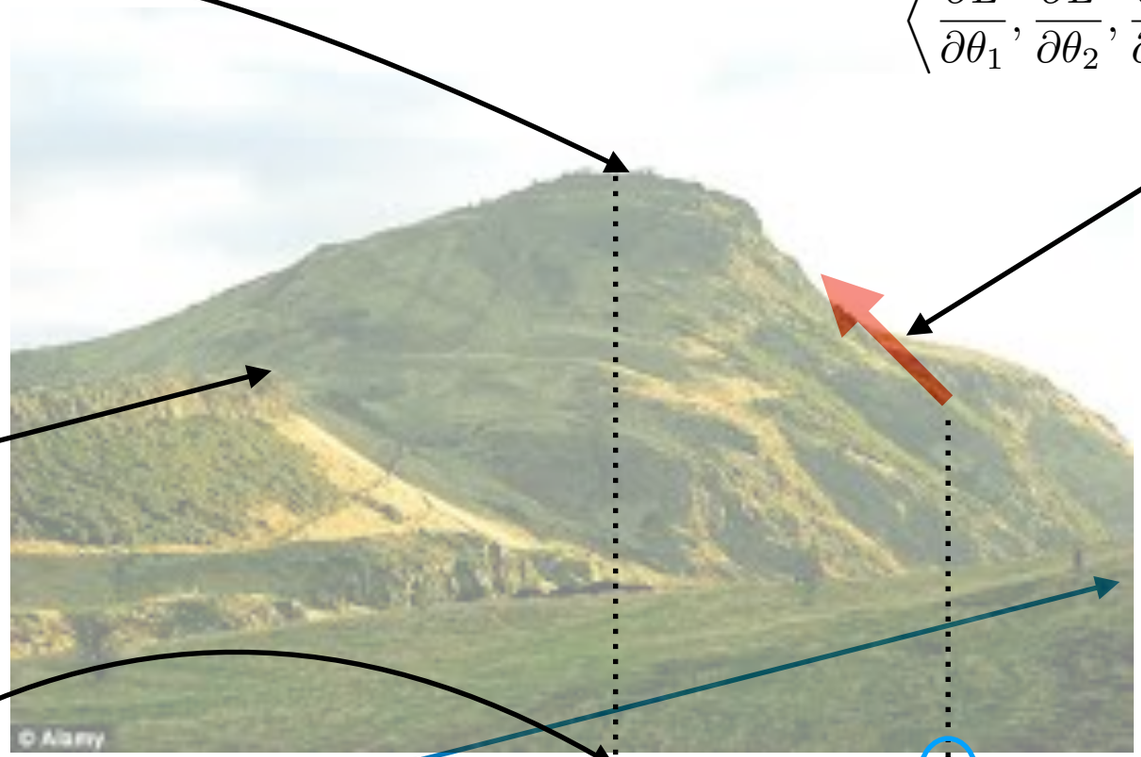
Gradient-based Optimization



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Gradient-based Optimization is Hard

- Lots of local maxima for deep networks
- In particular lots of saddle points
- Exploding and vanishing gradients

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Approximate Minimization

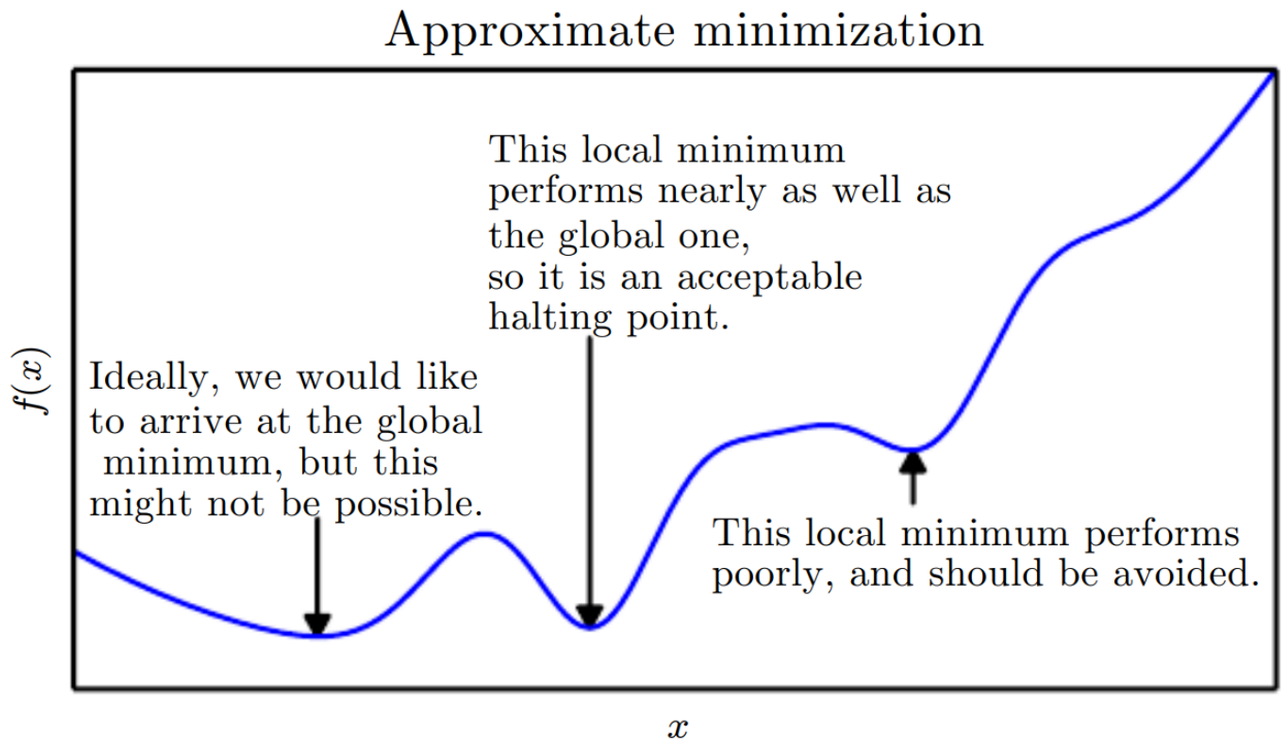

Figure 4.3: Optimization algorithms may fail to find a global minimum when there are multiple local minima or plateaus present. In the context of deep learning, we generally accept such solutions even though they are not truly minimal, so long as they correspond to significantly low values of the cost function.

p.85 of the Deep Learning textbook

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Calculating Gradients

- Back-propagation is an efficient algorithm for computing gradients in a neural network: "*Back-propagation is an algorithm that computes the chain rule [of calculus], with a specific order of operations that is highly efficient.*"

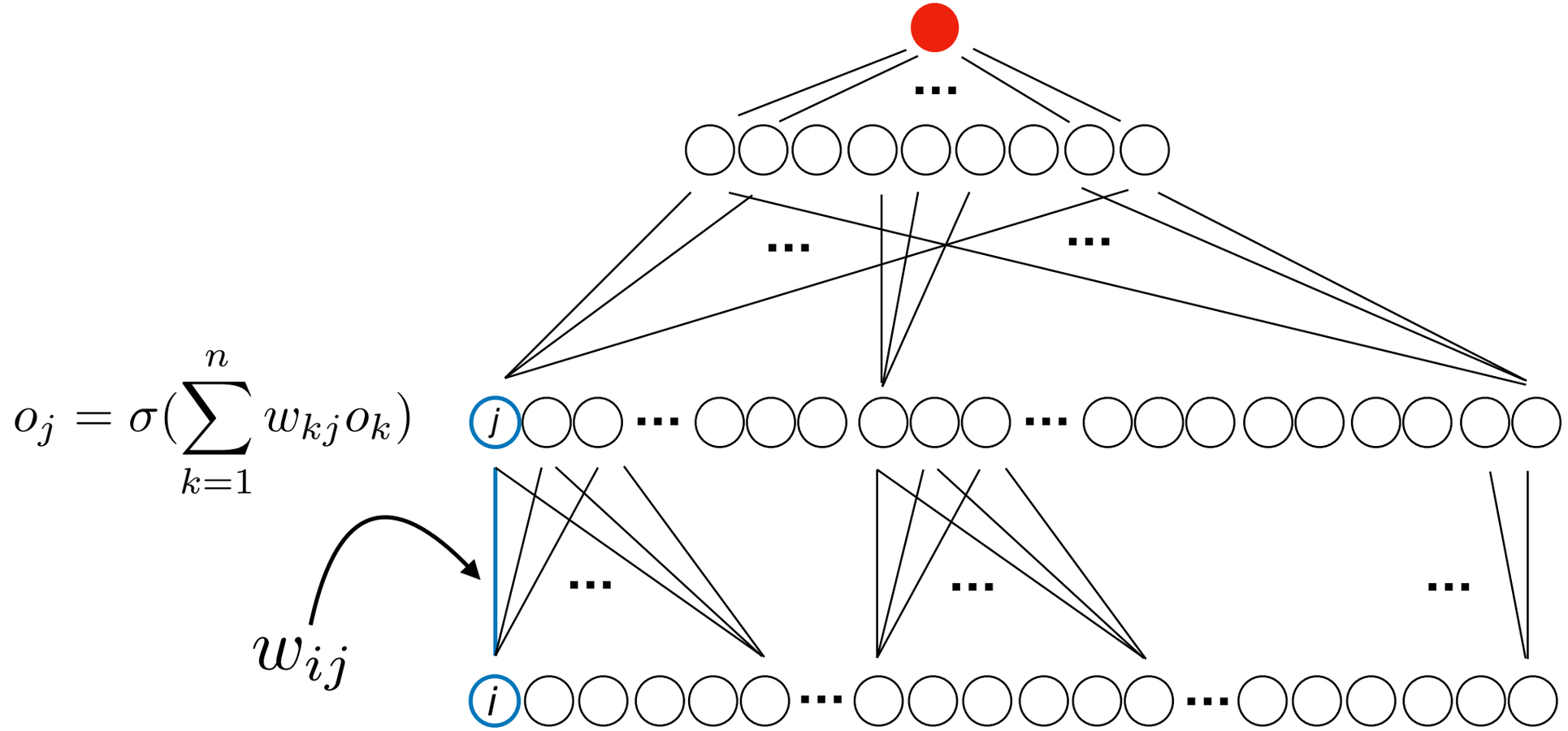
- Another algorithm, such as SGD, uses those gradients to perform learning (optimization)

- Back-propagation is not specific to MLPs, but in principle can compute derivatives of any function

p.198 of the Deep Learning textbook

Backprop for MLPs

$L = \frac{1}{2}(t - y)^2$ where t is the target output, y is the actual output



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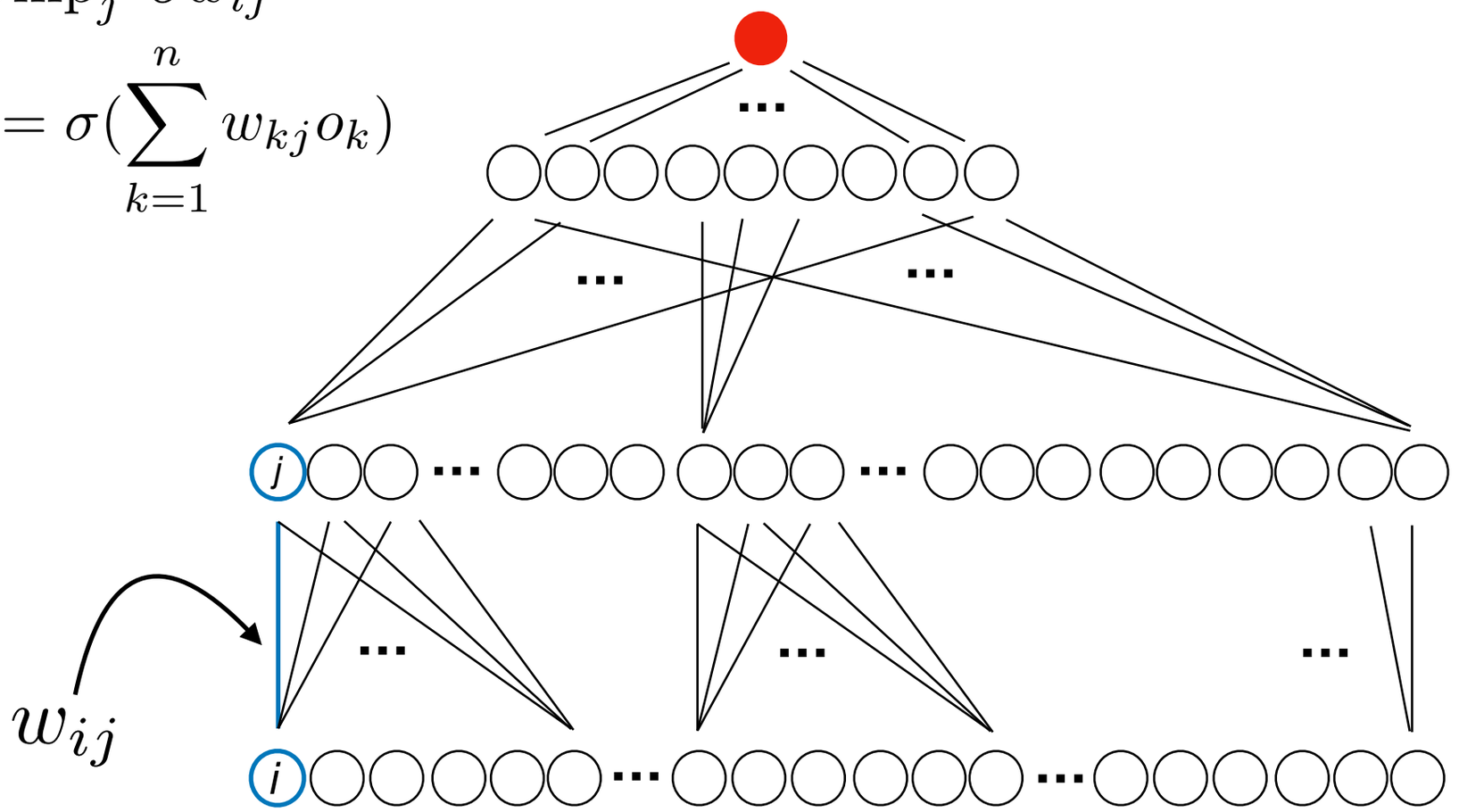
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial o_j} \frac{\partial o_j}{\partial \text{inp}_j} \frac{\partial \text{inp}_j}{\partial w_{ij}}$$

$$o_j = \sigma(\text{inp}_j) = \sigma(\sum_{k=1}^{n} w_{kj} o_k)$$

$$L = \frac{1}{2}(t - y)^2$$



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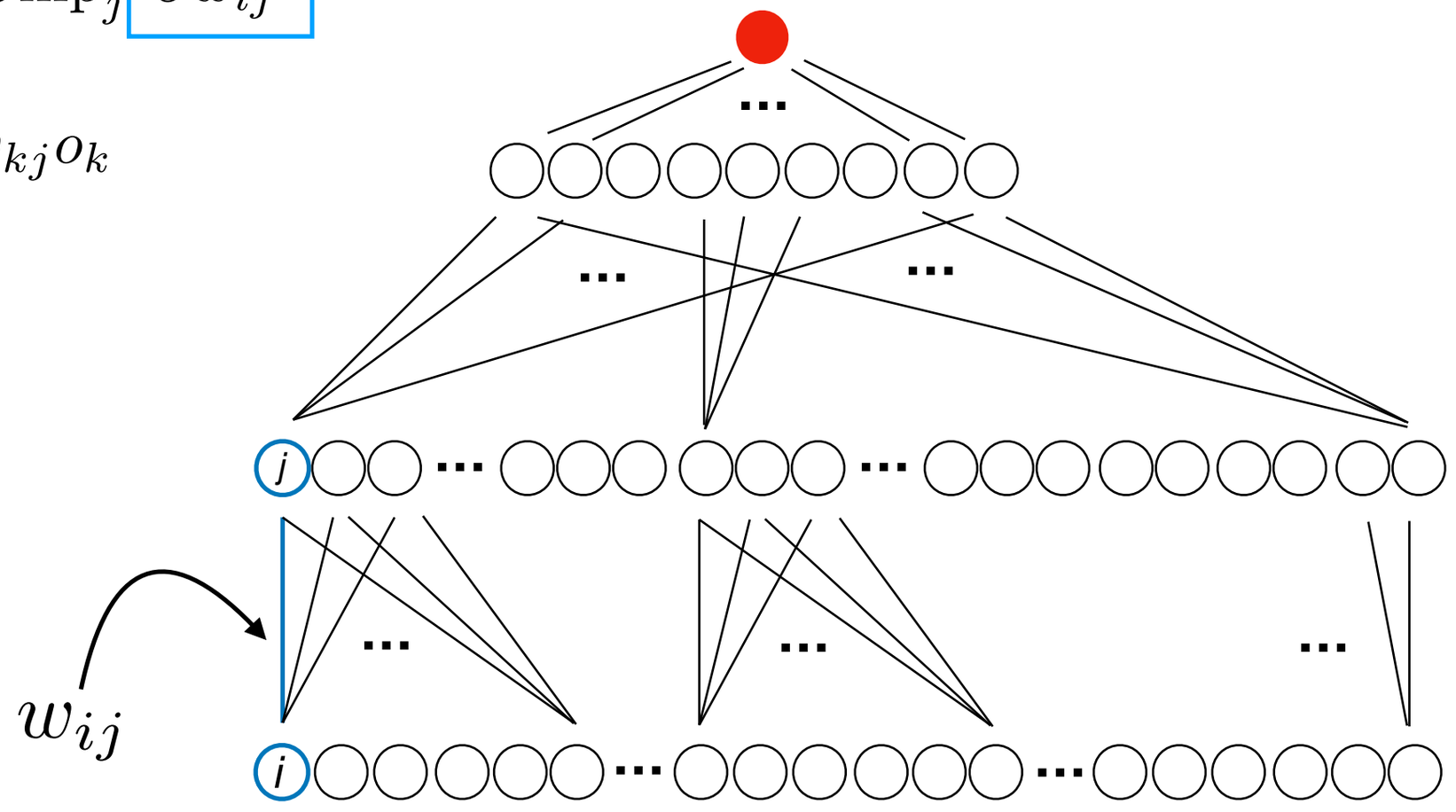
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial o_j}\frac{\partial o_j}{\partial \text{inp}_j}\boxed{\frac{\partial \text{inp}_j}{\partial w_{ij}}}$$

$$\text{inp}_j = \sum_{k=1}^{n} w_{kj} o_k$$

$$\frac{\partial \text{inp}_j}{\partial w_{ij}} = o_i$$

$$L = \frac{1}{2}(t-y)^2$$



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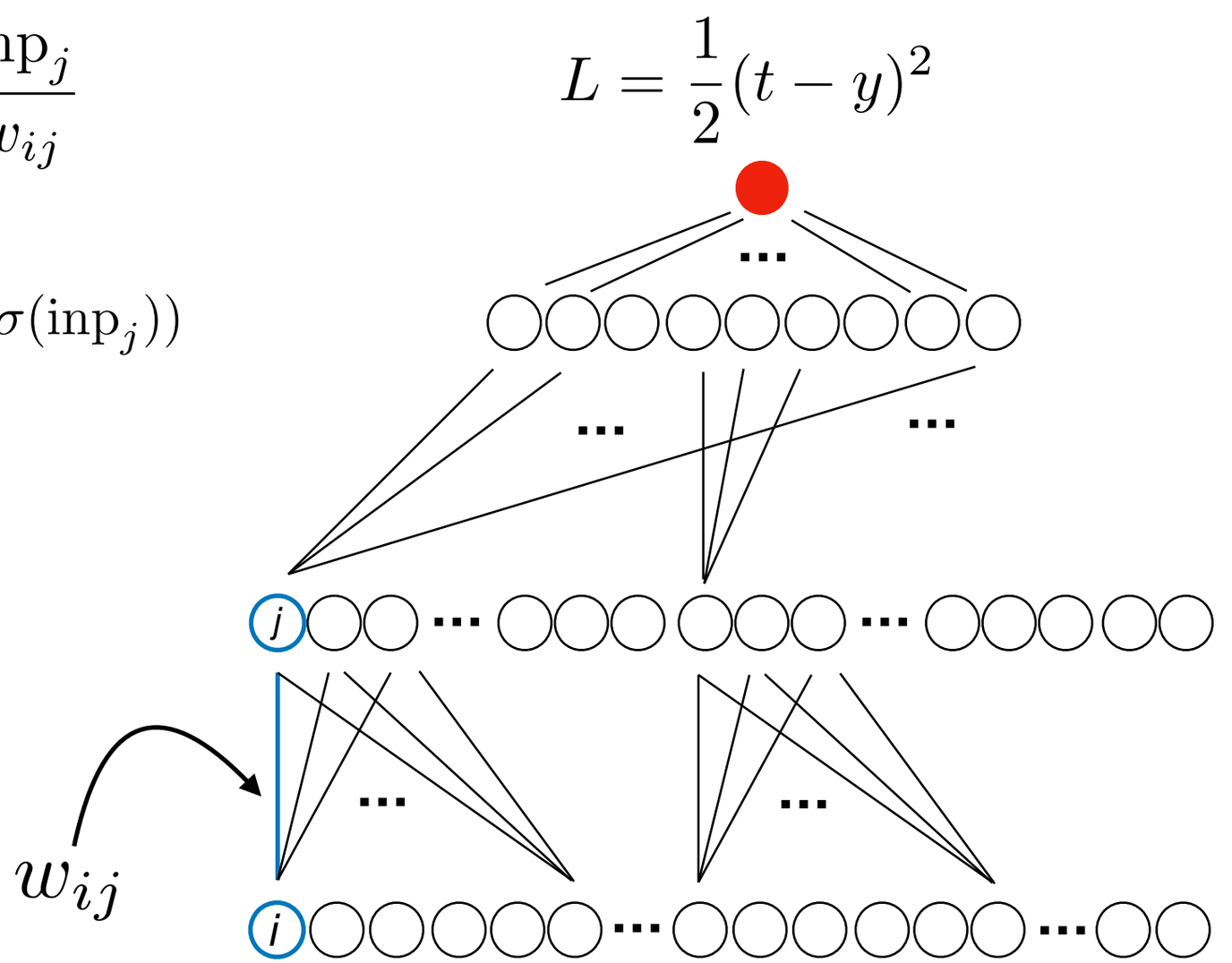
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial o_j}\frac{\partial o_j}{\partial \text{inp}_j}\frac{\partial \text{inp}_j}{\partial w_{ij}}$$

$$\frac{\partial \sigma(\text{inp}_j)}{\partial \text{inp}_j} = \sigma(\text{inp}_j)(1-\sigma(\text{inp}_j))$$
$$= o_j(1-o_j)$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{d\sigma(z)}{dz} = \sigma(z)(1-\sigma(z))$$



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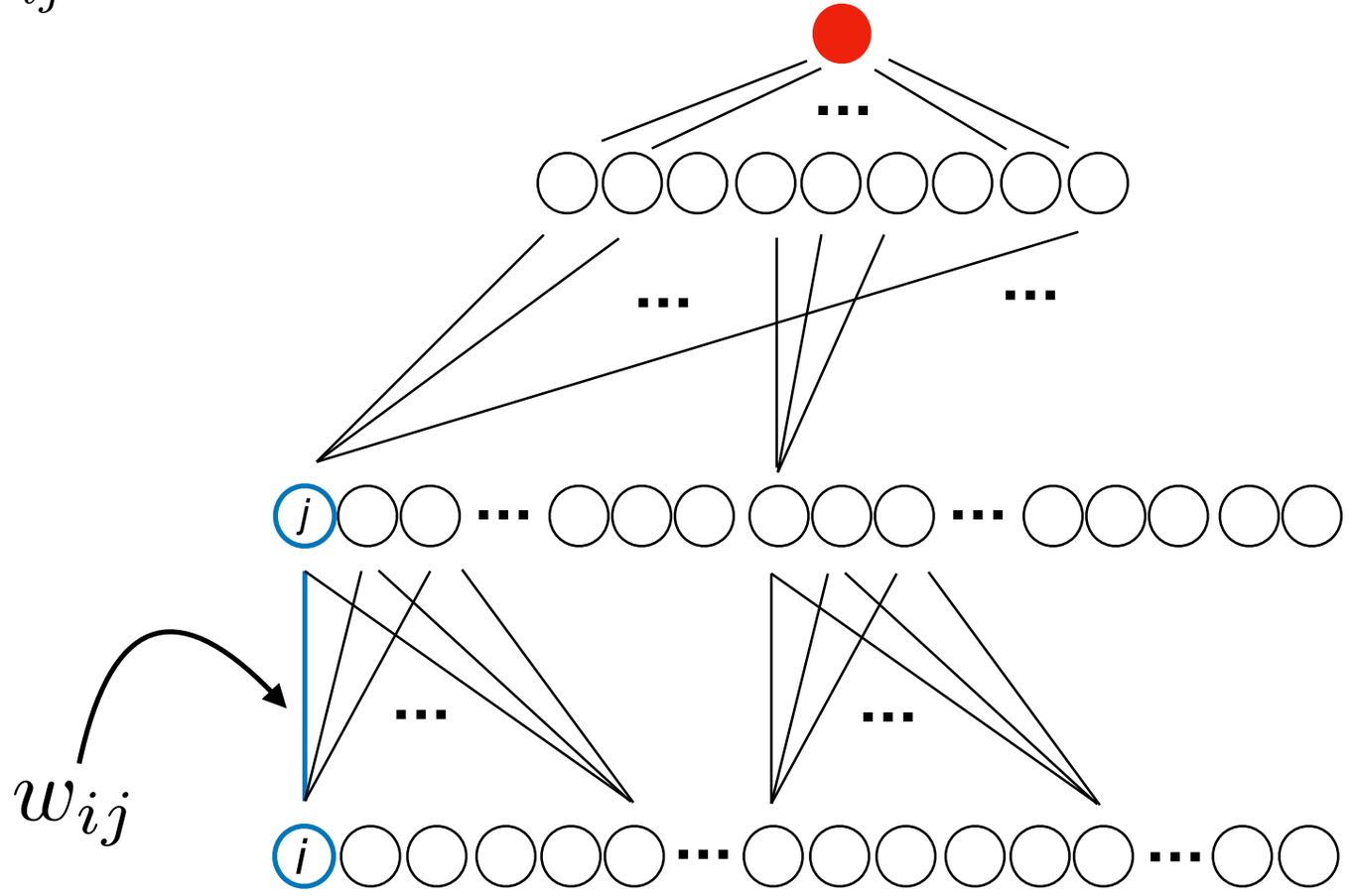
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \boxed{\frac{\partial L}{\partial o_j}} \frac{\partial o_j}{\partial \text{inp}_j} \frac{\partial \text{inp}_j}{\partial w_{ij}}$$

$$L = \frac{1}{2}(t - y)^2$$



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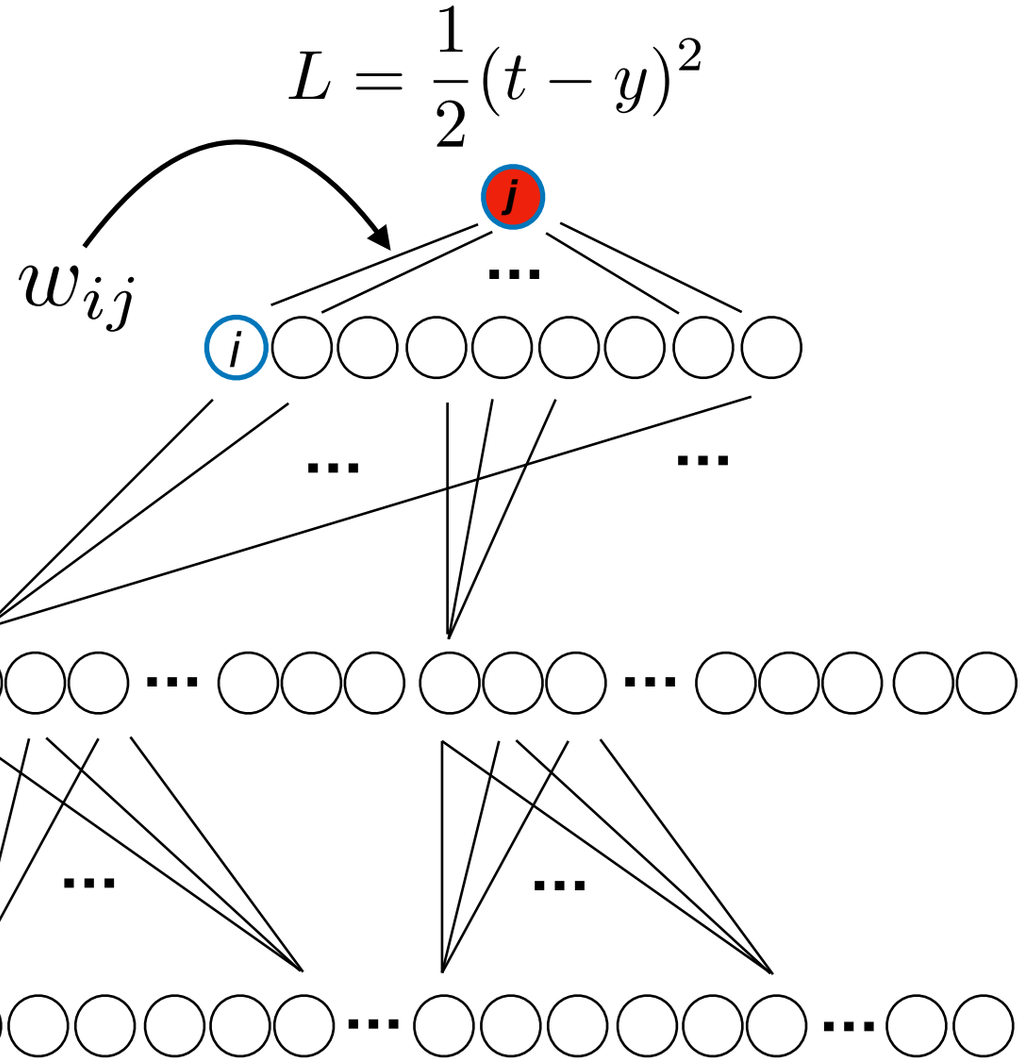
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \boxed{\frac{\partial L}{\partial o_j}} \frac{\partial o_j}{\partial \text{inp}_j} \frac{\partial \text{inp}_j}{\partial w_{ij}}$$

$$o_j = y$$

$$\frac{\partial L}{\partial y} = y - t$$

$$L = \frac{1}{2}(t-y)^2$$



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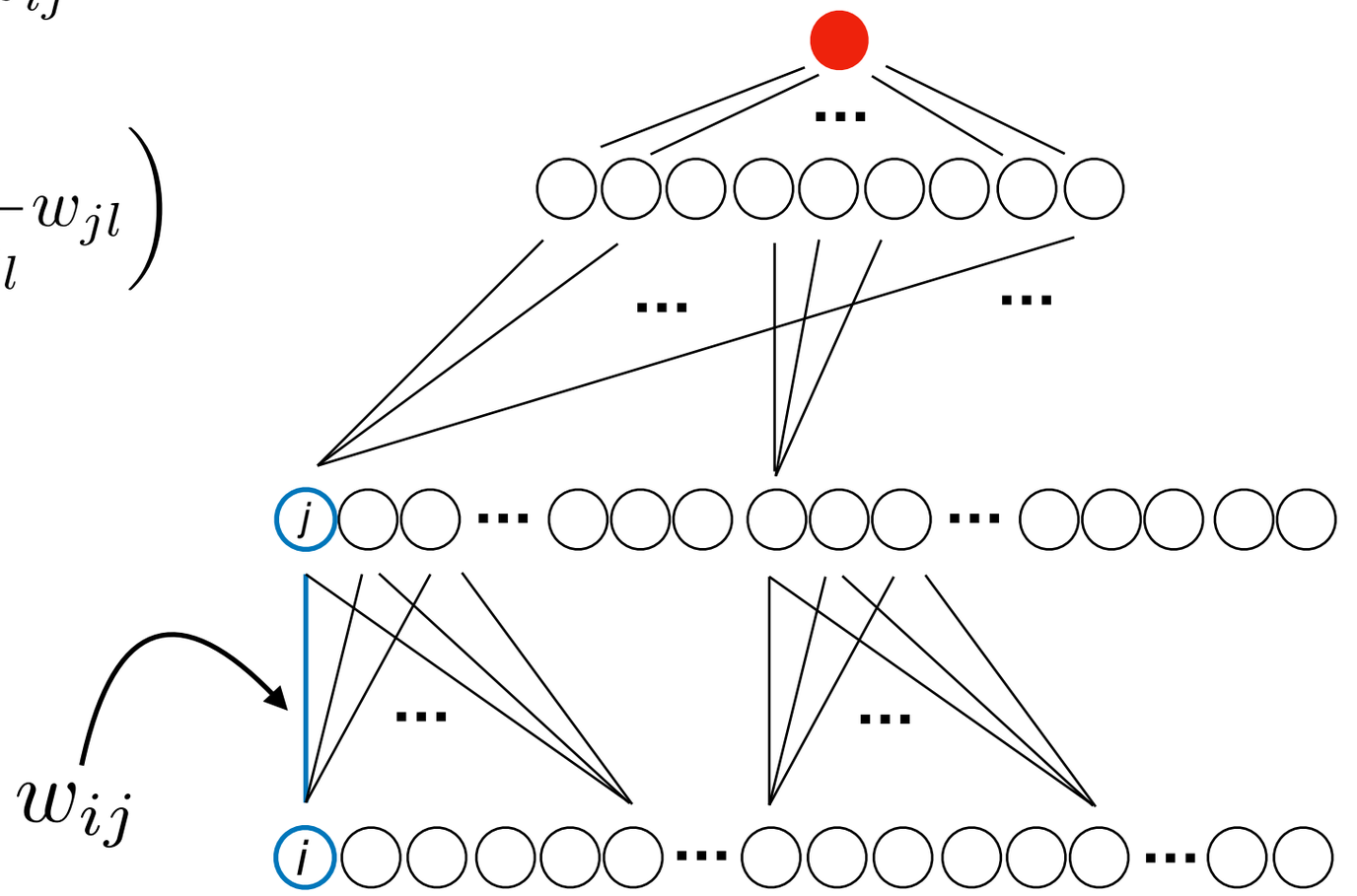
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = \boxed{\frac{\partial L}{\partial o_j}} \frac{\partial o_j}{\partial \text{inp}_j} \frac{\partial \text{inp}_j}{\partial w_{ij}}$$

$$\frac{\partial L}{\partial o_j} = \sum_l \left(\frac{\partial L}{\partial o_l} \frac{\partial o_l}{\partial \text{inp}_l} w_{jl} \right)$$

$$L = \frac{1}{2}(t - y)^2$$



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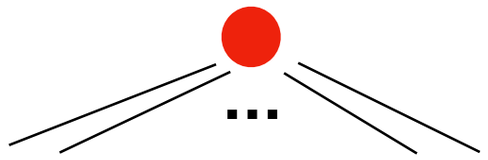
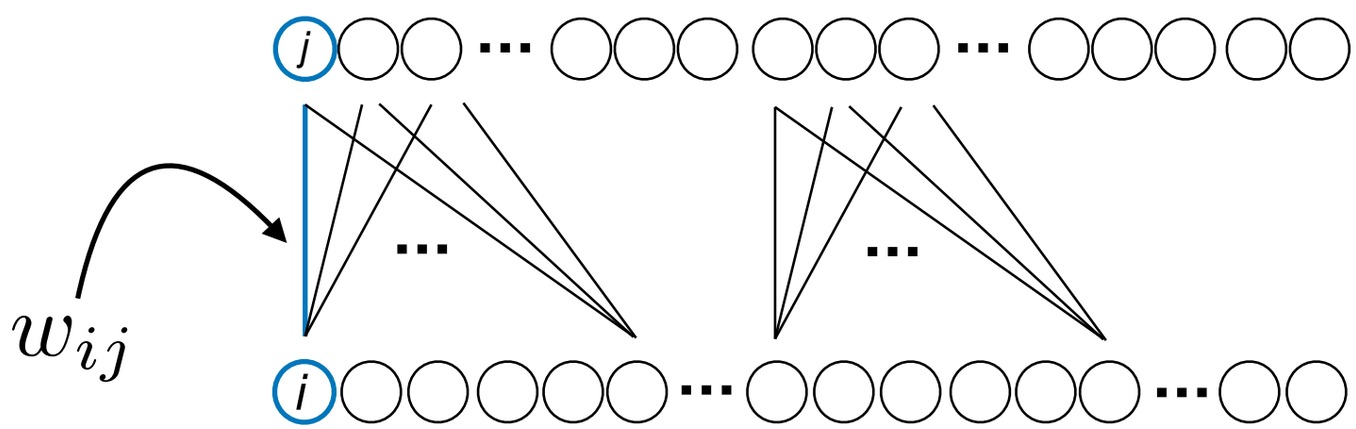
Backprop for MLPs

$$\frac{\partial L}{\partial w_{ij}} = o_i \delta_j$$

$$L = \frac{1}{2}(t - y)^2$$



$$\delta_j = \begin{cases} (o_j - t) o_j (1 - o_j) & \text{if } j \text{ is an output neuron} \\ \left(\sum_l \delta_l w_{jl}\right) o_j (1 - o_j) & \text{otherwise} \end{cases}$$



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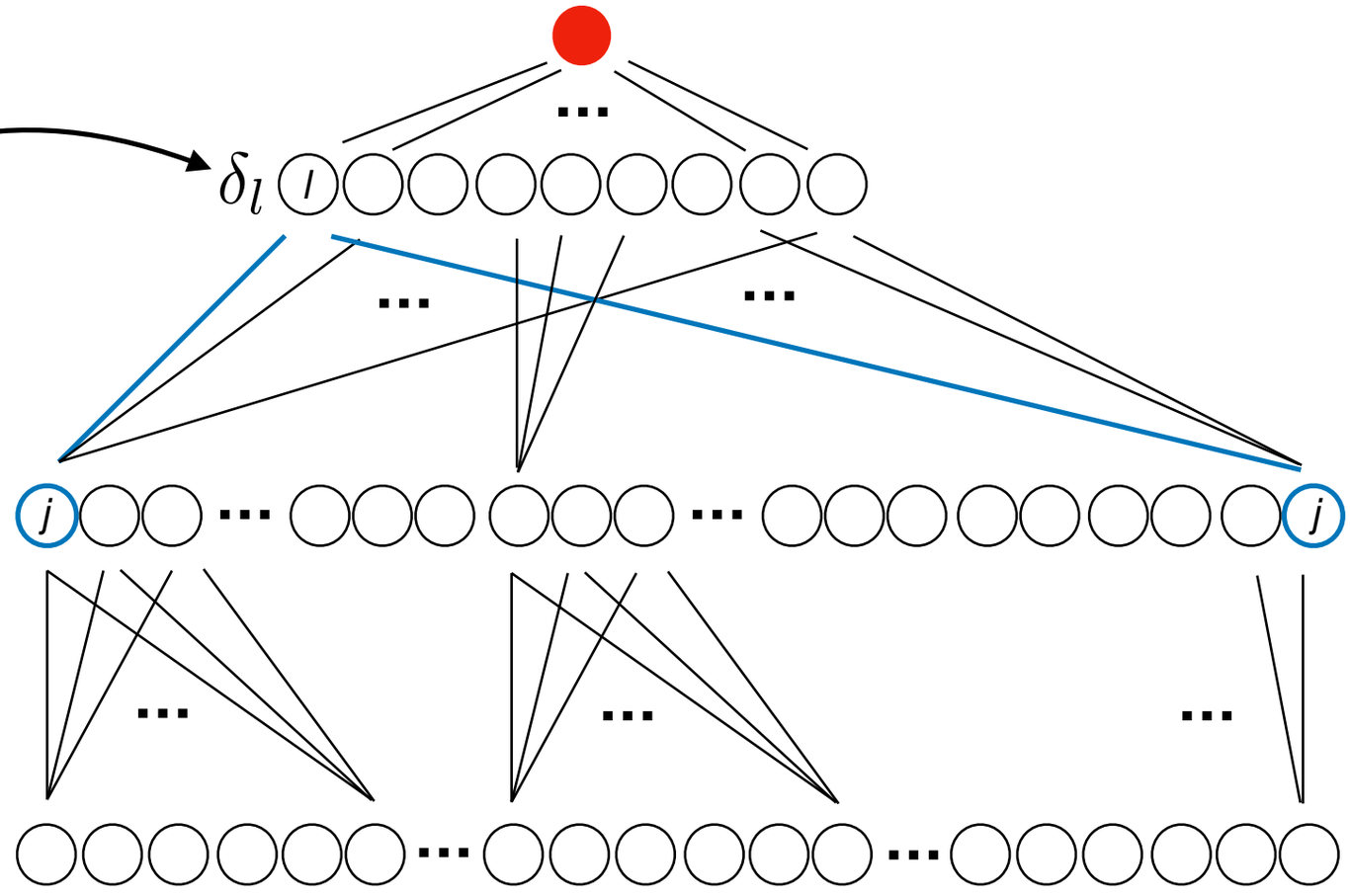
Backprop for MLPs

$$\delta_j = \begin{cases} (o_j - t_j) o_j (1 - o_j) & \text{if } j \text{ is an output neuron} \\ \left(\sum_l \delta_l w_{jl}\right) o_j (1 - o_j) & \text{otherwise} \end{cases}$$

Only compute this once and store in a DP table



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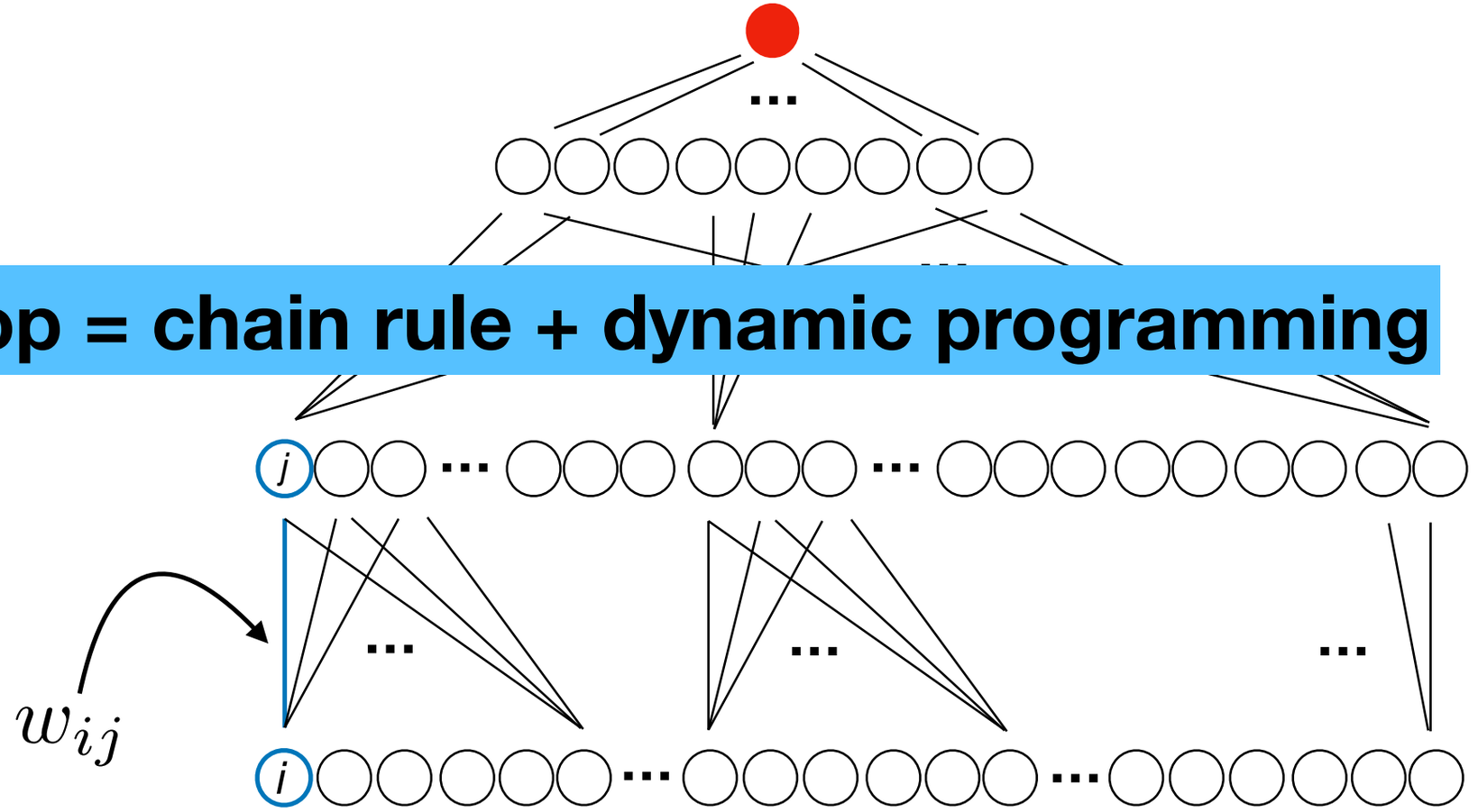
Backprop for MLPs

$$\delta_j = \begin{cases} (o_j - t_j) o_j (1 - o_j) & \text{if } j \text{ is an output neuron} \\ \left(\sum_l \delta_l w_{jl}\right) o_j (1 - o_j) & \text{otherwise} \end{cases}$$

Backprop = chain rule + dynamic programming



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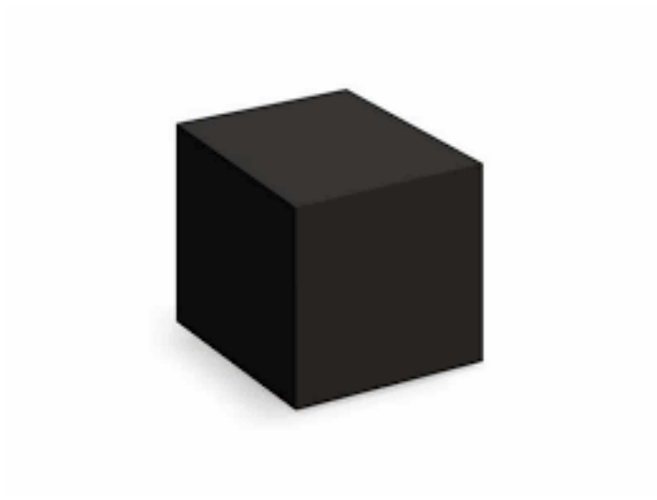
Backprop in Tensorflow



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Estimating Gradients

$$g = \nabla_\theta J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta L(\mathbf{x}^{(i)}, y^{(i)}, \theta)$$ Batch learning

$$\hat{g} = \frac{1}{|B|} \sum_{(\mathbf{x}^{(j)}, y^{(j)}) \in B} \nabla_\theta L(\mathbf{x}^{(j)}, y^{(j)}, \theta)$$ Mini-batch learning

In “pure” SGD, $|B| = 1$ (!)

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Selecting the Mini-batch

- Larger batches give more accurate estimate of the gradient, but with less than linear returns (p.271 DL book)

- Multicore architectures are usually underutilized by extremely small batch sizes (p.272 DL book)

- Small batches can offer a regularizing effect, perhaps due to the noise they add to the learning process (p.272)

- In practice, shuffle the training data, then cycle through the same mini-batches on each epoch

Stochastic Gradient Descent (SGD)

Algorithm 8.1 Stochastic gradient descent (SGD) update

Require: Learning rate schedule $\epsilon_1, \epsilon_2, \ldots$
Require: Initial parameter $\boldsymbol{\theta}$
  $k \leftarrow 1$
  while stopping criterion not met **do**
    Sample a minibatch of m examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient estimate: $\hat{\boldsymbol{g}} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \epsilon_k \hat{\boldsymbol{g}}$
    $k \leftarrow k + 1$
  end while

$$\epsilon_k = (1 - \alpha)\epsilon_0 + \alpha\epsilon_\tau \text{ with } \alpha = \max(k/\tau, 1)$$

p.286 of the Deep Learning textbook

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Lots of Other Alternatives

- Gradient descent optimization algorithms
  - Momentum
  - Nesterov accelerated gradient
  - Adagrad
  - Adadelta
  - RMSprop
  - Adam
  - AdaMax
  - Nadam
  - .....

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Optimization in Tensorflow

```
self._lr = tf.Variable(0.0, trainable=False)
tvars = tf.trainable_variables()
grads, _ = tf.clip_by_global_norm(tf.gradients(self._cost, tvars),
                                  config.max_grad_norm)
optimizer = tf.train.GradientDescentOptimizer(self._lr)
self._train_op = optimizer.apply_gradients(
    zip(grads, tvars),
    global_step=tf.train.get_or_create_global_step())

self._new_lr = tf.placeholder(
    tf.float32, shape=[], name="new_learning_rate")
self._lr_update = tf.assign(self._lr, self._new_lr)
```

from the Tensorflow RNN LM tutorial

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Parameter Initialization (matters)

Imagine you're trying to get to the top of Arthur's Seat in the fog (*haar*); where you start will be crucial to success

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Parameter Initialization (matters)

- Typically the weights are initialized randomly, biases are set to heuristically chosen constants
- Random weights are drawn from Gaussian or Uniform
  - e.g. $U\left(-\frac{1}{\sqrt{m}}, \frac{1}{\sqrt{m}}\right)$ where m is the number of inputs

p.292 of the Deep Learning textbook

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Regularization

$$\tilde{J}(\theta; \mathbf{x}, y) = J(\theta; \mathbf{x}, y) + \alpha\Omega(\theta)$$

$$\Omega(\theta) = \frac{1}{2}\|\mathbf{w}\|_2^2 = \frac{1}{2}\mathbf{w}^\top\mathbf{w}$$

L^2 parameter norm penalty (weight decay)

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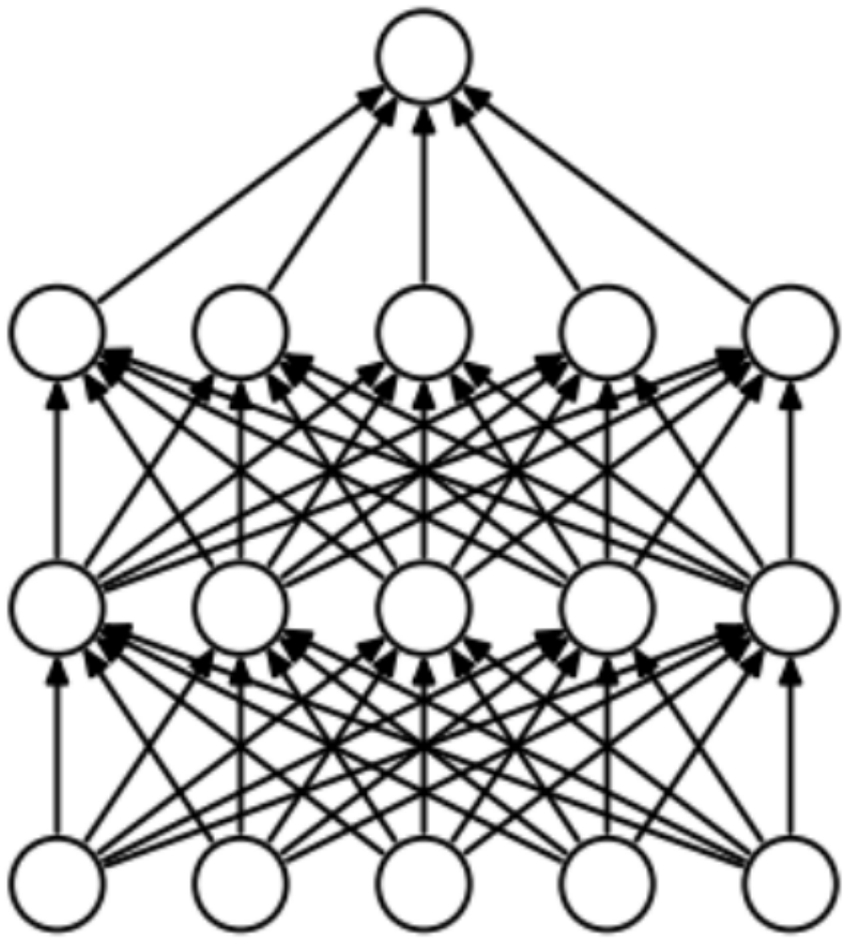
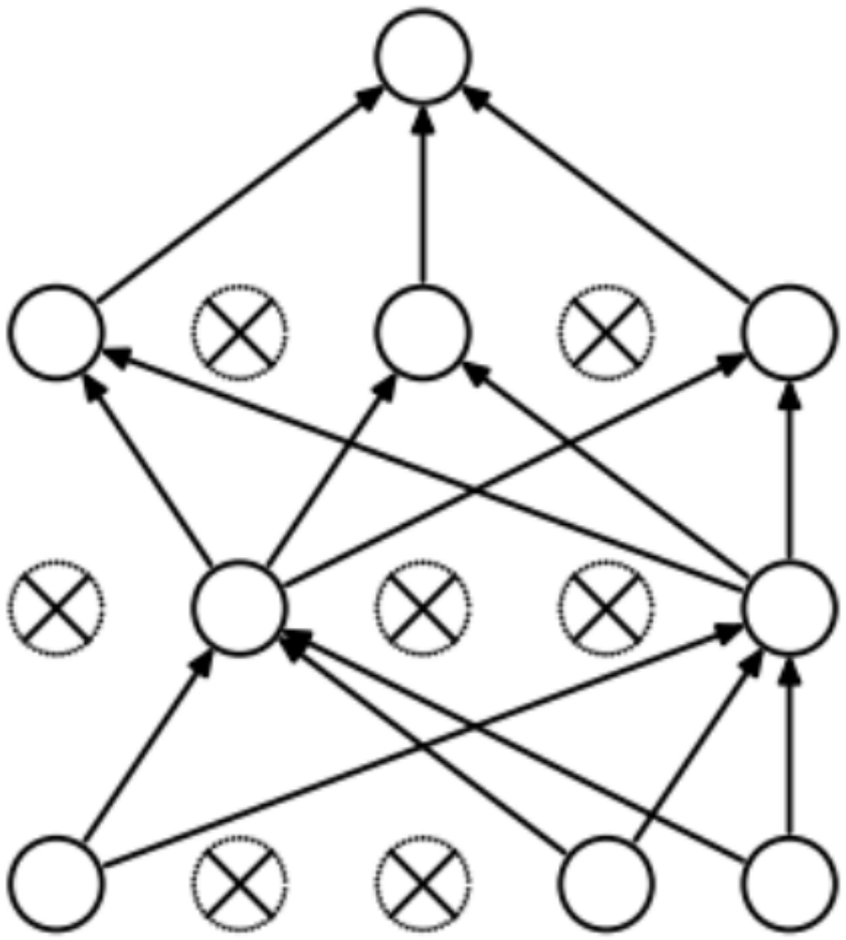
Dropout



(a) Standard Neural Net



(b) After applying dropout.

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References

- *How the backpropagation algorithm works*, Michael Nielsen

  http://neuralnetworksanddeeplearning.com/chap2.html

- colah's blog

  http://colah.github.io/posts/2015-08-Backprop/

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