Social and Technological Network Analysis

Lecture 6: Network Robustness and Applications

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In This Lecture

- We revisit power-law networks and define the concept of robustness
- We show the effect of random and targeted attacks on power law networks versus random networks
- We discuss applications to various networks
• Autonomous System (AS): a collection of networks under the same administration
• 2009: 25,000 ASs in the Internet
• By reading the routing tables of some gateways connected ASs, Internet topology information could be gathered
• October 08:
  – Over 30,000 ASs (including repeated entries)
  – Over 100,000 edges
Degree distribution of ASs: A scale free network!
Properties

- The top AS is connected to almost 10% of all ASs
- This connectedness drops rapidly
- Very high clustering coefficient for top 1000 hubs: an almost complete graph
- Most paths no longer than 3-4 hops
- Most ASs separated by shortest paths of maximum length of 6
The Internet Now [Sigcomm10]

• They monitored inter-domain traffic for 2 years
  – 3095 Routers
  – 110 ISPs
    • 18 Global
    • 38 Regional
    • 42 Consumer
  – 12 Terabits per second
  – 200 Exabytes total \( (200,000,000,000,000,000,000) \)
  – \(~25\%\) all inter-domain traffic.

• Inspect packets and classify them.
Internet 2007

[Diagram showing the structure of the internet with layers from National Backbone Operators to Customer IP Networks, including NAPs (Network Access Points), ISPs, and Consumers and business customers, with specifications for Settlement Free, Pay for BW, and Pay for access BW.]
Internet 2009

- Flatter and much more densely interconnected Internet
- Disintermediation between content and “eyeball” networks
- New commercial models between content, consumer and transit
In 2007, thousands of ASNs contributed 50% of content
In 2009, 150 ASNs contribute 50% of all Internet traffic
Robustness

• If a fraction of nodes or edges are removed:
  – How large are connected components?
  – What is the average distance between nodes in the components?
• This is related to Percolation
  – each edge/node is removed with probability (1-p)
    • Corresponds to random failure
  – Targeted attacks: remove nodes with high degree, or edges with high betweenness.
• The formation or dissolution of a giant component defines the percolation threshold
How Robust are These?
Edge (or Bond) Percolation

- 50 nodes, 116 edges, average degree 4.64
- after 25% edge removal
- 76 edges, average degree 3.04 – still well above percolation threshold
Percolation threshold: how many edges have to be removed before the giant component disappears?

As the average degree increases to 1, a giant component suddenly appears.

Edge removal is the opposite process – at some point the average degree drops below 1 and the network becomes disconnected.
Barabasi-Yeong-Albert’s study (2000)

- Given 2 networks (one exponential one scale free) with same number of nodes and links
- Remove a small number of nodes and study changes in average shortest path to see if information communication has been disrupted and how much.
Let's look at the blue lines

- Random graph: increasing monotonically
- SF: remains unchanged until at least 5%

Fraction of deleted nodes
Let’s look at the red lines

- Random graph: same behaviour if nodes with most links are chosen first
- SF: with 5% nodes removed the diameter is doubled
Effect of attacks and failure on WWW and Internet

Network diameter

Fraction of deleted nodes
Effect on Giant Component

Fraction of deleted nodes
Internet and WWW: Effect on Giant Component

Fraction of deleted nodes
Scale-free networks are resilient with respect to random attack

- Example: Gnutella network, 20% of nodes

574 nodes in giant component

427 nodes in giant component
Targeted attacks are affective against scale-free networks

- Example: same Gnutella network, 22 most connected nodes removed (2.8% of the nodes)

574 nodes in giant component

301 nodes in giant component
• Graph shows fraction of GC size over fraction of nodes randomly removed.
• Robustness of the Internet ($\gamma$ is the exponent of PL).
  – $\gamma = 2.5$ Virtually no threshold exists which means a GC is always present.
  – For $\gamma = 3.5$ there is a threshold around 0.4.
Skewness of power-law networks and effects and targeted attack

% of nodes removed, from highest to lowest degree

$\gamma = 2.7$ only 1% nodes removed leads to no GC

Kmax needs to be very low (10) to destroy the GC

$k_{\text{max}}$ is the highest degree among the remaining nodes
Percolation: let’s get formal

• Percolation process:
  • Occupation probability $\phi = \text{number of nodes in the network [ie not removed]}
  • It can be proven that the critical threshold depends on the degree:

$$\phi_c = \frac{<k>}{<k^2> - <k>}$$

• This tells us the minimum fraction of nodes which must exist for a GC to exist.
Threshold for Random Graphs

• For Random networks $\varphi_{\text{critical}} = 1/c$ where $c$ is the mean degree
  • If $c$ is large the network can withstand the loss of many vertices
  • $c=4$ then $\frac{1}{4}$ of vertices are enough to have a GC [3/4 of the vertices need to be destroyed to destroy the GC]
Threshold for Scale Free Networks

- For the Internet and Scale Free networks with $2<\alpha<3$
  - Finite mean $\langle k \rangle$ however $\langle k^2 \rangle$ diverges (in theory)
  - Then $\varphi_{\text{critical}}$ is zero: no matter how many vertices we remove there will always be a GC
  - In practice $\langle k^2 \rangle$ is never infinite for a finite network, although it can be very large, resulting in very small $\varphi_{\text{critical}}$, so still highly robust networks
Non random removal

• The threshold models we have presented hold for random node removal but not for targeted attacks [ie removal of high degree nodes first]

• The equation for non random removal cannot be solved analytically
Robustness Study and Improvements

- A method to improve network resilience
- Percolation threshold $q$ ignores situation when the network is very damaged but not collapsing.
- Robustness:

$$R = \frac{1}{N} \sum_{Q=1}^{N} S(Q)$$

- $R$ ranges values from star and fully connected graph.

$S(Q) = \text{nodes in the connected component after removing } Q = qN \text{ nodes}$
Improve Robustness

- Add links until network is fully connected: not practical.
- Swap edges of 2 random nodes so that \( R' > R \)
  - Repeat until no substantial improvement (a value delta);
- Some additional constraints could be introduced (limit the geographical length of new edges for economic reasons).
Robustness Improvement over edge changes

Robustness improved by 55-45% with ~5% link change. Percolation threshold remains unchanged.
Best Network for Robustness

• How do we design a robust network with a fixed degree distribution?
• Scale free N=100 edges=300, exponent=2.5

• Onion-like structure!
Robustness of Technological and Social Network

• Targeted attacks on high degree nodes are lethal to a technological and a biological as well as transport network.

• However as seen in Lecture 2, for social systems it is the bridges and weak ties which make a difference...
References