Reflection models and radiometry

Advanced Graphics

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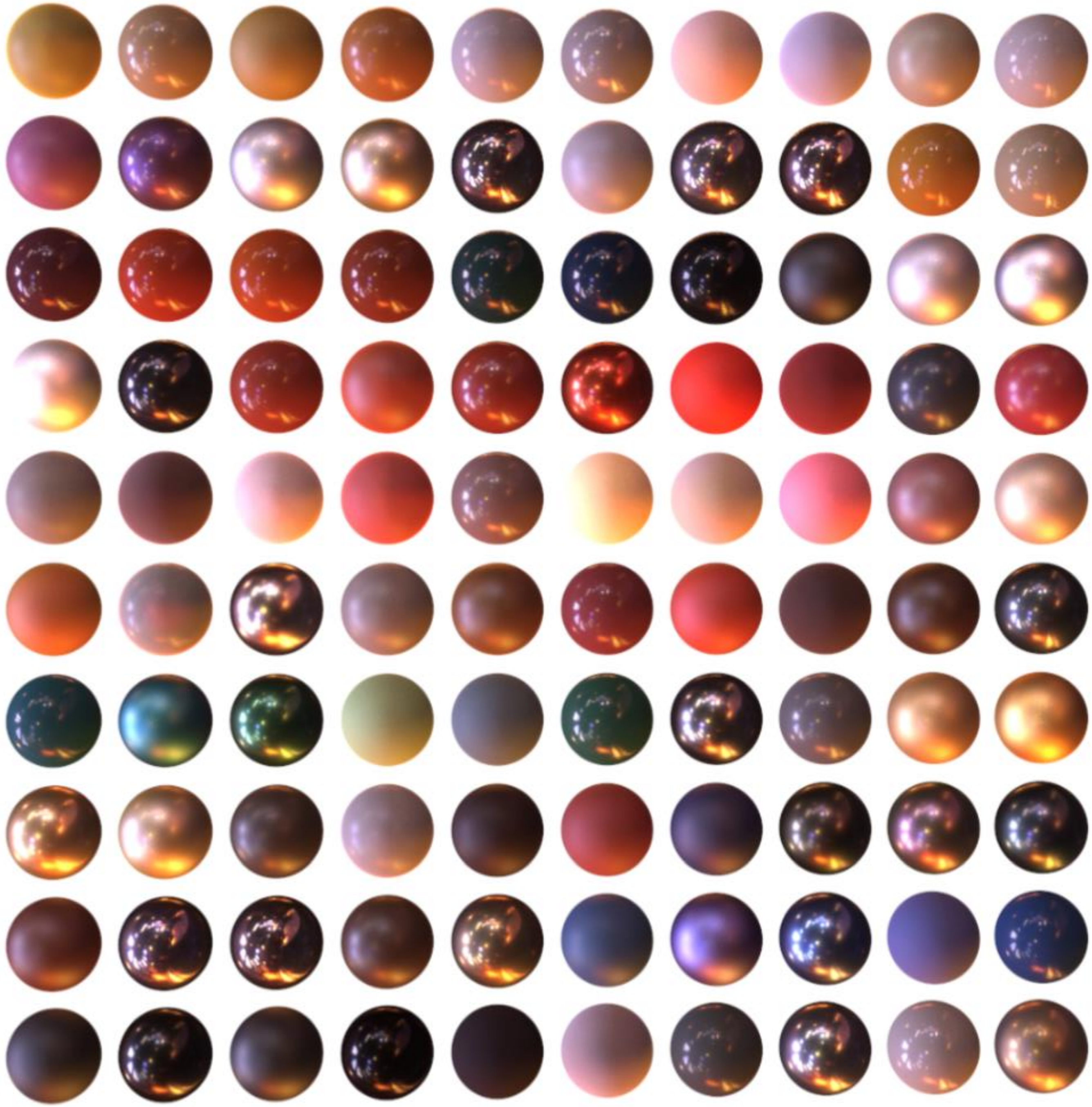
Applications

- To render realistic looking materials
- Applications also in computer vision, optical engineering, remote sensing, etc.
  - To understand how surfaces reflect light



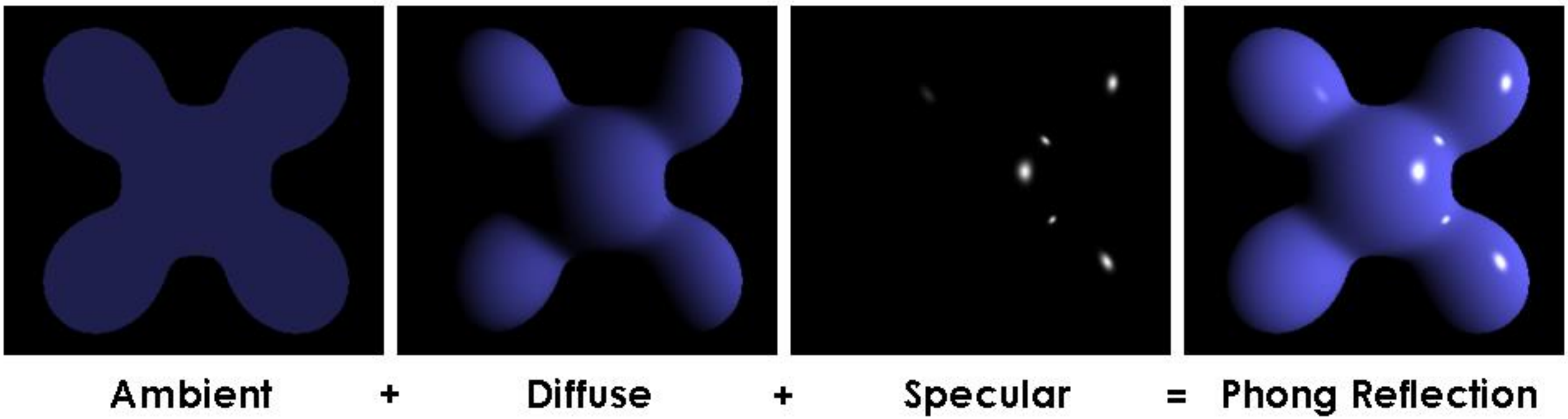
Applications

- Many applications require faithful reproduction of material appearance

Source: http://ikea.com/

Source: http://www.mercedes-benz.co.uk/

Last year you learned about Phong reflection model



Good for plastic objects, not very accurate for other materials

Pictures courtasy of MIT, Lecture Notes 6.837

Rendering equation

- Most rendering methods require solving an (approximation) of the rendering equation:

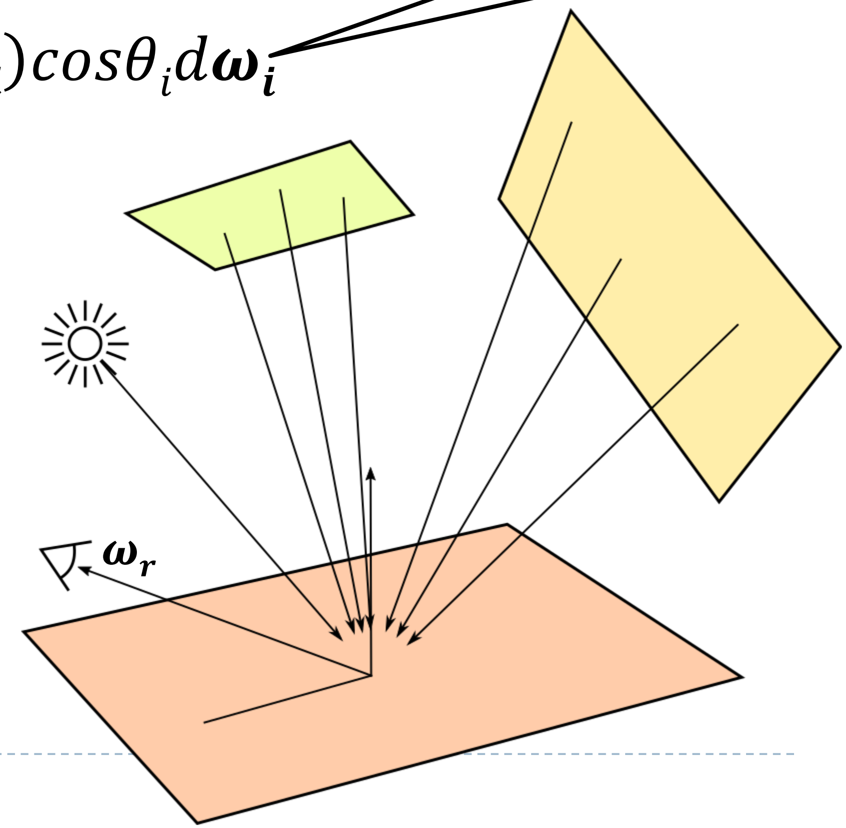
Incident light

Integral over the hemisphere of incident light

$$L_r(\boldsymbol{\omega_r}) = \int_{\Omega} \rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r}) L_i(\boldsymbol{\omega_i}) cos\theta_i d\boldsymbol{\omega_i}$$

Reflected light

BRDF



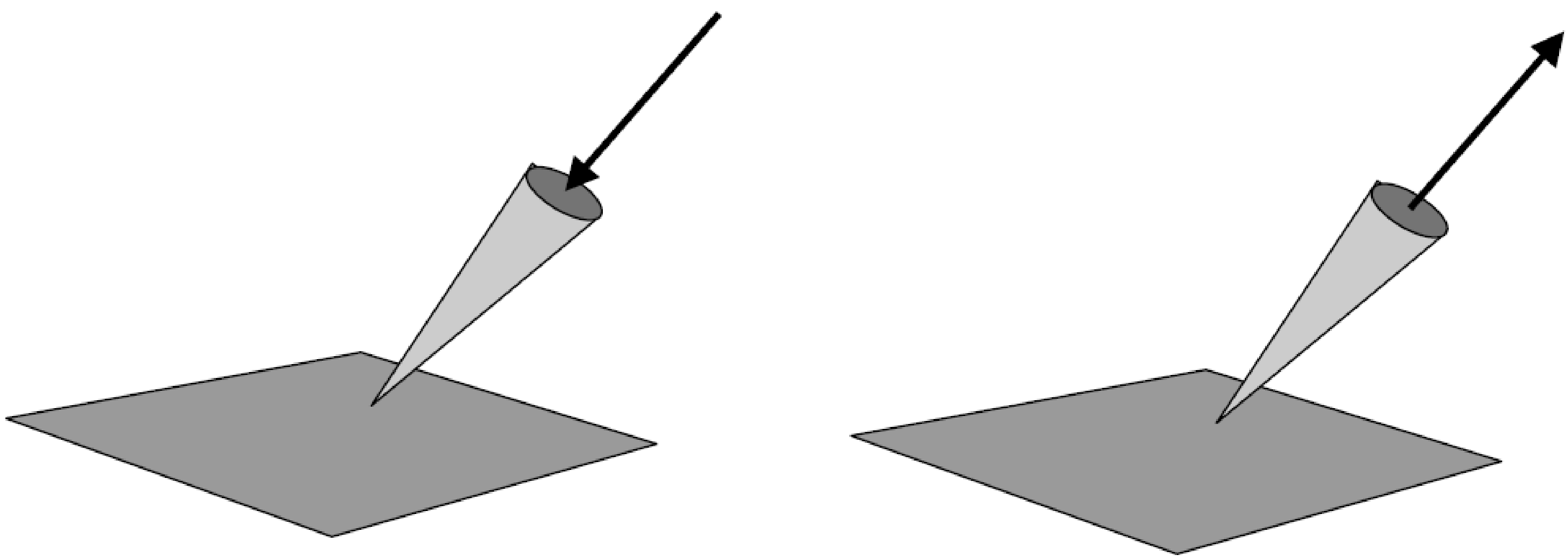
- The solution is trivial for point light sources
- Much harder to estimate the contribution of other surfaces

$$\boldsymbol{\omega_i} = [\phi_i, \theta_i]$$

Radiance

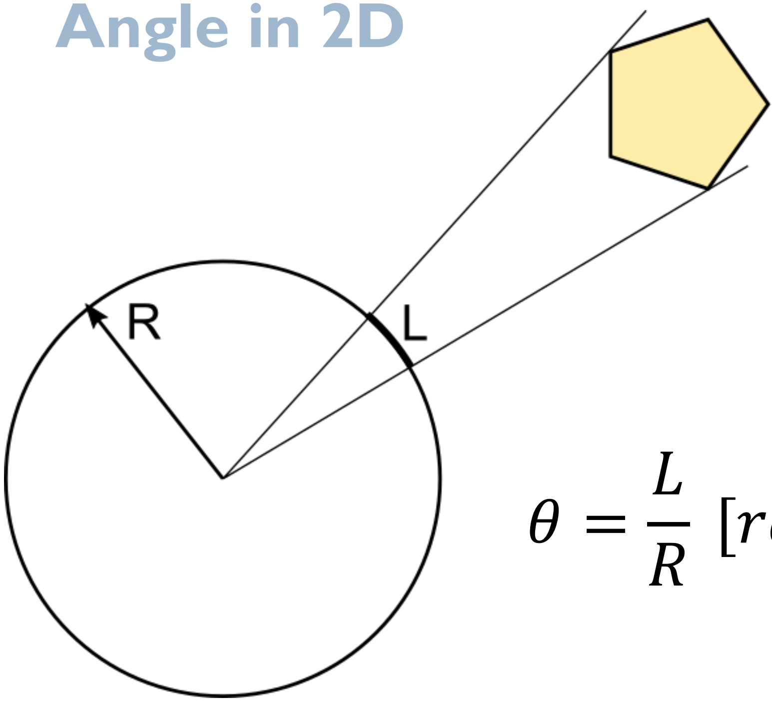
- Power of light per unit projected area per unit solid angle
- Symbol: $L(\boldsymbol{x}, \boldsymbol{\omega}_i)$
- Units: $\dfrac{W}{m^2 sr}$

Steradian



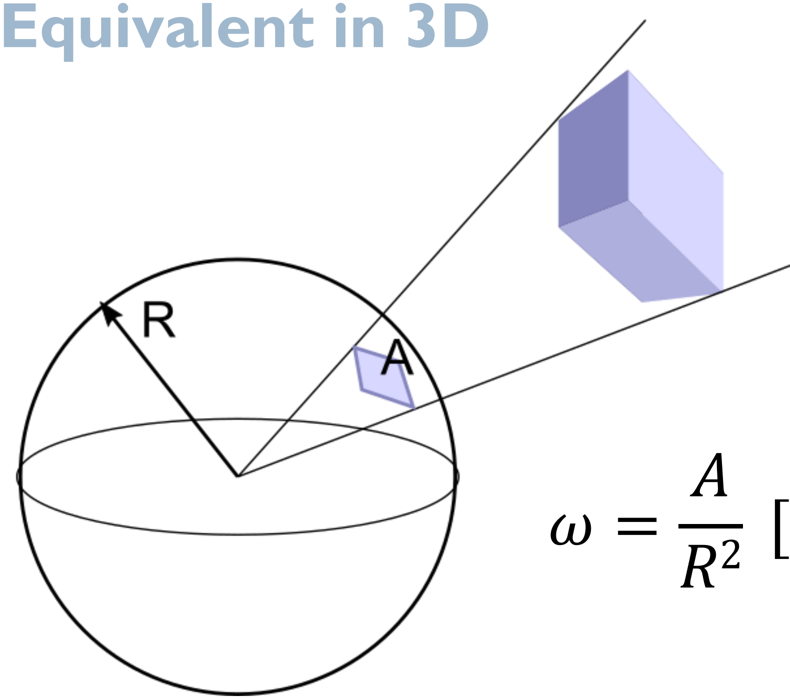
Solid angle

Angle in 2D



$$\theta = \frac{L}{R}\ [rad]$$

Equivalent in 3D



$$\omega = \frac{A}{R^2}\ [sr]$$

Full circle = 2π radians

- Full sphere = 4 π steradians

Radiance

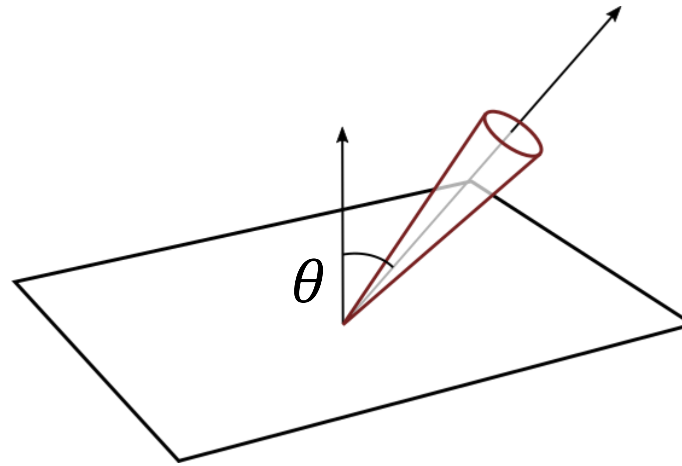
Position

Power of light

$$L(\boldsymbol{x},\boldsymbol{\omega}_i) = \frac{d\phi}{d\omega\; dA\; cos\theta}$$

Incoming direction

Projected area

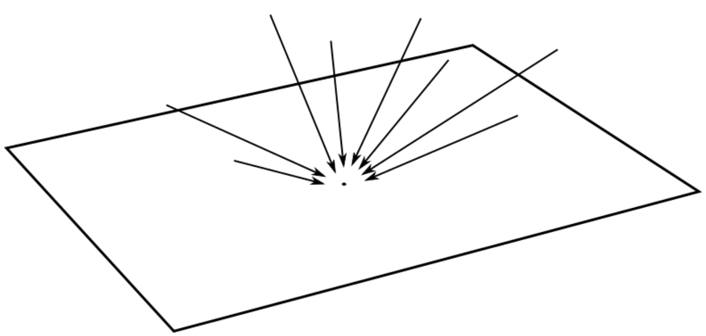


- Power per solid angle per projected surface area
- Invariant along the direction of propagation (in vacuum)
- Response of a camera sensor or a human eye is related to radiance
- Pixel values in image are related to radiance (projected along the view direction)

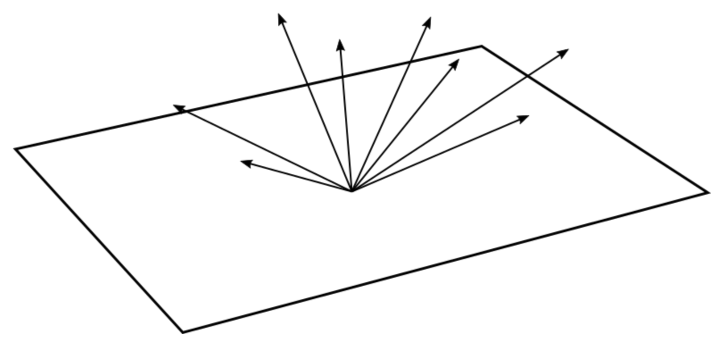
Irradiance and Exitance

- Power per unit area
- Irradiance: H(x) – incident power per unit area
- Exitance / radiosity: E(x) – exitant power per unit area
- Units: $\dfrac{W}{m^2}$

Irradiance



Exitance / Radiosity



Relation between Irradiance and Radiance

- Irradiance is an integral over all incoming rays
  - Integration over a hemisphere Ω:

$$H = \int_{\Omega} L(\boldsymbol{x}, \boldsymbol{\omega_i}) \, cos\theta \, d\boldsymbol{\omega}$$

  - In the spherical coordinate system, the differential solid angle is:

$$d\boldsymbol{\omega} = \sin\theta d\theta \, d\phi$$

  - Therefore:

$$H = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\frac{\pi}{2}} L(\boldsymbol{x}, \boldsymbol{\omega_i}) \, cos\theta \, sin\theta \, d\theta \, d\phi$$

  - For constant radiance:

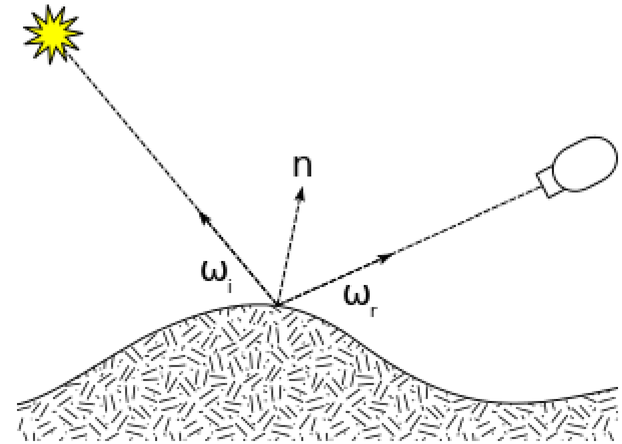
$$H = \pi L$$

BRDF: Bidirectional Reflectance Distribution Function

Differential radiance of reflected light

$$\rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r}) = \frac{dL_r(\boldsymbol{\omega_r})}{dH_i(\boldsymbol{\omega_i})} = \frac{dL_r(\boldsymbol{\omega_r})}{L_i(\boldsymbol{\omega_i}) cos\theta_i d\boldsymbol{\omega_i}}$$

Differential irradiance of incoming light

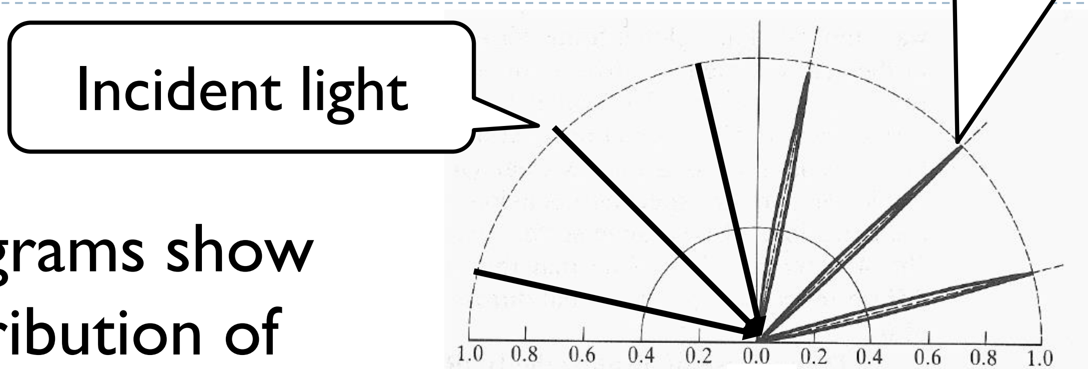


Source: Wikipedia

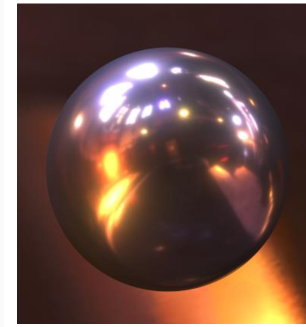
- BRDF is measured as a ratio of reflected radiance to irradiance
  - Because it is difficult to measure $L_i(\boldsymbol{\omega_i})$, BRDF is not defined as the ratio $L_r(\boldsymbol{\omega_r})/L_i(\boldsymbol{\omega_i})$

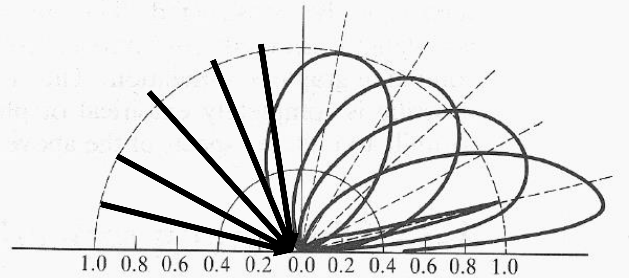
BRDF of various materials

- The diagrams show the distribution of reflected light for the given incoming direction
- The material samples are close but not accurate matches for the diagrams

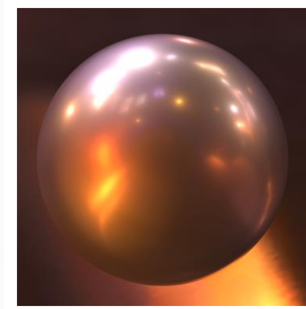


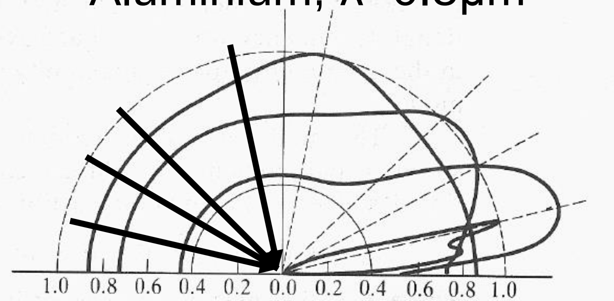
Aluminium; λ=2.0μm





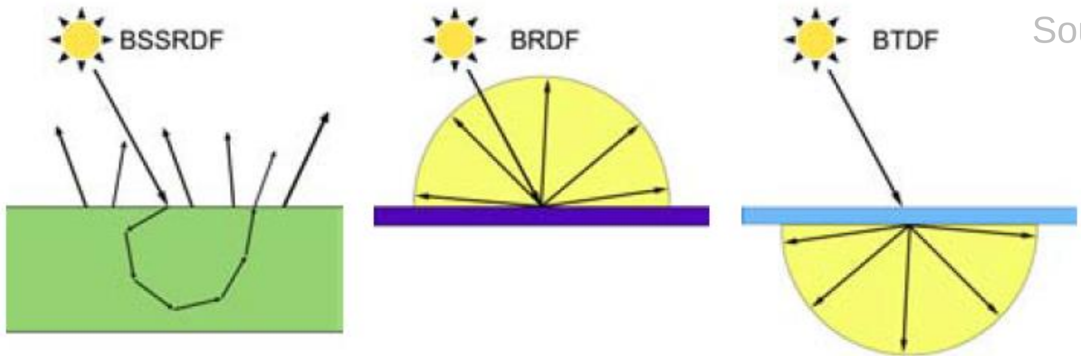
Aluminium; λ=0.5μm





Magnesium alloy; λ=0.5μm

Other material models

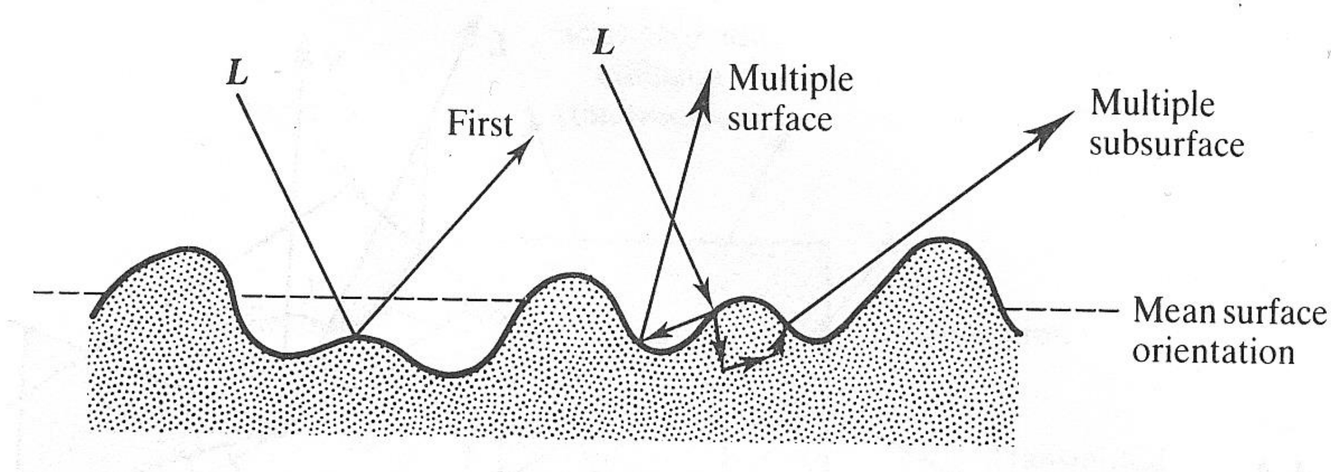


Source: Guarnera et al. 2016

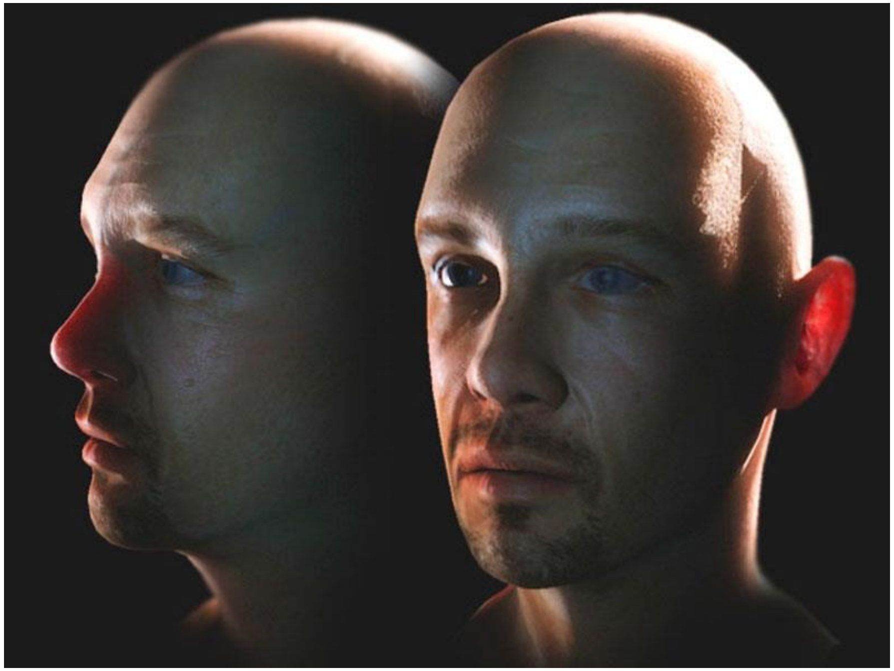
- Bidirectional Scattering Surface Reflectance Distribution F.
- **Bidirectional Reflectance Distribution Function**
- Bidirectional Transfer Distribution Function
- But also: BTF, SVBRDF, BSDF
- In this lecture we will focus mostly on BRDF

Sub-surface scattering

- Light enters material and is scattered several times before it exits
  - Examples - human skin: hold a flashlight next to your hand and see the color of the light
- The effect is expensive to compute
  - But approximate methods exist



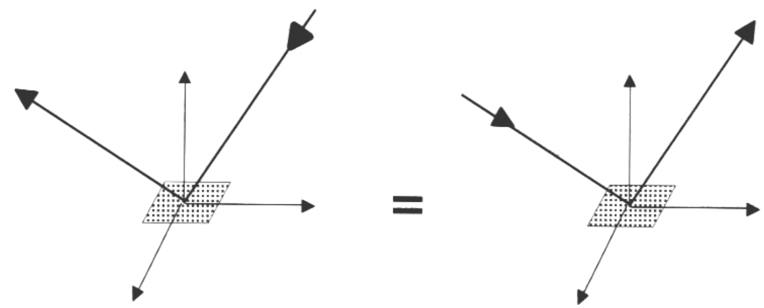
Subsurface scattering - examples



BRDF Properties

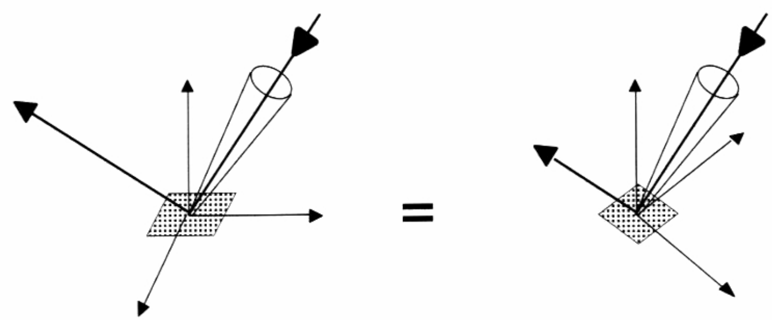
- Helmholtz reciprocity principle
  - BRDF remains unchanged if incident and reflected directions are interchanged

$$\rho(\boldsymbol{\omega_r}, \boldsymbol{\omega_i}) = \rho(\boldsymbol{\omega_i}, \boldsymbol{\omega_r})$$



- Smooth surface: isotropic BRDF
  - reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$\rho(\theta_i, \theta_r, \phi_r - \phi_i)$$



BRDF Properties

- Characteristics
  - BRDF units [1/sr]
    - Not intuitive
  - Range of values:
    - From 0 (absorption) to ∞ (reflection, δ -function)
  - Energy conservation law

$$\int_{\Omega} \rho(\omega_r, \omega_i) cos\theta_i d\omega_i \leq 1$$

    - No self-emission
    - Possible absorption
  - Reflection only at the point of entry ($x_i = x_r$)
    - No subsurface scattering

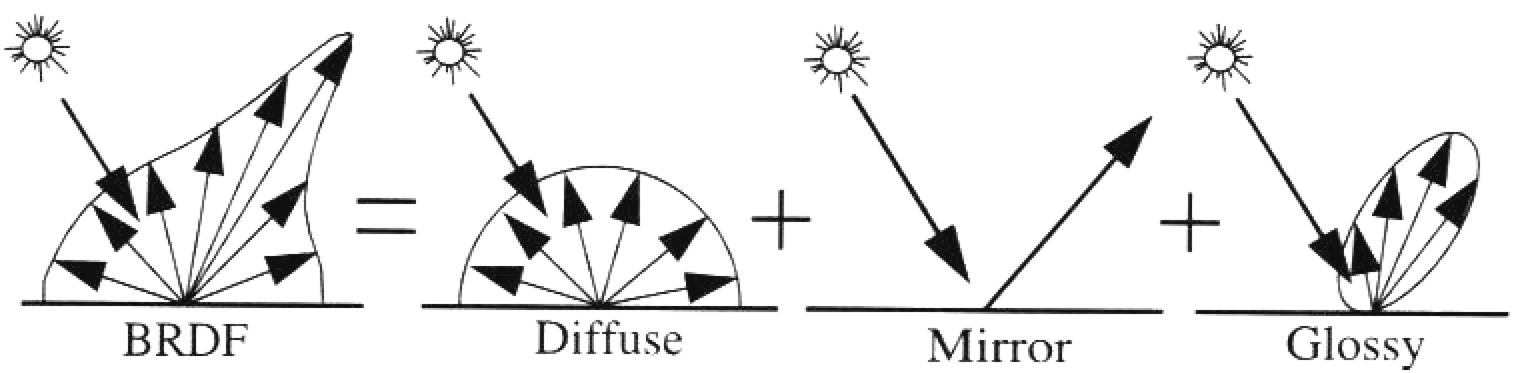
BRDF Measurement

- Gonio-Reflectometer
- BRDF measurement
  - point light source position (θ,φ)
  - light detector position (θ_o, φ_o)
- 4 directional degrees of freedom
- BRDF representation
  - m incident direction samples (θ,φ)
  - n outgoing direction samples (θ_o, φ_o)
  - $m*n$ reflectance values (large!!!)

Stanford light gantry

BRDF Modeling

- It is common to split BRDF into diffuse, mirror and glossy components
- Ideal diffuse reflection
  - Lambert's law
  - Matte surfaces
- Ideal specular reflection
  - Reflection law
  - Mirror
- Glossy reflection
  - Directional diffuse
  - Shiny surfaces

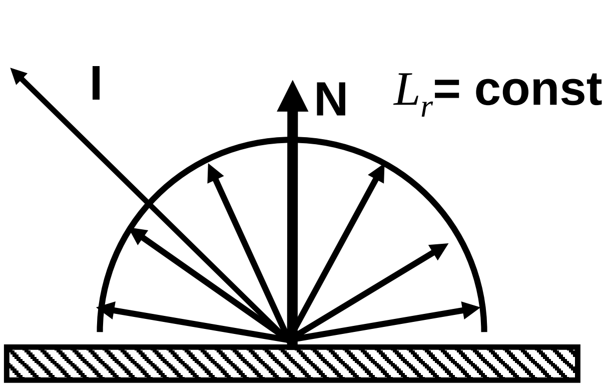


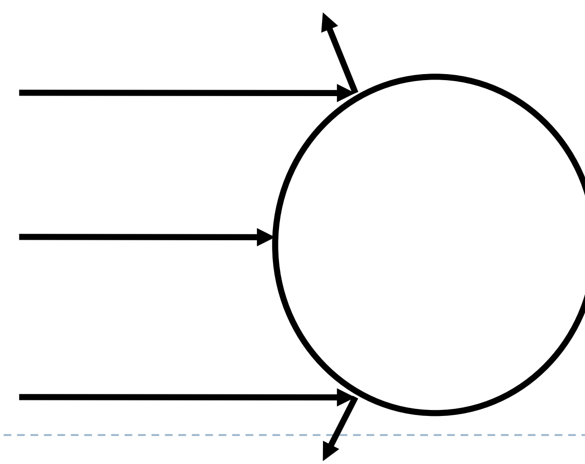
Diffuse Reflection

- Light equally likely to be reflected in any outgoing direction (independent of incident direction)
- Constant BRDF $\rho(\omega_r, \omega_i) = k_d = const$

$$L_r(\omega_r) = \int_{\Omega} k_d L_i(\omega_i) cos\theta_i d\omega_i = k_d \int_{\Omega} L_i(\omega_i) \cos\theta_i d\omega_i = k_d H$$

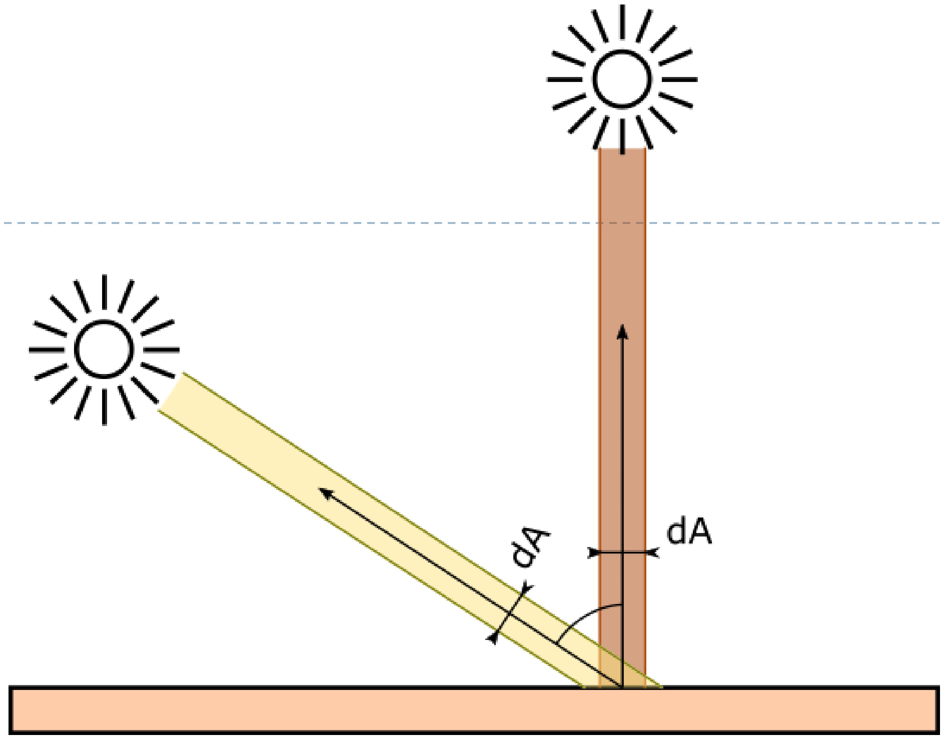
  - k_d: diffuse coefficient, material property [1/sr]

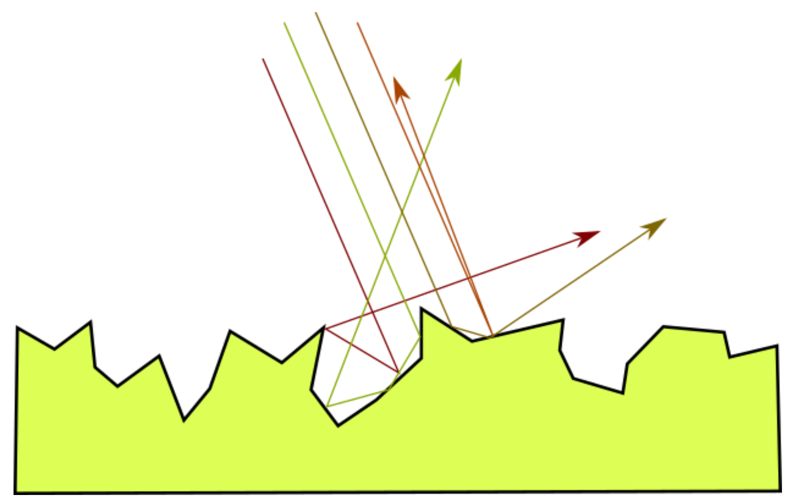




Diffuse reflection

- Cosine term
  - The surface receives less light per unit area at steep angles of incidence
- Rough, irregular surface results in diffuse reflection
  - Light is reflected in random direction
  - (this is just a model, light interaction is more complicated)



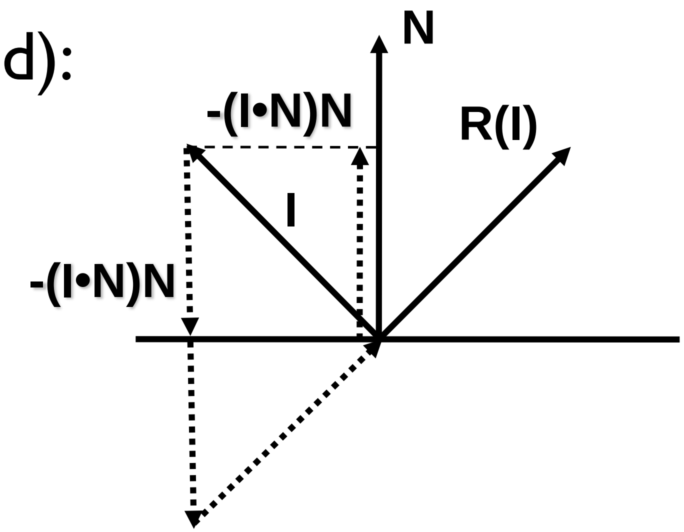


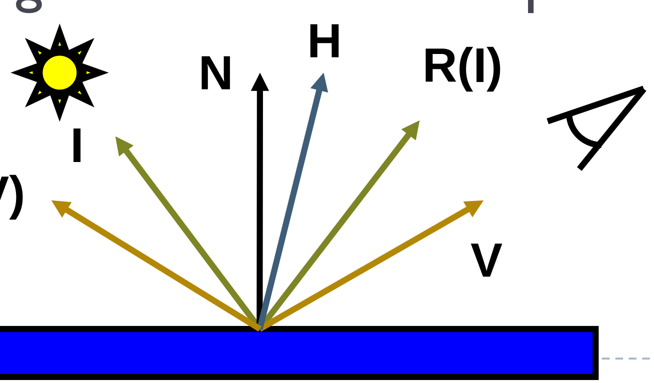
Reflection Geometry

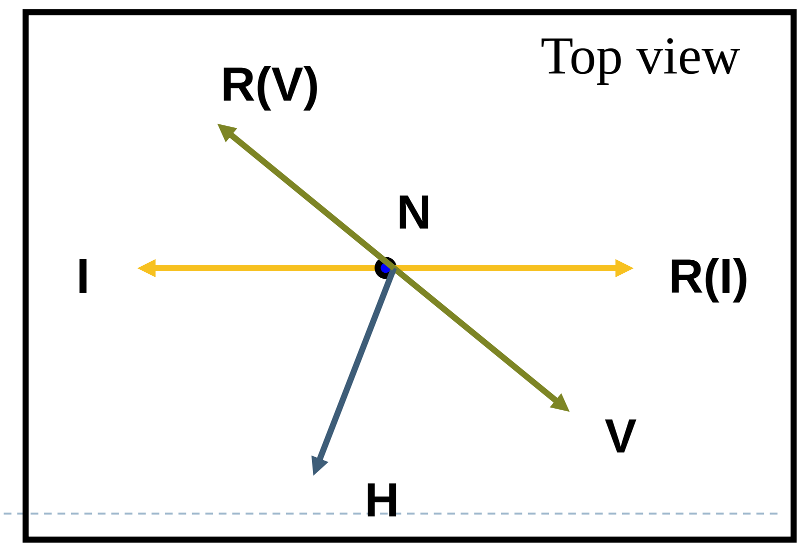
- Direction vectors (all normalized):
  - N: surface normal
  - I: vector to the light source
  - V: viewpoint direction vector
  - H: halfway vector

    H= (I + V) / |I + V|
  - R(I): reflection vector

    R(I)= I - 2(I•N)N
  - Tangential surface: local plane



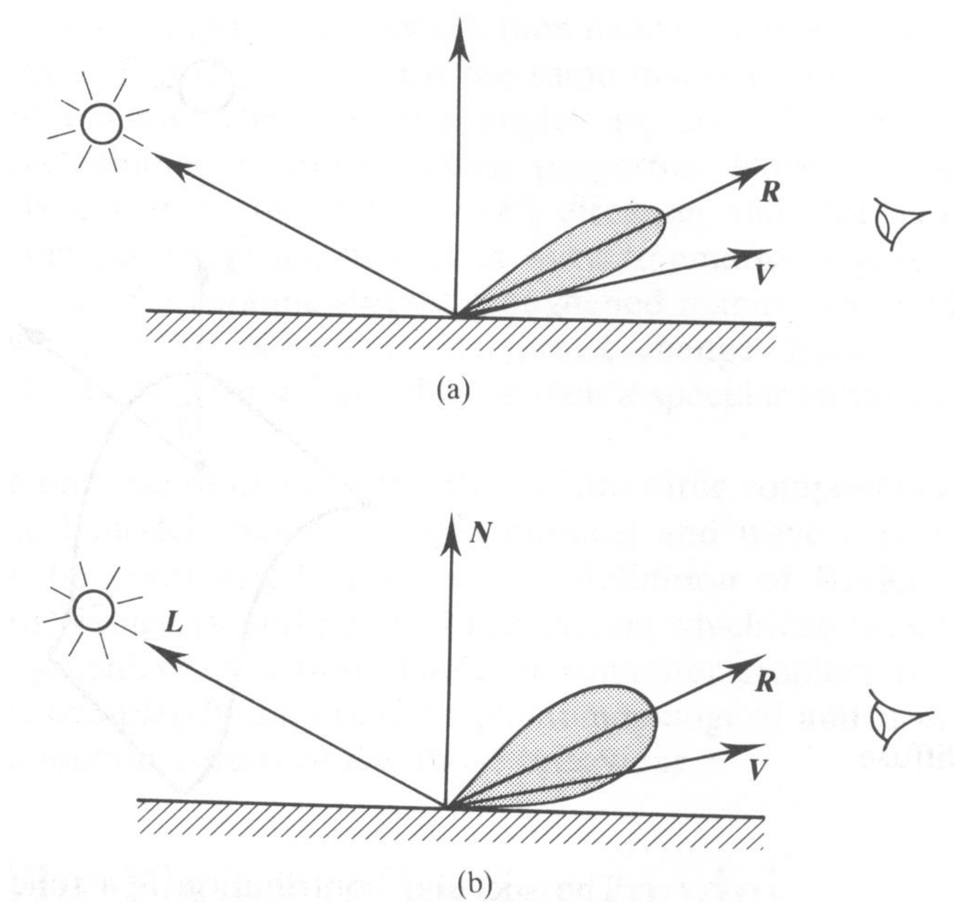




Glossy Reflection

Glossy Reflection

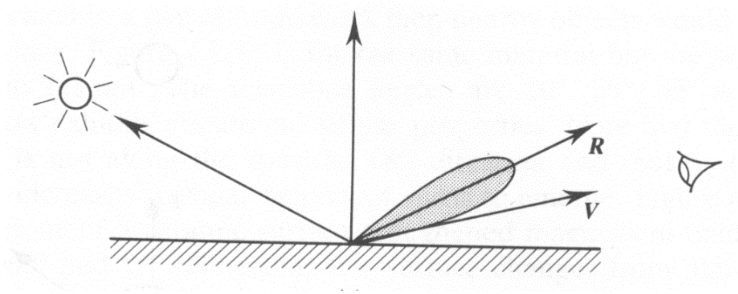
- Due to surface roughness
- Empirical models
  - Phong
  - Blinn-Phong
- Physical models
  - Blinn
  - Cook & Torrance



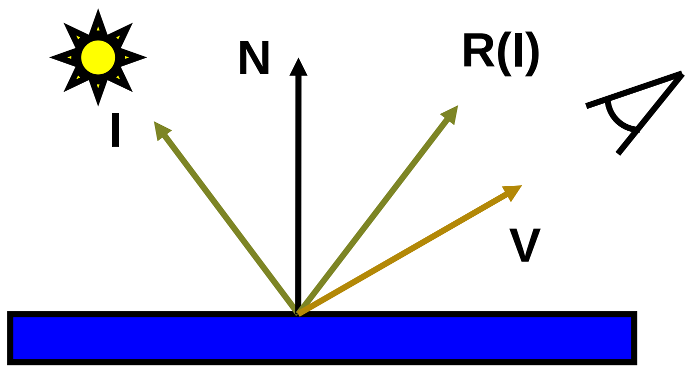
Phong Reflection Model

- Cosine power lobe

$$\rho(\omega_r, \omega_i) = k_s(R \cdot V)^{k_e}$$



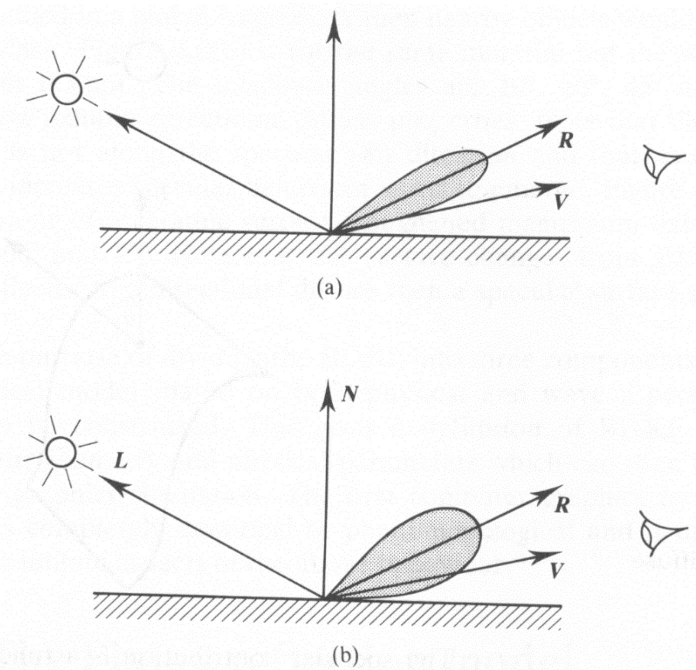
- Dot product & power
- Not energy conserving/reciprocal
- Plastic-like appearance

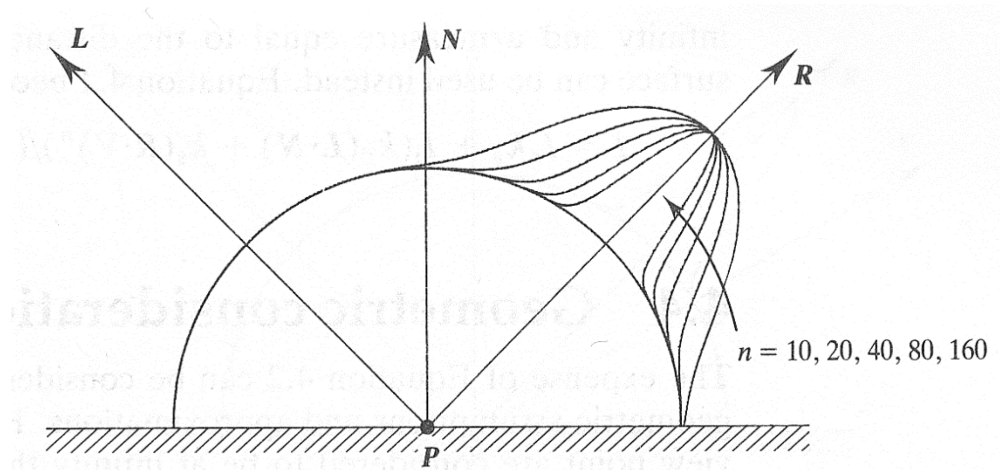


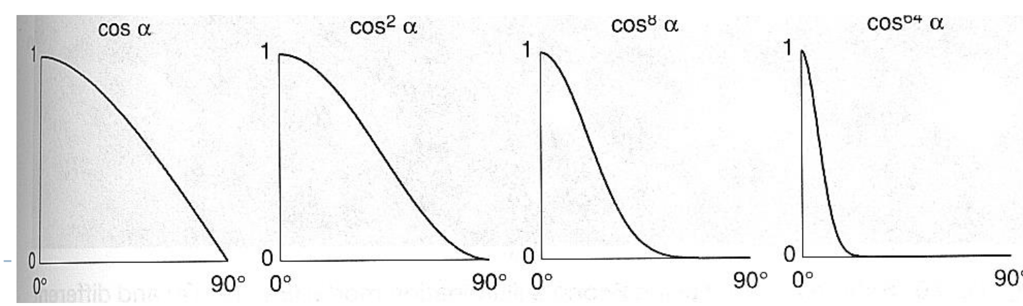
Phong Exponent k_e

$$\rho(\omega_r, \omega_i) = k_s (R \cdot V)^{k_e}$$

- Determines the size of highlight







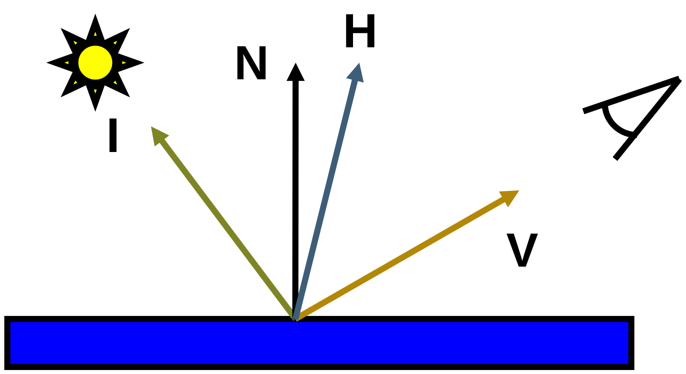
Blinn-Phong Reflection Model

- Blinn-Phong reflection model

$$\rho(\omega_r, \omega_i) = k_s (H \cdot N)^{k_e}$$

  - Light source, viewer far away
  - I, R constant: H constant

    H less expensive to compute



Phong Illumination Model

- Extended light sources: *l* point light sources

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(Phong)}$$

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(Blinn)}$$

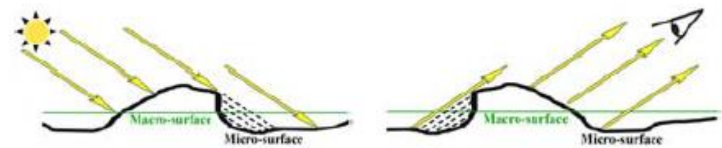
- Colour of specular reflection equal to light source
- Heuristic model
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- Often: light sources & viewer assumed to be far away

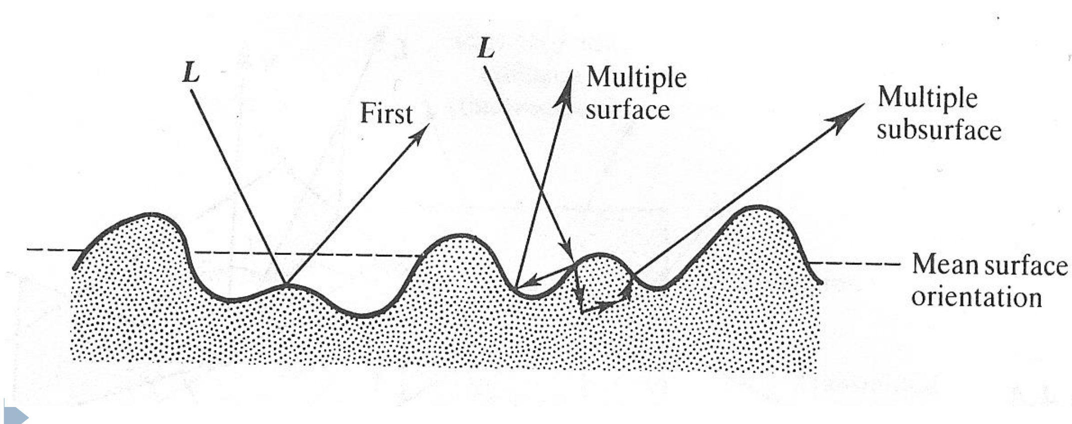
Micro-facet model

- We can assume that at small scale materials are made up of small facets
  - Facets can be described by the distribution of their sizes and directions D
  - Some facets are occluded by other, hence there is also a geometrical attenuation term G
  - And we need to account for Fresnel reflection (see next slides)

$$\rho(\omega_i, \omega_r) = \frac{D \cdot G \cdot F}{4 \cos \theta_i \theta_r}$$

Rendering of glittery materials [Jakob et al. 2014]



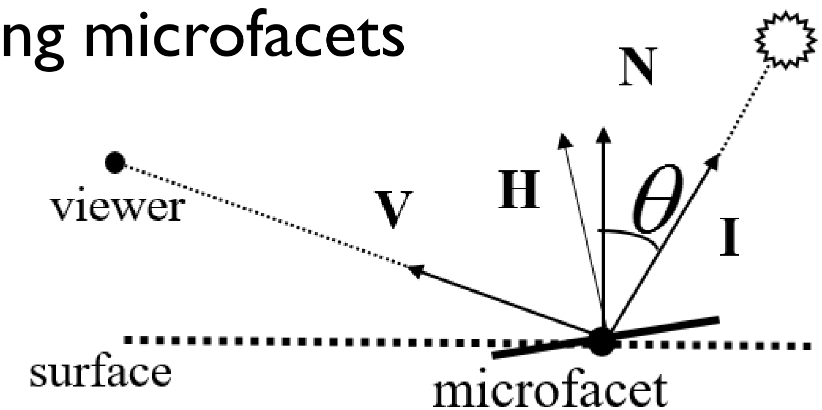


Ward Reflection Model

- BRDF

$$\rho = \frac{k_d}{\pi} + k_s \frac{1}{\sqrt{(I \cdot N)(V \cdot N)}} \frac{\exp\left(-\tan^2 \frac{\angle(H,N)}{\sigma^2}\right)}{2\pi\sigma^2}$$

- σ standard deviation (RMS) of surface slope
- Simple expansion to anisotropic model (σ_x, σ_y)
- Empirical, not physics-based
- Inspired by notion of reflecting microfacets
- Convincing results
- Good match to measured data



Cook-Torrance model

- Can model metals and dielectrics
- Sum of diffuse and specular components

$$\rho(\omega_i, \omega_r) = \rho_d(\omega_i) + \rho_s(\omega_i, \omega_r)$$

- Specular component:

$$\rho(\omega_i, \omega_r) = \frac{D(h)\, G(I, V)\, F(\omega_i)}{\pi \cos\theta_i \cos\theta_r}$$

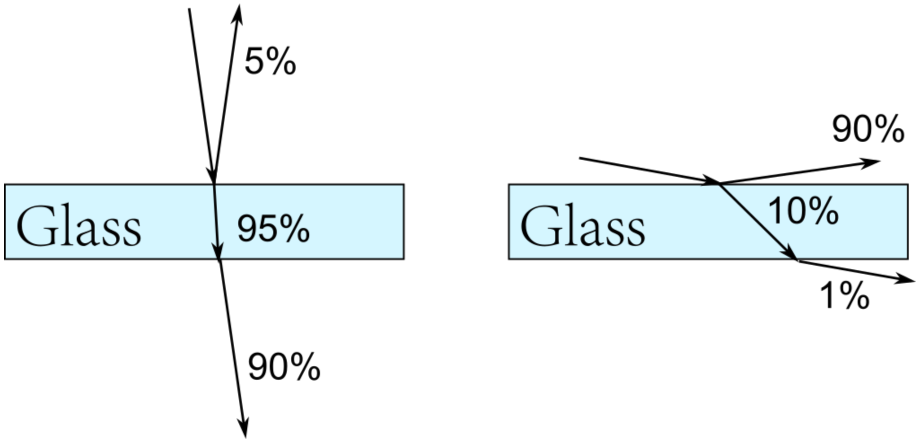
- Distribution of microfacet orientations: $D(h) = \cos\theta_r\, e^{-\left(\frac{\alpha}{m}\right)^2}$

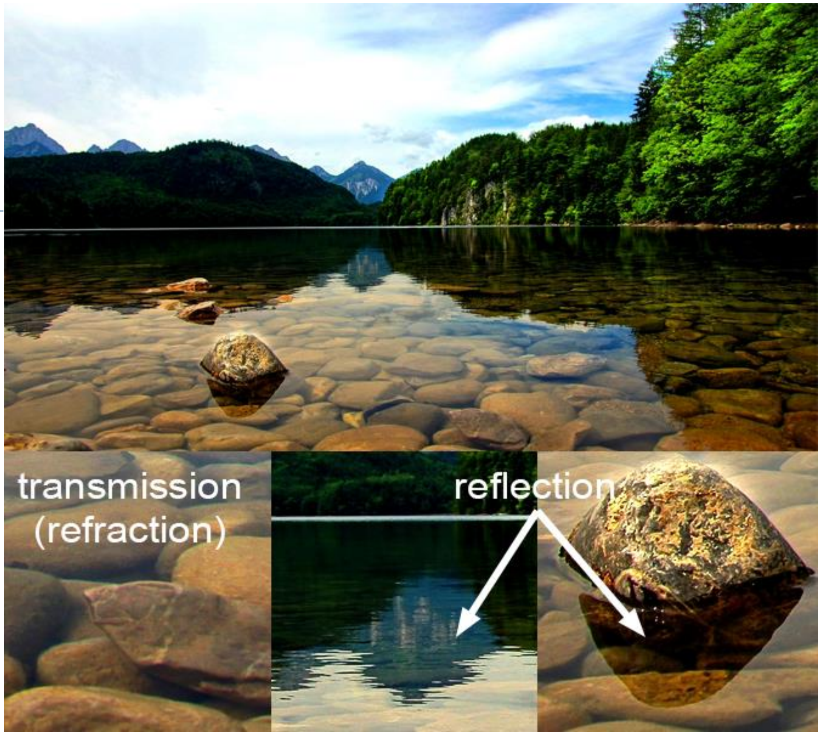
Roughness parameter

- Geometrical attenuation factor
  - To account to self-masking and shadowning

$$G(I, V) = min\left\{1, \frac{2(N \cdot H)(N \cdot V)}{V \cdot H}, \frac{2(N \cdot H)(N \cdot I)}{V \cdot H}\right\}$$

Fresnel term





Example from: https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-shading/reflection-refraction-fresnel

- The light is more likely to be reflected rather than transmitted near grazing angles
- The effect is modelled by Fresnel equation: it gives the probability that a photon is reflected rather than transmitted (or absorbed)

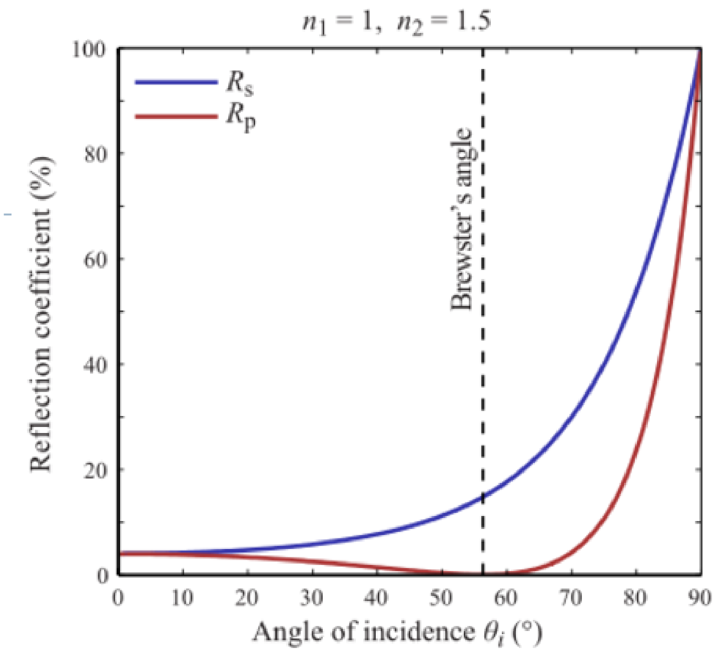
Fresnel equations

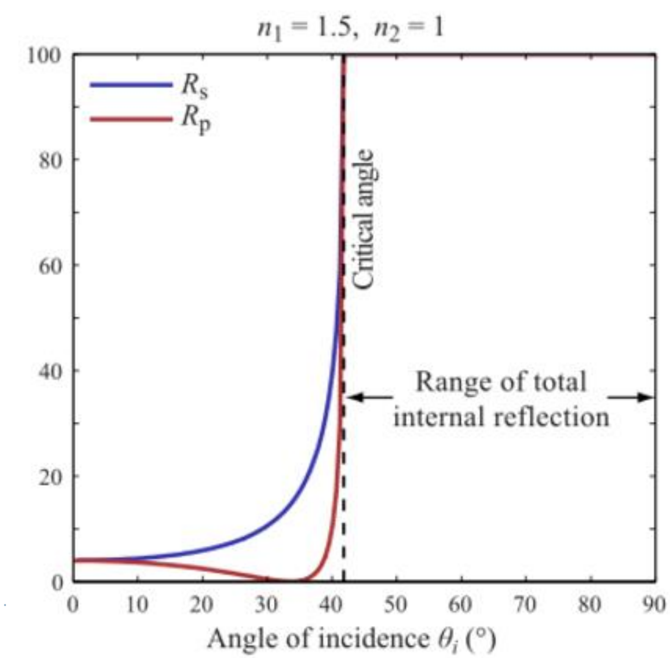
- Reflectance for s-polarized light:

$$R_s = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2}} \right|^2,$$

- Reflectance for p-polarized light:

$$R_p = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2 = \left| \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2} - n_2 \cos\theta_i}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_i\right)^2} + n_2 \cos\theta_i} \right|^2.$$





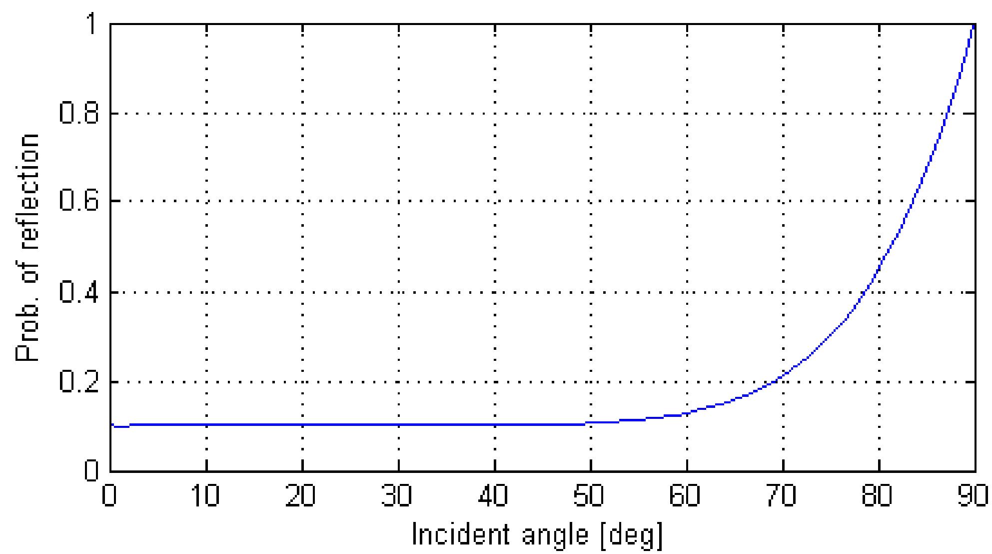
Fresnel term

- In Computer Graphics the Fresnel equation is approximated by Schlick's formula [Schlick, 94]:

$$R(\theta,\lambda) = R_0(\lambda) + \big(1 - R_0(\lambda)\big)(1 - cos\theta)^5$$

- where $R_0(\lambda)$ is reflectance at normal incidence and λ is the wavelength of light
- For dielectrics (such as glass):

$$R_0(\lambda) = \left(\frac{n(\lambda) - 1}{n(\lambda) + 1}\right)^2$$



Which one is Phong / Cook-Torrance ?

Image based lighting (IBL)

1. Capture an HDR image of a light probe

2. Create an illumination (cube) map

3. Use the illumination map as a source of light in the scene

The scene is surrounded by a cube map

Source: Image-based lighting, Paul Debevec, HDR Symposium 2009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Blender monkeys + IBL (path tracing)

Further reading

- A. Watt, 3D Computer Graphics
  - Chapter 7: Simulating light-object interaction: local reflection models
- Eurographics 2016 tutorial
  - D. Guarnera, G. C. Guarnera, A. Ghosh, C. Denk, and M. Glencross
  - BRDF Representation and Acquisition
  - **DOI:** 10.1111/cgf.12867

- Some slides have been borrowed from Computer Graphics lecture by Hendrik Lensch
  - http://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG07-Brdf+Texture.pdf