Super advanced functional programming

Or: dependently-typed programming in Agda

Dr. Dominic Mulligan

Programming, Logic, and Semantics Group,
University of Cambridge

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System $F_\omega$

Very expressive type theory:

- Used as a compiler intermediate language (e.g. GHC)
- Can embed almost all useful (co)datatypes within it
- Can express common programming abstractions with higher-kinds
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But:

- The monadic abstraction has associated laws
- Must be checked by hand, on pen-and-paper
How can we internalise this checking of laws?
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Requires an embedded higher-order logic

Requires types that depend on terms:

\[ \forall x : \mathbb{N}. \; x + 0 = x \]

is a type, and its inhabitants are proofs
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Moved from left plane to right plane of \( \lambda \)-cube
Agda can be seen as both a programming language and a proof checker.
What’s the advantage?

Agda can be seen as both a programming language and a proof checker

Agda:

- Allows us to encode very powerful invariants in types that guarantee program correctness
- Acts as a foundation for mathematics, based not on sets, but on functions and types
Rest of this lecture: an interactive introduction to Agda...