

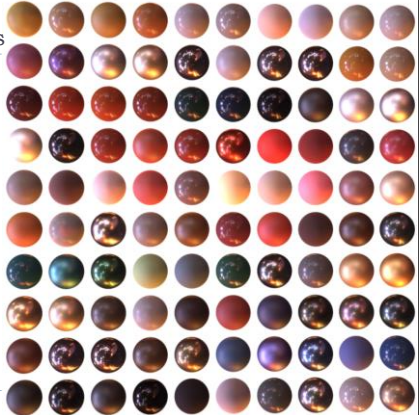
Reflection models and radiometry

Advanced Graphics

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Applications

- ▶ To render realistic looking materials
- ▶ Applications also in computer vision, optical engineering, remote sensing, etc.
 - ▶ To understand how surfaces reflect light




Applications

- ▶ Many applications require faithful reproduction of material appearance

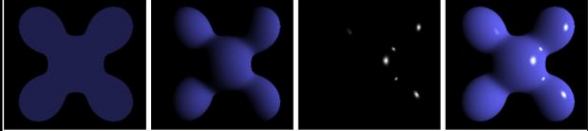


Source: <http://ikea.com/>



Source: <http://www.mercedes-benz.co.uk/>

Last year you learned about Phong reflection model



Ambient + Diffuse + Specular = Phong Reflection

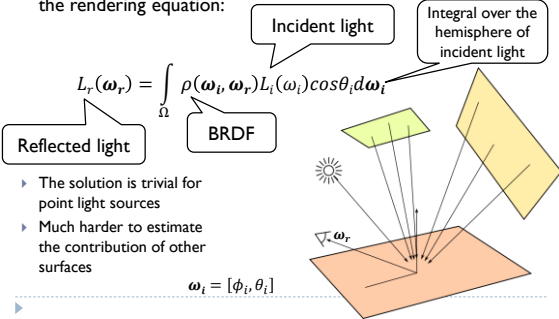
Good for plastic objects, not very accurate for other materials

Rendering equation

- ▶ Most rendering methods require solving an (approximation) of the rendering equation:

$$L_r(\omega_r) = \int_{\Omega} \rho(\omega_i, \omega_r) L_i(\omega_i) \cos\theta_i d\omega_i$$

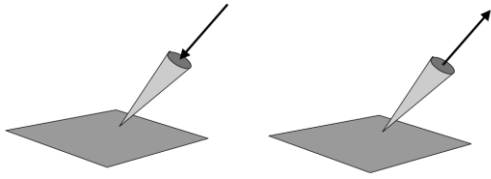
$\omega_i = [\phi_i, \theta_i]$



- ▶ The solution is trivial for point light sources
- ▶ Much harder to estimate the contribution of other surfaces

Radiance

- ▶ Power of light per unit projected area per unit solid angle
- ▶ Symbol: $L(x, \omega_i)$
- ▶ Units: $\frac{W}{m^2 sr}$ Steradian



Solid angle

Angle in 2D

$\theta = \frac{L}{R} [rad]$

Equivalent in 3D

$\omega = \frac{A}{R^2} [sr]$

Full circle = 2π radian ▶ Full sphere = 4π steradians

Radiance

Position

Power of light

Incoming direction

Projected area

$$L(x, \omega_i) = \frac{d\phi}{d\omega dA \cos\theta}$$

- ▶ Power per solid angle per projected surface area
- ▶ Invariant along the direction of propagation (in vacuum)
- ▶ Response of a camera sensor or a human eye is related to radiance
- ▶ Pixel values in image are related to radiance (projected along the view direction)

Irradiance and Exitance

- ▶ Power per unit area
- ▶ Irradiance: $H(x)$ – incident power per unit area
- ▶ Exitance / radiosity: $E(x)$ – exitant power per unit area
- ▶ Units: $\frac{W}{m^2}$

Irradiance

Exitance / Radiosity

Relation between Irradiance and Radiance

- ▶ Irradiance is an integral over all incoming rays
- ▶ Integration over a hemisphere Ω :

$$H = \int_{\Omega} L(x, \omega_i) \cos\theta d\omega$$

- ▶ In the spherical coordinate system, the differential solid angle is:

$$d\omega = \sin\theta d\theta d\phi$$

- ▶ Therefore:

$$H = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} L(x, \omega_i) \cos\theta \sin\theta d\theta d\phi$$

- ▶ For constant radiance:

$$H = \pi L$$

BRDF: Bidirectional Reflectance Distribution Function

Differential radiance of reflected light

$$\rho(\omega_i, \omega_r) = \frac{dL_r(\omega_r)}{dH_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i) \cos\theta_i d\omega_i}$$

Differential irradiance of incoming light

Source: Wikipedia

- ▶ BRDF is measured as a ratio of reflected radiance to irradiance
- ▶ Because it is difficult to measure $L_i(\omega_i)$, BRDF is not defined as the ratio $L_r(\omega_r)/L_i(\omega_i)$

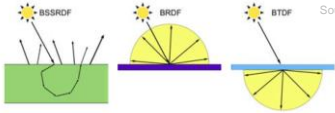
BRDF of various materials

Incident light

Reflected light

- ▶ The diagrams show the distribution of reflected light for the given incoming direction
- ▶ The material samples are close but not accurate matches for the diagrams

Other material models

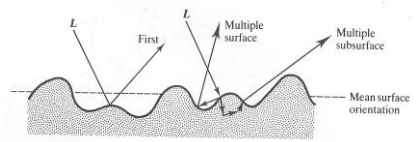


Source: Guarnera et al. 2016

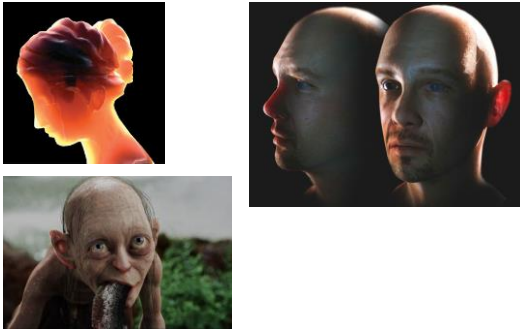
- ▶ Bidirectional Scattering Surface Reflectance Distribution F.
- ▶ Bidirectional Reflectance Distribution Function
- ▶ Bidirectional Transfer Distribution Function
- ▶ But also: BTF, SVBRDF, BSDF
- ▶ In this lecture we will focus on BRDF only

Sub-surface scattering

- ▶ Light enters material and is scattered several times before it exits
- ▶ Examples - human skin: hold a flashlight next to your hand and see the color of the light
- ▶ The effect is expensive to compute
- ▶ But approximate methods exist



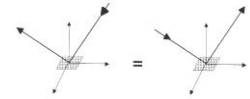
Subsurface scattering - examples



BRDF Properties

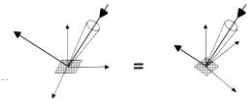
- ▶ Helmholtz reciprocity principle
- ▶ BRDF remains unchanged if incident and reflected directions are interchanged

$$\rho(\omega_r, \omega_i) = \rho(\omega_i, \omega_r)$$



- ▶ Smooth surface: isotropic BRDF
- ▶ reflectivity independent of rotation around surface normal
- ▶ BRDF has only 3 instead of 4 directional degrees of freedom

$$\rho(\theta_i, \theta_r, \phi_r - \phi_i)$$



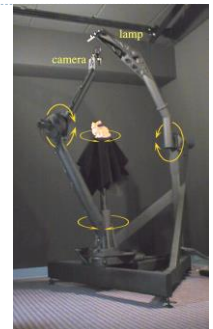
BRDF Properties

- ▶ Characteristics
 - ▶ BRDF units [1/sr]
 - ▶ Not intuitive
- ▶ Range of values:
 - ▶ From 0 (absorption) to ∞ (reflection, δ -function)
- ▶ Energy conservation law

$$\int_{\Omega} \rho(\omega_r, \omega_i) \cos\theta_i d\omega_i \leq 1$$
 - ▶ No self-emission
 - ▶ Possible absorption
- ▶ Reflection only at the point of entry ($x_i = x_r$)
 - ▶ No subsurface scattering

BRDF Measurement

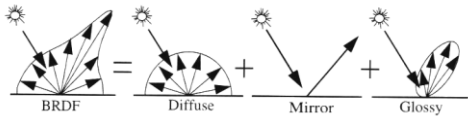
- ▶ Gonio-Reflectometer
- ▶ BRDF measurement
 - ▶ point light source position (θ, φ)
 - ▶ light detector position (θ_o, φ_o)
- ▶ 4 directional degrees of freedom
- ▶ BRDF representation
 - m incident direction samples (θ, φ)
 - n outgoing direction samples (θ_o, φ_o)
 - $m*n$ reflectance values (large!!!)



Stanford light gantry

BRDF Modeling

- ▶ It is common to split BRDF into diffuse, mirror and glossy components
- ▶ Ideal diffuse reflection
 - ▶ Lambert's law
 - ▶ Matte surfaces
- ▶ Ideal specular reflection
 - ▶ Reflection law
 - ▶ Mirror
- ▶ Glossy reflection
 - ▶ Directional diffuse
 - ▶ Shiny surfaces

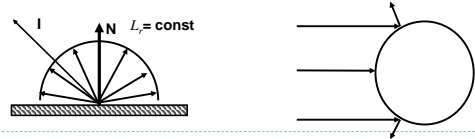


Diffuse Reflection

- ▶ Light equally likely to be reflected in any outgoing direction (independent of incident direction)
- ▶ Constant BRDF $\rho(\omega_r, \omega_i) = k_d = const$

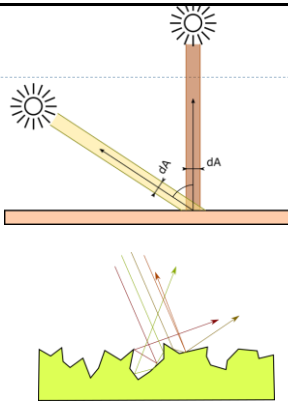
$$L_r(\omega_r) = \int_{\Omega} k_d L_i(\omega_i) \cos \theta_i d\omega_i = k_d \int_{\Omega} L_i(\omega_i) \cos \theta_i d\omega_i = k_d H$$

- ▶ k_d : diffuse coefficient, material property [1/sr]



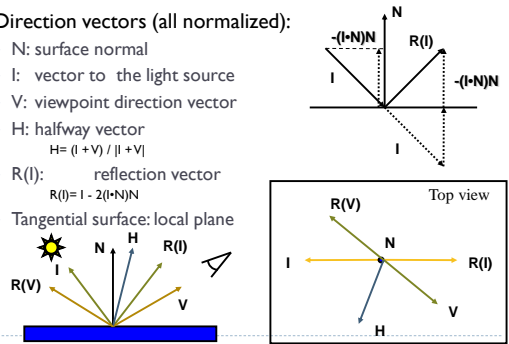
Diffuse reflection

- ▶ Cosine term
 - ▶ The surface receives less light per unit area at steep angles of incidence
- ▶ Rough, irregular surface results in diffuse reflection
 - ▶ Light is reflected in random direction
 - ▶ (this is just a model, light interaction is more complicated)



Reflection Geometry

- ▶ Direction vectors (all normalized):
 - ▶ N : surface normal
 - ▶ I : vector to the light source
 - ▶ V : viewpoint direction vector
 - ▶ H : half-way vector $H = (I + V) / \|I + V\|$
 - ▶ $R(I)$: reflection vector $R(I) = I - 2(I \cdot N)N$
- ▶ Tangential surface: local plane

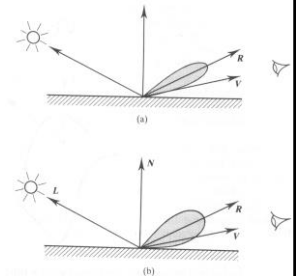


Glossy Reflection



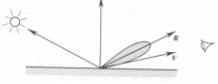
Glossy Reflection

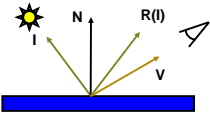
- ▶ Due to surface roughness
- ▶ Empirical models
 - ▶ Phong
 - ▶ Blinn-Phong
- ▶ Physical models
 - ▶ Blinn
 - ▶ Cook & Torrance



Phong Reflection Model

- ▶ Cosine power lobe

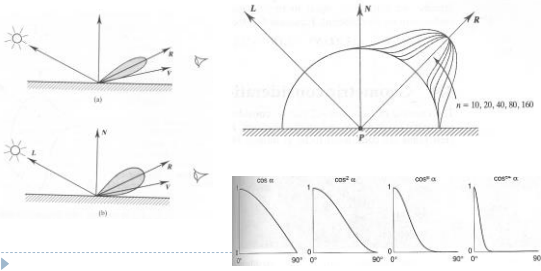
$$\rho(\omega_r, \omega_i) = k_s (R \cdot V)^{k_e}$$

- ▶ Dot product & power
- ▶ Not energy conserving/reciprocal
- ▶ Plastic-like appearance



Phong Exponent k_e

$$\rho(\omega_r, \omega_i) = k_s (R \cdot V)^{k_e}$$

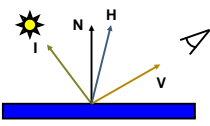
- ▶ Determines the size of highlight



Blinn-Phong Reflection Model

- ▶ Blinn-Phong reflection model

$$\rho(\omega_r, \omega_i) = k_s (H \cdot N)^{k_e}$$
 - ▶ Light source, viewer far away
 - ▶ I, R constant: H constant
 - ▶ H less expensive to compute



Phong Illumination Model

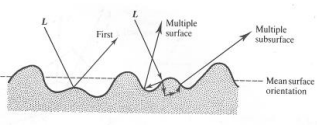

- ▶ Extended light sources: l point light sources

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e}$$
 (Phong)

$$L_r = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e}$$
 (Blinn)
- ▶ Colour of specular reflection equal to light source
- ▶ Heuristic model
 - ▶ Contradicts physics
 - ▶ Purely local illumination
 - ▶ Only direct light from the light sources
 - ▶ No further reflection on other surfaces
 - ▶ Constant ambient term
- ▶ Often: light sources & viewer assumed to be far away

Micro-facet model

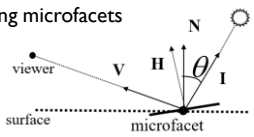
- ▶ We can assume that at small scale materials are made up of small facets
 - ▶ Facets can be described by the distribution of their sizes and directions D
 - ▶ Some facets are occluded by other, hence there is also a geometrical attenuation term G
 - ▶ And we need to account for Fresnel reflection (see next slides)

$$\rho(\omega_i, \omega_r) = \frac{D \cdot G \cdot F}{4 \cos \theta_i \theta_r}$$



Ward Reflection Model

- ▶ BRDF

$$\rho = \frac{k_d}{\pi} + k_s \frac{1}{\sqrt{(I \cdot N)(V \cdot N)}} \frac{\exp\left(-\tan^2 \frac{\angle(H, N)}{\sigma^2}\right)}{2\pi\sigma^2}$$
- ▶ σ standard deviation (RMS) of surface slope
- ▶ Simple expansion to anisotropic model (σ_x, σ_y)
- ▶ Empirical, not physics-based
- ▶ Inspired by notion of reflecting microfacets
- ▶ Convincing results
- ▶ Good match to measured data



Cook-Torrance model

- ▶ Can model metals and dielectrics
- ▶ Sum of diffuse and specular components

$$\rho(\omega_i, \omega_r) = \rho_d(\omega_i) + \rho_s(\omega_i, \omega_r)$$

- ▶ Specular component: $\rho(\omega_i, \omega_r) = \frac{D(h) G(\omega_i, \omega_r) F(\omega_i)}{\pi \cos \theta_i \cos \theta_r}$

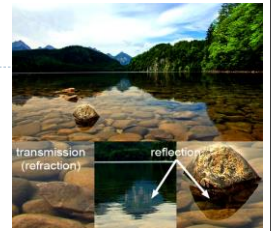
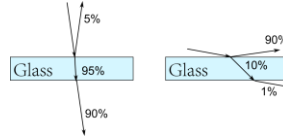
- ▶ Distribution of microfacet orientations: $D(h) = \cos \theta_r e^{-\left(\frac{\alpha}{m}\right)^2}$

- ▶ Geometrical attenuation factor
 - ▶ To account to self-masking and shadowing

$$G(\omega_i, \omega_r) = \min\left\{1, \frac{2(n \cdot h)(n \cdot \omega_r)}{\omega_r \cdot h}, \frac{2(n \cdot h)(n \cdot \omega_i)}{\omega_r \cdot h}\right\}$$

Roughness parameter

Fresnel term



Example from: <https://www.scratchapixel.com/lessons/3d-basic-rendering/introduction-to-shading/reflection-refraction-fresnel>

- ▶ The light is more likely to be reflected rather than transmitted near grazing angles
- ▶ The effect is modelled by Fresnel equation: it gives the probability that a photon is reflected rather than transmitted (or absorbed)

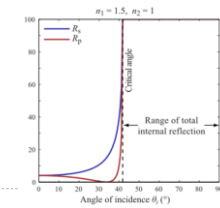
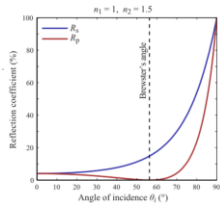
Fresnel equations

- ▶ Reflectance for s-polarized light:

$$R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2 = \left| \frac{n_1 \cos \theta_i - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}}{n_1 \cos \theta_i + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2}} \right|^2$$

- ▶ Reflectance for p-polarized light:

$$R_p = \left| \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} \right|^2 = \left| \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2} - n_2 \cos \theta_i}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_i\right)^2} + n_2 \cos \theta_i} \right|^2$$



Fresnel term

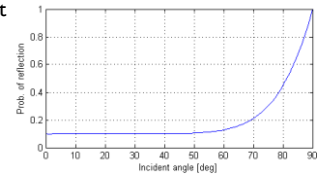
- ▶ In Computer Graphics the Fresnel equation is approximated by Schlick's formula [Schlick, 94]:

$$R(\theta, \lambda) = R_0(\lambda) + (1 - R_0(\lambda))(1 - \cos \theta)^5$$

- ▶ where $R_0(\lambda)$ is reflectance at normal incidence and λ is the wavelength of light

- ▶ For dielectrics (such as glass):

$$R_0(\lambda) = \left(\frac{n(\lambda) - 1}{n(\lambda) + 1} \right)^2$$



Which one is Phong / Cook-Torrance ?



Image based lighting (IBL)

1. Capture an HDR image of a light probe

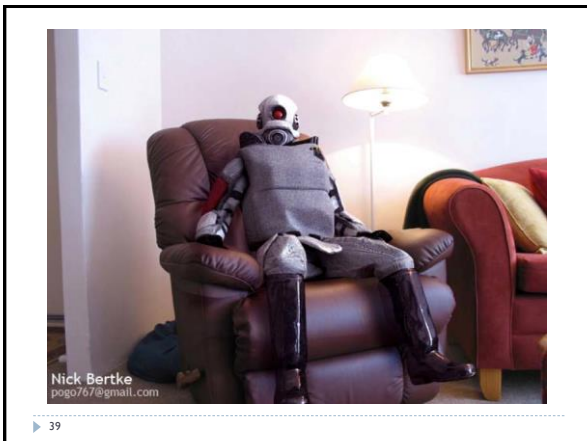
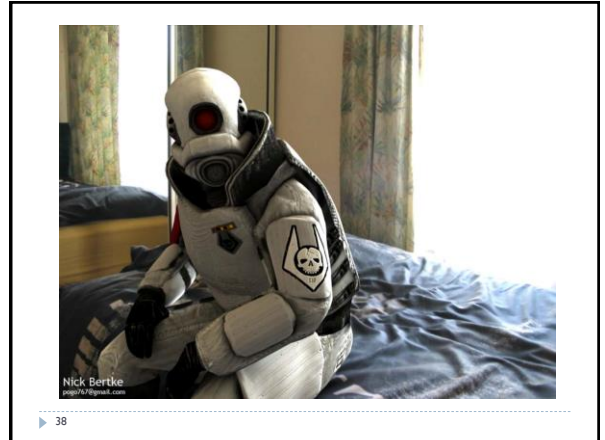
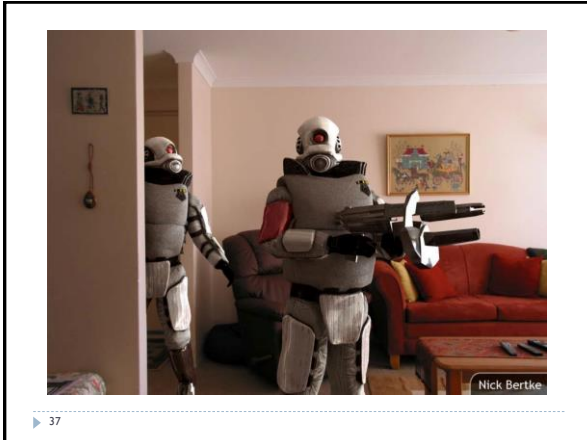


2. Create an illumination (cube) map



3. Use the illumination map as a source of light in the scene

The scene is surrounded by a cube map



Further reading

- ▶ **A. Watt, 3D Computer Graphics**
 - ▶ Chapter 7: Simulating light-object interaction: local reflection models
- ▶ **Eurographics 2016 tutorial**
 - ▶ D. Guarnera, G. C. Guarnera, A. Ghosh, C. Denk, and M. Glencross
 - ▶ BRDF Representation and Acquisition
 - ▶ DOI: 10.1111/cgf.12867
- ▶ Some slides have been borrowed from Computer Graphics lecture by Hendrik Lensch
 - ▶ <http://resources.mpi-inf.mpg.de/departments/d4/teaching/ws200708/cg/slides/CG07-Brdf+Texture.pdf>