Mobile Processes

Two forms of process mobility (movement):

Mobility of commonication links

Two mechanisms:

E.g. SPL

Pi-Calculus

Name generation

You at a party

'Virtual' mobility thro' (gradual) change of comm. links.

Relocation

E.g. Process passing

Process passing

Ambient Calculus

You on a flight

'Real' mobility thro' (abrupt) change of context.

The π-calculus

CCS + Channel passing + Channel creation

in CCS, only basic values (numbers) can be sent

Restriction / Channel passing.

new a. a?x ~ nil

Transitions

new a.(register!a. a?y)
| register?y. request!y
| request?z. z!v

$\longrightarrow$ new a(a?y | request!a) | request?z. z!v

$\longrightarrow$ new a(a?y | a!v)

scope extrusion

HOPLA

~ Higher-Order Process LAnguage ~

* Combines nondeterminism with higher-order processes.

$\lambda x.\ t$	Lambda-abstraction
$t\ u$	Application
$a\,t$	Label transitions
$\cdot t$	Action step
$\sum_{i \in I} b_i$	Non-deterministic choice
$\mathrm{rec}\ x\ t$	Recursion
$[u > \cdot x \Rightarrow t]$	Match
$\pi_a(t)$	Selection

Overview:

* Processes have types to describe their possible behaviour

  $t : \mathbb{P}$ e.g. $\lambda y.\ cy : a. + b. \to ca. + cb.$

* Actions:

  $p \xrightarrow{\overbrace{a_1 \ldots a_n}^{\text{string}} \cdot} p'$ (string followed by $\cdot$)

  + higher-order actions

  $\lambda y.cy \xrightarrow{a.+b. \mapsto ca.} nil$

⇝ Types for actions

HOPLA

Types (of computations of processes)

$$\mathbb{P}, \mathbb{Q} ::= .\mathbb{P} \;\Big|\; \mathbb{P} \to \mathbb{Q} \;\Big|\; \sum_{a \in A} a\mathbb{P}_a \;\Big|\; P \;\Big|\; \mu_j \vec{P}\,\vec{\mathbb{P}}$$

finite sum $a_1\mathbb{P}_1 + \cdots + a_k\mathbb{P}_k$

empty sum $\mathbb{O}$

$\mu_j P_1, \ldots, P_k \quad \mathbb{P}_1, \ldots, \mathbb{P}_k$

Process terms

$$t, u ::= x \;\Big|\; \mathrm{rec}\, x\, t \;\Big|\; \sum_{i \in I} t_i \;\Big|\; .t \;\Big|\; [u > .x \Rightarrow t] \;\Big|$$
$$\lambda x\, t \;\Big|\; t\,u \;\Big|\; a\,t \;\Big|\; \pi_a(t)$$

finite sum $t_1 + \cdots + t_k$

empty sum nil

subject to type judgements

$$x_1 : \mathbb{P}_1, \ldots, x_k : \mathbb{P}_k \vdash t : \mathbb{Q}$$

$$\frac{\Gamma(x) \vdash \mathbb{P}}{\Gamma \vdash x : \mathbb{P}} \qquad \frac{\Gamma, x:\mathbb{P} \vdash t:\mathbb{P}}{\Gamma \vdash \mathrm{rec}\, x\, t : \mathbb{P}} \qquad \frac{\Gamma \vdash t_j : \mathbb{P} \quad j \in I}{\Gamma \vdash \sum_{i \in I} t_i : \mathbb{P}}$$

$$\frac{\Gamma \vdash t : \mathbb{P}}{\Gamma \vdash {}_{\bullet}t : {}_{\bullet}\mathbb{P}} \qquad \frac{\Gamma \vdash u : {}_{\bullet}\mathbb{P} \qquad \Gamma, x:\mathbb{P} \vdash t : \mathbb{Q}}{\Gamma \vdash [u > {}_{\bullet}x \Rightarrow t] : \mathbb{Q}}$$

$$\frac{\Gamma, x:\mathbb{P} \vdash t : \mathbb{Q}}{\Gamma \vdash \lambda x\, t : \mathbb{P} \to \mathbb{Q}} \qquad \frac{\Gamma \vdash t : \mathbb{P} \to \mathbb{Q} \quad \Gamma \vdash u : \mathbb{P}}{\Gamma \vdash t\,u : \mathbb{Q}}$$

$$\frac{\Gamma \vdash t : \mathbb{P}_b \qquad b \in A}{\Gamma \vdash b\,t : \sum_{a \in A} a\mathbb{P}_a} \qquad \frac{\Gamma \vdash t : \sum_{a \in A} a\mathbb{P}_a \qquad b \in A}{\Gamma \vdash \pi_b(t) : \mathbb{P}_b}$$

$$\frac{\Gamma \vdash t : \mathbb{P}_j[\mu \vec{P}\vec{\mathbb{P}}/\vec{P}]}{\Gamma \vdash t : \mu_j \vec{P}\vec{\mathbb{P}}} \qquad \frac{\Gamma \vdash t : \mu_j \vec{P}\vec{\mathbb{P}}}{\Gamma \vdash t : \mathbb{P}_j[\mu \vec{P}\vec{\mathbb{P}}/\vec{P}]}$$

Actions

$$p ::= . \mid u \mapsto p \mid a\,p$$

Action types $\mathbb{P} : p : \mathbb{Q}$

$$.\mathbb{P} : . : \mathbb{P} \qquad \frac{u : \mathbb{P} \qquad \mathbb{Q} : q : \mathbb{Q}'}{\mathbb{P} \to \mathbb{Q} : (u \mapsto q) : \mathbb{Q}'} \qquad \frac{\mathbb{P}_a : p : \mathbb{P}'}{\Sigma_{a\in A} a\mathbb{P}_a : a\,p : \mathbb{P}'} \qquad \frac{\mathbb{P}_j[\mu\vec{P}\vec{\mathbb{P}}/\vec{P}] : p : \mathbb{P}'}{\mu_j\vec{P}\vec{\mathbb{P}} : p : \mathbb{P}'}$$

Transition rules $\mathbb{P} : t \xrightarrow{p} t'$

$\leadsto$ implicitly require $\vdash t : \mathbb{P}$ and $\mathbb{P} : p : \mathbb{P}'$ for some $\mathbb{P}'$

$$\frac{\mathbb{P} : t[rec\,x\,t/x] \xrightarrow{p} t'}{\mathbb{P} : rec\,x\,t \xrightarrow{p} t'} \qquad \frac{\mathbb{P} : t_j \xrightarrow{p} t'}{\mathbb{P} : \Sigma_{i\in I} t_i \xrightarrow{p} t'}\ j \in I$$

$$\frac{}{.\mathbb{P} : .t \xrightarrow{.} t} \qquad \frac{.\mathbb{P} : u \xrightarrow{.} u' \qquad \mathbb{Q} : t[u'/x] \xrightarrow{q} t'}{\mathbb{Q} : [u > .x \Rightarrow t] \xrightarrow{q} t'}$$

$$\frac{\mathbb{Q} : t[u/x] \xrightarrow{p} t'}{\mathbb{P} \to \mathbb{Q} : \lambda x\,t \xrightarrow{u\mapsto p} t'} \qquad \frac{\mathbb{P} \to \mathbb{Q} : t \xrightarrow{u\mapsto p} t'}{\mathbb{Q} : t\,u \xrightarrow{p} t'}$$

$$\frac{\mathbb{P}_a : t \xrightarrow{p} t'}{\Sigma_{a\in A} a\mathbb{P}_a : a\,t \xrightarrow{a\,p} t'} \qquad \frac{\Sigma_{a\in A} a\mathbb{P}_a : t \xrightarrow{a\,p} t'}{\mathbb{P}_a : \pi_a(t) \xrightarrow{p} t'}$$

$$\frac{\mathbb{P}_j[\mu\vec{P}\vec{\mathbb{P}}/\vec{P}] : t \xrightarrow{p} t'}{\mu_j\vec{P}\vec{\mathbb{P}} : t \xrightarrow{p} t'}$$

"Type preservation"

Proposition 8.2 *Suppose* $\mathbb{P} : t \xrightarrow{p} t'$ *where* $\mathbb{P} : p : \mathbb{P}'$. *Then* $t' : \mathbb{P}'$.

Proof: By a simple induction on derivations.

Bisimulation

A relation R is **type-respecting** if it is a family of relations $R_{\mathbb{P}}$ s.t. if

$t \; R_{\mathbb{P}} \; u$ then $t : \mathbb{P}$ and $u : \mathbb{P}$

closed terms

R is a **bisimulation** if whenever $t \; R_{\mathbb{P}} \; u$:

① if $\mathbb{P} : t \xrightarrow{p} t' : \mathbb{P}'$ then $\exists u'$ s.t.
$\mathbb{P} : u \xrightarrow{p} u' : \mathbb{P}'$ and $t' \; R_{\mathbb{P}'} \; u'$

② if $\mathbb{P} : u \xrightarrow{p} u' : \mathbb{P}'$ then $\exists t'$ s.t.
$\mathbb{P} : t \xrightarrow{p} t' : \mathbb{P}'$ and $t' \; R_{\mathbb{P}'} \; u'$

Bisimilarity:

$t \sim u$ iff $\exists$ bisimulation R s.t. $t \; R_{\mathbb{P}} \; u$ for some $\mathbb{P}$.

Products

For types $\mathbb{P}$, $\mathbb{Q}$, define the product type

$$\mathbb{P} \& \mathbb{Q} \equiv 1\mathbb{P} + 2\mathbb{Q}$$

For terms $t : \mathbb{P}$ and $u : \mathbb{Q}$

$$(t, u) \equiv 1t + 2q$$

Projections

$$fst(v) \equiv \pi_1(v) \qquad snd(v) \equiv \pi_2(v)$$

Actions : For actions p of type $\mathbb{P}$ and q of type $\mathbb{Q}$

$$(p, -) \equiv 1p \qquad (-, q) \equiv 2q$$

Rules.

$$\frac{\Gamma \vdash t : \mathbb{P} \quad \Gamma \vdash u : \mathbb{Q}}{\Gamma \vdash (t, u) : \mathbb{P} \& \mathbb{Q}} \qquad \frac{\Gamma \vdash v : \mathbb{P} \& \mathbb{Q}}{\Gamma \vdash fst(v) : \mathbb{P}} \qquad \frac{\Gamma \vdash v : \mathbb{P} \& \mathbb{Q}}{\Gamma \vdash snd(v) : \mathbb{Q}}$$

$$\frac{\mathbb{P} : p : \mathbb{P}'}{\mathbb{P} \& \mathbb{Q} : (p, -) : \mathbb{P}'} \qquad \frac{\mathbb{Q} : q : \mathbb{Q}'}{\mathbb{P} \& \mathbb{Q} : (-, q) : \mathbb{Q}'}$$

General patterns

$$p ::= .x \mid u \mapsto p \mid a\,p$$

$$[u > a\,p \Rightarrow t] \underset{\text{def}}{\equiv} [\pi_a(u) > p \Rightarrow t]$$

$$[v > (u \mapsto p) \Rightarrow t] \underset{\text{def}}{\equiv} [(v\,u) > p \Rightarrow t]$$

$$\frac{\mathbb{P} : u \xrightarrow{p} u' \qquad \mathbb{Q} : t[u'/x] \xrightarrow{q} t'}{\mathbb{Q} : [u > p \Rightarrow t] \xrightarrow{q} t'}$$

var. x

is derivable (elide variable in p when an action)

— induction on p.

Type rule

$$\frac{\Gamma \vdash u : \mathbb{P} \qquad \Gamma, x : \mathbb{R} \vdash t : \mathbb{Q}}{\Gamma \vdash [u > p \Rightarrow t] : \mathbb{Q}}$$

where $\mathbb{P} : p : \mathbb{R}$

Pure CCS

§ 8.6.1

Types:

$$\mathbb{P} \stackrel{\text{def}}{=} \tau.\mathbb{P} + \sum_{n\in N} n.\mathbb{P} + \sum_{n\in N} \bar{n}.\mathbb{P}$$

Interpretation:

HOPLA

$$[\![x]\!] = x \qquad [\![\mathrm{rec}\ x\ P]\!] = \mathrm{rec}\ x\ [\![P]\!]$$

CCS

$$[\![\alpha.P]\!] = \alpha.[\![P]\!] \qquad [\![\sum_{i\in I} P_i]\!] = \sum_{i\in I} [\![P_i]\!]$$

$$[\![P \mid Q]\!] = \mathit{Par}\ [\![P]\!]\ [\![Q]\!]$$

$$[\![P \backslash S]\!] = \ldots$$

$$\begin{aligned} \mathit{Par} \stackrel{\text{def}}{=} \mathrm{rec}\ p\ \lambda x\ \lambda y\ & \sum_{\alpha} [x > \alpha.x' \Rightarrow \alpha.(p\ x'\ y)] \\ & + \sum_{\alpha} [y > \alpha.y' \Rightarrow \alpha.(p\ x\ y')] \\ & + \sum_{l} [x > l.x' \Rightarrow [y > \bar{l}.y' \Rightarrow \\ & \qquad \tau.(p\ x'\ y')]] \end{aligned}$$

Higher-order!

CCS with value passing:

$$\mathbb{P} = \overset{\tau.t}{\tau.\mathbb{P}} + \overset{a?v.t}{\sum_{a \in L} a? \sum_{v \in V} v.\mathbb{P}} + \overset{a!v.t}{\sum_{a \in L} a! \sum_{v \in V} v.\mathbb{P}}$$

[In exercise restrict booleans to tests for equality]

'Early' ↗

'Late' ↘

$$\mathbb{P} = \tau.\mathbb{P} + \overset{a?.f}{\sum_{a} a?.\left(\sum_{v} v.\mathbb{P}\right)} + \sum_{a} a! \sum_{v} v.\mathbb{P}$$