Lecture 5: Language Modelling in Information Retrieval and Classification

Information Retrieval
Computer Science Tripos Part II

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[1]Adapted from Simone Teufel's original slides

Upcoming today

- Query-likelihood method in IR

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- Document Language Modelling

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- Classification

Overview

1. Query Likelihood
2. Estimating Document Models
3. Smoothing
4. Naive Bayes Classification

Language Model

- A model for how humans generate language
- Used in many language orientated-tasks (MT, word prediction, IR)
- Usually probabilistic in nature (e.g. multinomial, neural)

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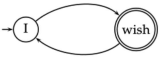

I wish
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I wish I wish I wish
I wish I wish I wish I wish I wish I wish
...

CANNOT GENERATE: wish I wish

▶ **Figure 12.1** A simple finite automaton and some of the strings in the language it generates. → shows the start state of the automaton and a double circle indicates a (possible) finishing state.

What is a document language model?

- A model for how an author generates a document on a particular topic
- The document itself is just one sample from the model (i.e. ask the author to write the document again and he/she will invariably write something similar, but not exactly the same)
- A probabilistic generative model for documents

Two Document Models

Model M_1		Model M_2	
the	0.2	the	0.15
a	0.1	a	0.12
frog	0.01	frog	0.0002
toad	0.01	toad	0.0001
said	0.03	said	0.03
likes	0.02	likes	0.04
that	0.04	that	0.04
dog	0.005	dog	0.01
cat	0.003	cat	0.015
monkey	0.001	monkey	0.002
...	...	...	...

► **Figure 12.3** Partial specification of two unigram language models.

$$\sum_{t \in V} P(t|M_d) = 1 \tag{1}$$

Query Likelihood Method (I)

- Users often pose queries by thinking of words that are likely to be in *relevant* documents
- The query likelihood approach uses this idea as a principle for ranking documents
- Given a query string q, we rank documents by the likelihood of their document *models* M_d generating q

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Note: $P(d)$ is uniform if we have no reason a priori to favour one document over another. Useful priors (based on aspects such as authority, length, novelty, freshness, popularity, click-through rate) could easily be incorporated.

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$$\tag{6}$$

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$$P(q|M_1) > P(q|M_2) \tag{7}$$

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Maximum likelihood estimates

click=0.5, go=0.125, the=0.125, shears=0.125, boys=0.125,

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Sample query

$P(shears\ boys\ hair|M_d) =$

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What if the query is long?

Make sure no non-zero probabilities

- Only assign a zero probability when something *cannot* happen
- Remember that the document model is a generative explanation
- If a person was to rewrite the document he/she may include *hair* or indeed some other words

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Some type of smoothing

click=0.4, go=0.1, the=0.1, shears=0.1, boys=0.1, hair=0.01, man=0.01, the=0.001, bacon=0.0001, ...

Overview

How to smooth

ML estimates

$$\hat{P}(t|M_d) = \frac{tf_t}{|d|} \tag{8}$$

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Linear Smoothing

$$\hat{P}(t|M_d) = \lambda \frac{tf_t}{|d|} + (1-\lambda)\hat{P}(t|M_c) \tag{9}$$

where λ is a smoothing parameter between 0 and 1, and $\hat{P}(t|M_c) = \frac{cf_t}{|c|}$ is the estimated probability of seeing t in general (i.e. ct_t is the frequency of t in the entire document collection of $|c|$ tokens).

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$$\hat{P}(t|M_d) = \lambda \frac{tf_t}{|d|} + (1 - \lambda)\frac{cf_t}{|c|} \tag{10}$$

Dirichlet Smoothing has been found to be more effective in IR where λ is $\frac{|d|}{\alpha+|d|}$. Plugging this in yields:

$$\hat{P}(t|M_d) = \frac{|d|}{\alpha + |d|}\frac{tf_t}{|d|} + \frac{\alpha}{\alpha + |d|}\frac{cf_t}{|c|} \tag{11}$$

where α is interpreted as the background mass (pseudo-counts).

Bayesian Intuition

We should have more trust (belief) in ML estimates that are derived from longer documents (see the $\frac{|d|}{\alpha+|d|}$ factor).

Putting this all together

Rank documents according to:

$$P(q|d) = \prod_{t \in q} \left(\frac{|d|}{\alpha + |d|} \frac{tf_t}{|d|} + \frac{\alpha}{\alpha + |d|} \frac{cf_t}{|c|} \right) \qquad (12)$$

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or

$$log\ P(q|d) = \sum_{t \in q} log \left(\frac{|d|}{\alpha + |d|} \frac{tf_t}{|d|} + \frac{\alpha}{\alpha + |d|} \frac{cf_t}{|c|} \right) \tag{13}$$

Pros and Cons

- It is principled, intuitive, simple, and extendable
- Aspects of *tf* and *idf* are incorporated quite naturally
- It is computationally efficient for large scale corpora
- More complex language models (markov-models) can be adopted and priors can be added
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- Both documents and queries are modelled as simple strings of symbols
- No formal treatment of relevance
- Therefore model does not handle relevance feedback automatically

Extensions

- Relevance-based language models (very much related to Naive-Bayes classification) incorporate the idea of relevance and are useful for capturing feedback
- Treating the query as being drawn from a query model (useful for long queries)
- Markov-chain models for document modelling
- Use different generative distributions (e.g. replacing the multinomial with neural models)

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- Our goal in Naive Bayes classification is to find the "best" class.
- The best class is the most likely or maximum a posteriori (MAP) class c_{map}:

$$c_{\text{map}} = \arg\max_{c \in \mathbb{C}} \hat{P}(c|d) = \arg\max_{c \in \mathbb{C}} \ \hat{P}(c) \prod_{1 \leq k \leq n_d} \hat{P}(t_k|c)$$

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- Since $\log(xy) = \log(x) + \log(y)$, we can sum log probabilities instead of multiplying probabilities.
- Since log is a monotonic function, the class with the highest score does not change.
- So what we usually compute in practice is:

$$c_{\text{map}} = \underset{c \in \mathbb{C}}{\arg\max} \, [\log \hat{P}(c) + \sum_{1 \leq k \leq n_d} \log \hat{P}(t_k|c)]$$

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  - We select the class with the most evidence.

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- T_{ct} is the number of tokens of t in training documents from class c (includes multiple occurrences)
- We've made a Naive Bayes independence assumption here: $\hat{P}(t_{k_1}|c) = \hat{P}(t_{k_2}|c)$, independent of positions k_1, k_2

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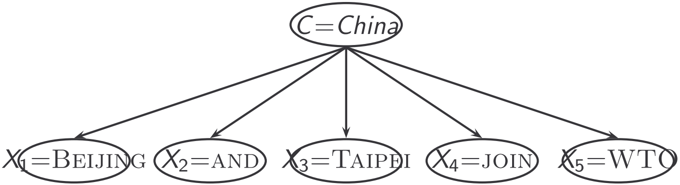

$$\begin{aligned} P(China|d) \quad \propto \quad & P(China) \cdot P(\text{BEIJING}|China) \cdot P(\text{AND}|China) \\ & \cdot P(\text{TAIPEI}|China) \cdot P(\text{JOIN}|China) \cdot P(\text{WTO}|China) \end{aligned}$$

- If WTO never occurs in class China in the train set:

$$\hat{P}(\text{WTO}|China) = \frac{T_{China,\text{WTO}}}{\sum_{t' \in V} T_{China,t'}} = \frac{0}{\sum_{t' \in V} T_{China,t'}} = 0$$

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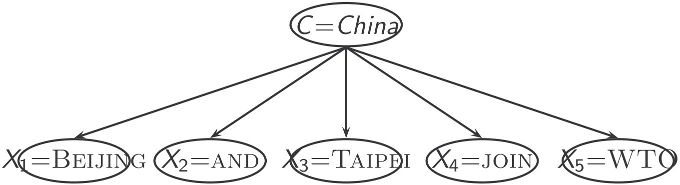

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\begin{aligned}
P(\mathit{China}|d) \quad \propto \quad & P(\mathit{China}) \cdot P(\textrm{Beijing}|\mathit{China}) \cdot P(\textrm{and}|\mathit{China}) \\
& \cdot P(\textrm{Taipei}|\mathit{China}) \cdot P(\textrm{join}|\mathit{China}) \cdot P(\textrm{WTO}|\mathit{China})
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- $\rightarrow$ We will get $P(China|d) = 0$ for any document that contains WTO!

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- Now: Add one to each count to avoid zeros:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V}(T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

To avoid zeros: Add-one smoothing

- Before:

$$\hat{P}(t|c) = \frac{T_{ct}}{\sum_{t' \in V} T_{ct'}}$$

- Now: Add one to each count to avoid zeros:

$$\hat{P}(t|c) = \frac{T_{ct} + 1}{\sum_{t' \in V} (T_{ct'} + 1)} = \frac{T_{ct} + 1}{(\sum_{t' \in V} T_{ct'}) + B}$$

- B is the number of bins – in this case the number of different words or the size of the vocabulary $|V| = M$

Example

	docID	words in document	in $c = China$?
training set	1	Chinese Beijing Chinese	yes
	2	Chinese Chinese Shanghai	yes
	3	Chinese Macao	yes
	4	Tokyo Japan Chinese	no
test set	5	Chinese Chinese Chinese Tokyo Japan	?

- Estimate parameters of Naive Bayes classifier
- Classify test document

$|text_c| = 8$
$|text_{\overline{c}}| = 3$
B=6 (vocabulary)

Example: Parameter estimates

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Priors: $\hat{P}(c) = 3/4$ and $\hat{P}(\overline{c}) = 1/4$

Conditional probabilities:

$$
\begin{aligned}
\hat{P}(\text{CHINESE}|c) &= (5+1)/(8+6) = 6/14 = 3/7 \\
\hat{P}(\text{TOKYO}|c) = \hat{P}(\text{JAPAN}|c) &= (0+1)/(8+6) = 1/14 \\
\hat{P}(\text{CHINESE}|\overline{c}) &= (1+1)/(3+6) = 2/9 \\
\hat{P}(\text{TOKYO}|\overline{c}) = \hat{P}(\text{JAPAN}|\overline{c}) &= (1+1)/(3+6) = 2/9
\end{aligned}
$$

The denominators are $(8+6)$ and $(3+6)$ because the lengths of $text_c$ and $text_{\overline{c}}$ are 8 and 3, respectively, and because the constant B is 6 as the vocabulary consists of six terms.

Example: Classification

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$$\begin{aligned}\hat{P}(c|d_5) &\propto 3/4 \cdot (3/7)^3 \cdot 1/14 \cdot 1/14 \approx 0.0003 \\ \hat{P}(\overline{c}|d_5) &\propto 1/4 \cdot (2/9)^3 \cdot 2/9 \cdot 2/9 \approx 0.0001\end{aligned}$$

Thus, the classifier assigns the test document to $c = China$. The reason for this classification decision is that the three occurrences of the positive indicator CHINESE in d_5 outweigh the occurrences of the two negative indicators JAPAN and TOKYO.

Time complexity of Naive Bayes

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testing	$\Theta(L_{\mathrm{a}} + \lvert\mathbb{C}\rvert M_{\mathrm{a}}) = \Theta(\lvert\mathbb{C}\rvert M_{\mathrm{a}})$

- L_{ave}: average length of a training doc, L_{a}: length of the test doc, M_{a}: number of distinct terms in the test doc, $\mathbb{D}$: training set, V: vocabulary, $\mathbb{C}$: set of classes

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- Generally: $\lvert\mathbb{C}\rvert\lvert V\rvert < \lvert\mathbb{D}\rvert L_{\text{ave}}$
- Test time is also linear (in the length of the test document).
- Thus: Naive Bayes is linear in the size of the training set (training) and the test document (testing). This is optimal.

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- Very fast; low storage requirements

Not covered

- Derivation of NB formula
- Evaluation of text classification

Summary

- Query-likelihood as a general principle for ranking documents in an unsupervised manner
  - Treat queries as strings
  - Rank documents according to their models
- Document language models
  - Know the difference between the document and the document model
  - Multinomial distribution is simple but effective
- Smoothing
  - Reasons for, and importance of, smoothing
  - Dirichlet (Bayesian) smoothing is very effective
- Classification
  - Text classification is supervised learning
  - Naive Bayes: simple baseline text classifier

Reading

- Manning, Raghavan, Schütze: Introduction to Information Retrieval (MRS), chapter 12: Language models for information retrieval
- MRS chapters 13.1-13.4 for text classification