

# Topics in Concurrency: Problem sheet 2

Lent 2015

*You might find the questions marked \*\* quite difficult. Attempt them seriously, but don't be discouraged if you don't get very far with them.*

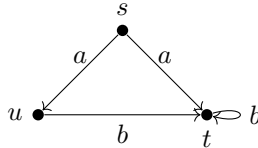
1. Describe, without proof, how to express maximum fixed points  $\nu Y.A$  in terms of minimum fixed points.
2. Describe, without proof, the meaning of the assertions
  - (a)  $\nu Z.\langle c \rangle Z$
  - (b)  $\mu Z.\langle c \rangle Z$
  - (c)  $\nu Z.(A \wedge ([c]F \vee \langle c \rangle Z))$
  - (d)  $\mu Z.(B \vee (A \wedge \langle c \rangle Z))$
  - (e)  $\nu Z.(B \vee (A \wedge \langle c \rangle Z))$
3. Prove that a finite-state process  $p$  satisfies

$$\nu Z.(B \vee (A \wedge [-]Z))$$

iff, for all paths  $\pi$  from  $p$ , either  $\pi_i \models A$  for all states  $\pi_i$  on the path or there exists  $n$  such that  $\pi_n \models B$  and  $\pi_i \models A$  for all  $i < n$ .

4. \*\* Show the function  $\varphi$  taking  $Z$ , a subset of states of a transition system, to  $B \vee (A \wedge \langle - \rangle Z)$  is  $\bigcup$ -continuous.
5. Use the local model checking algorithm to determine whether or not the state  $s$  in the labelled transition system below satisfies the assertion

$$[a]\nu Y.(\langle b \rangle T \wedge [b]Y).$$



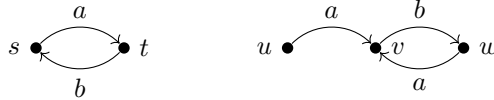
6. An algorithm for determining whether states in a finite transition system are bisimilar can be presented as

$$\begin{aligned}
 P \vdash s \sim t &\rightarrow \mathbf{true} && \text{if } (s, t) \in P \\
 P \vdash s \sim t &\rightarrow && \bigwedge_{\{s', a \mid s \xrightarrow{a} s'\}} \bigvee_{\{t' \mid t \xrightarrow{a} t'\}} P \cup \{(s, t)\} \vdash s' \sim t' \\
 &&& \wedge \bigwedge_{\{t', a \mid t \xrightarrow{a} t'\}} \bigvee_{\{s' \mid s \xrightarrow{a} s'\}} P \cup \{(s, t)\} \vdash s' \sim t' \\
 &&& \text{if } (s, t) \notin P
 \end{aligned}$$

in which  $P$  ranges over subsets of pairs of states.

Conjunctions and disjunctions can be reduced in any sensible manner. Recall that the empty conjunction is equivalent to **true** and the empty disjunction is equivalent to **false**.

- (a) Apply the algorithm to show that  $\emptyset \vdash s \sim u \rightarrow^* \mathbf{true}$  in the transition system



- (b) \*\* Briefly outline a proof that the algorithm determines whether two processes are bisimilar when starting with  $P = \emptyset$ . [Hint: write down a statement of correctness of the algorithm and consider what the proof techniques were when establishing that the local model checking algorithm for modal- $\mu$  is correct]
- (c) \*\* As well as the correctness of the algorithm above, what other theorems/lemmas would you need to show that if  $\emptyset \vdash p \sim q \rightarrow^* \mathbf{false}$  then there exists a formula  $A$  of modal- $\mu$  such that  $p \models A$  and  $q \models \neg A$  for finite-state processes  $p$  and  $q$ ?
7. \*\* The proof that (strong) bisimilarity and logical equivalence in Hennessy-Milner logic coincide made use of a possibly-infinite conjunction. Give, without proof, two non-bisimilar states of a transition system that cannot be distinguished by finite formulas

$$A ::= \langle a \rangle A \mid \neg A \mid A_1 \wedge A_2$$

[Hint: such a transition system is necessarily not finite. In fact, it is not *image finite*: there exists a state from which there is an infinite number of transitions. ]