

## The Meaning of Truth—In FOL!

For interpretation  $\mathcal{I}$  and valuation  $V$ , define  $\models_{\mathcal{I}, V}$  by recursion.

$\models_{\mathcal{I}, V} P(t)$       if  $I[P](\mathcal{I}_V[t])$  equals 1 (is true)

$\models_{\mathcal{I}, V} t = u$       if  $\mathcal{I}_V[t]$  equals  $\mathcal{I}_V[u]$

$\models_{\mathcal{I}, V} A \wedge B$       if  $\models_{\mathcal{I}, V} A$  and  $\models_{\mathcal{I}, V} B$

$\models_{\mathcal{I}, V} \exists x A$       if  $\models_{\mathcal{I}, V\{m/x\}} A$  holds for some  $m \in D$

Finally, we define

$\models_{\mathcal{I}} A$       if  $\models_{\mathcal{I}, V} A$  holds for all  $V$ .

A **closed** formula  $A$  is **satisfiable** if  $\models_{\mathcal{I}} A$  for some  $\mathcal{I}$ .

