Additional exercises

Exercise 1
Prove soundness of the partial correctness rule for conditionals by proving that if 
\[ \models \{P \land B\} \ C_1 \ \{Q\}\] and \( \models \{P \land \neg B\} \ C_2 \ \{Q\}\) then \( \models \{P\} \text{if } B \text{ then } C_1 \ \text{else } C_2 \ \{Q\}\).

Exercise 2
Provide a program \( C \) such that the following partial correctness triple holds or argue why such a \( C \) cannot exist:
\[ \{X = x \land Y = y \land x \neq y\} \ C \ \{x = y\}\]

Exercise 3
Show that the alternative assignment axiom, \( \vdash \{P\} \ V := E \ \{P[E/V]\}\), is unsound by providing a \( P, V \) and \( E \) such that \( \lnot (\models \{P\} \ V := E \ \{P[E/V]\})\).

Exercise 4
Prove that the following backwards-reasoning sequenced assignment rule is derivable from the normal proof rules of Hoare logic:
\[ \vdash \{P\} \ C \ \{Q[E/V]\}\]
\[ \vdash \{P\} \ C; V := E \ \{Q\}\]

Exercise 5
Propose a loop invariant for proving the following partial correctness triple:
\[ \{X = x \land Y = y \land Z = 0 \land A = 1 \land Y \geq 0\}\]
\[ \text{while } A \leq Y \text{ do } (Z := Z + X; A := A + 1) \]
\[ \{Z = x \cdot y\}\]

Exercise 6
Prove soundness of the separation logic assignment rule by proving that
\[ \models \{E_1 \mapsto \_\} \ [E_1] := E_2 \ \{E_1 \mapsto E_2\}\].
Exercise 7

Propose a loop invariant for proving the following partial correctness triple in Separation logic:

\[
\{(N \geq 0 \land X = 0) \land Y \rightarrow 0\}
\]

\[
\text{while } X < N \text{ do } (A := [Y]; X := X + 1; [Y] := A + X)
\]

\[
\{Y \rightarrow \sum_{i=1}^{N} i\}
\]