Topics in Concurrency: Problem sheet 2

*You might find the questions marked ** quite difficult. Attempt them seriously, but don't be discouraged if you don't get very far with them.*

1. This question is on (strong) bisimilarity $\sim$ and weak bisimilarity $\approx$.

   (a) Is every bisimulation a weak bisimulation? Either provide a brief proof or a counterexample.

   (b) Give a weak bisimulation to show that $p \approx \tau.p$ for any CCS process p. Give a brief proof showing that this is a weak bisimulation.

   (c) Show that weak bisimilarity is not a congruence by explaining briefly why $\mathbf{nil} + a.\mathbf{nil} \not\approx \tau.\mathbf{nil} + a.\mathbf{nil}$.

2. Desribe, without proof, how to express maximum fixed points $\nu Y.A$ in the modal-μ calculus in terms of minimum fixed points.

3. Describe, without proof, the meaning of the assertions

   (a) $\nu Z.\langle c\rangle Z$

   (b) $\mu Z.\langle c\rangle Z$

   (c) $\nu Z(A \wedge ([c]F \vee \langle c\rangle Z))$

   (d) $\mu Z.(B \vee (A \wedge \langle c\rangle Z))$

   (e) $\nu Z.(B \vee (A \wedge \langle c\rangle Z))$

4. Prove that a finite-state process p satisfies

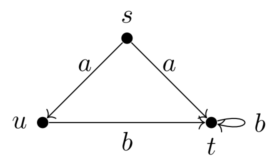
$$\nu Z.(B \vee (A \wedge [-]Z))$$

   iff, for all paths π from p, either $\pi_i \models A$ for all states π_i on the path or there exists n such that $\pi_n \models B$ and $\pi_i \models A$ for all $i < n$.

5. Show the function φ taking Z, a subset of states of a transition system, to $B \vee (A \wedge \langle -\rangle Z)$ is $\bigcup$-continuous.

6. Use the local model checking algorithm to determine whether or not the state s in the labelled transition system below satisfies the assertion

$$[a]\nu Y.(\langle b\rangle T \wedge [b]Y).$$



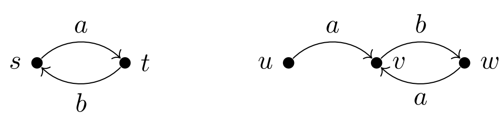
7. An algorithm for determining whether states in a finite transition system are bisimilar can be presented as

$$
\begin{array}{lcl}
P \vdash s \sim t & \rightarrow & \textbf{true} \qquad \text{if } (s,t) \in P \\
P \vdash s \sim t & \rightarrow & \bigwedge_{\{s',a \mid s \xrightarrow{a} s'\}} \bigvee_{\{t' \mid t \xrightarrow{a} t'\}} P \cup \{(s,t)\} \vdash s' \sim t' \\
 & & \wedge\ \bigwedge_{\{t',a \mid t \xrightarrow{a} t'\}} \bigvee_{\{s' \mid s \xrightarrow{a} s'\}} P \cup \{(s,t)\} \vdash s' \sim t' \\
 & & \text{if } (s,t) \notin P
\end{array}
$$

in which P ranges over subsets of pairs of states.

Conjunctions and disjunctions can be reduced in any sensible manner. Recall that the empty conjunction is equivalent to **true** and the empty disjunction is equivalent to **false**.

(a) Apply the algorithm to show that $\emptyset \vdash s \sim u \rightarrow^* \textbf{true}$ in the transition system



(b) ** Briefly outline a proof that the algorithm determines whether two processes are bisimilar when starting with $P = \emptyset$. [Hint: write down a statement of correctness of the algorithm and consider what the proof techniques were when establishing that the local model checking algorithm for modal-μ is correct]

(c) As well as the correctness of the algorithm above, what other theorems/lemmas would you need to show that if $\emptyset \vdash p \sim q \rightarrow^* \textbf{false}$ then there exists a formula A of modal-μ such that $p \models A$ and $q \models \neg A$ for finite-state processes p and q?

8. ** The proof that (strong) bisimilarity and logical equivalence in Hennessy-Milner logic coincide made use of a possibly-infinite conjunction. Give, without proof, two non-bisimilar states of a transition system that cannot be distinguished by finite formulas

$$A ::= \langle a \rangle A \mid \neg A \mid A_1 \wedge A_2$$

[Hint: such a transition system is necessarily not finite. In fact, it is not *image finite*: there exists a state from which there is an infinite number of transitions.]