

# Lecture 2: Datastructures and Algorithms for Indexing

Information Retrieval  
Computer Science Tripos Part II

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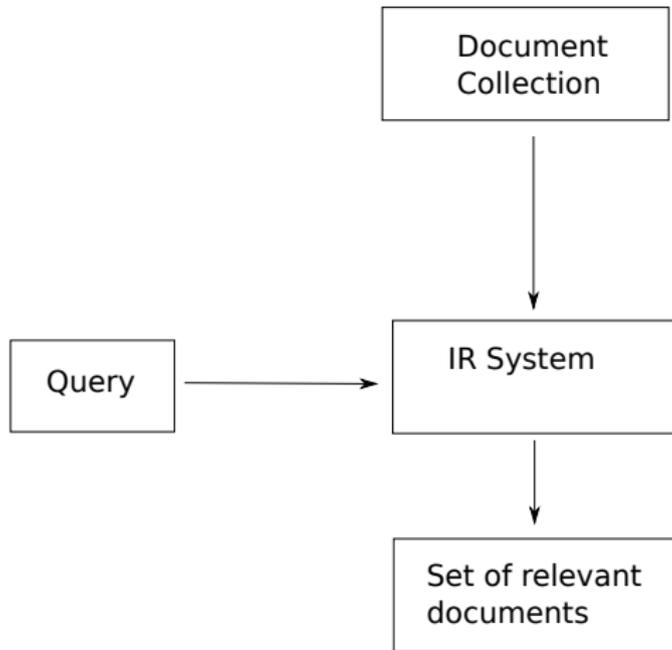
Natural Language and Information Processing (NLIP) Group



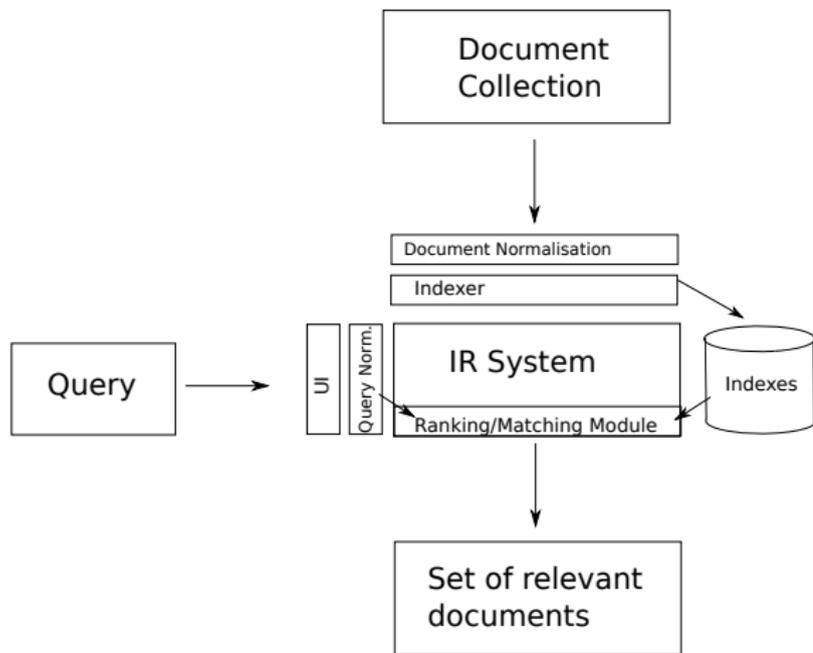
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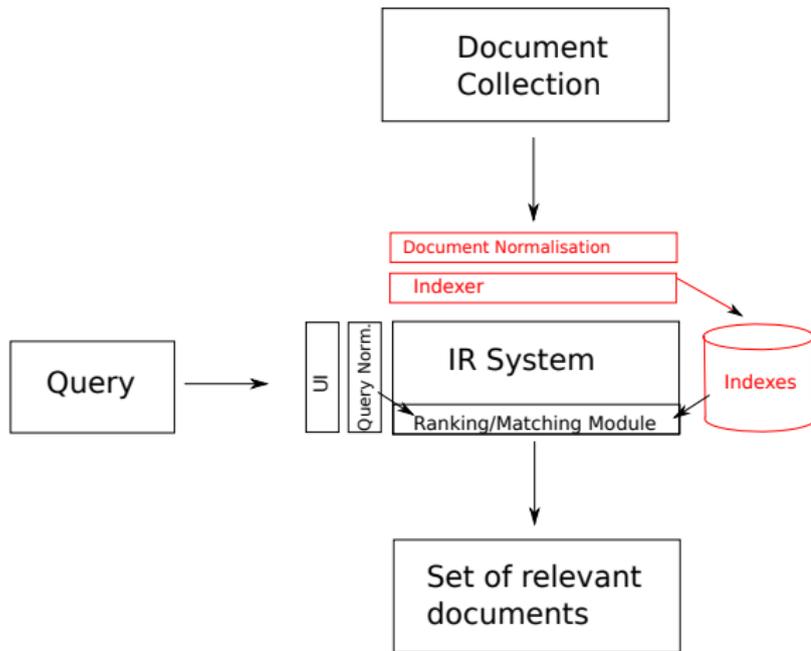
# IR System Components



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Today: The indexer

- 1 The inverted index
- 2 Processing Boolean Queries
- 3 Index construction
- 4 Document and Term Normalisation
  - Documents
  - Terms
  - Reuter RCV1 and Heap's Law

# Recap: Term-Document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
¬Calpurnia	1	0	1	1	1	1
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0
<b>AND</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>

- **Word**: a delimited string of characters as it appears in the text.
- **Term**: a “normalised” word (case, morphology, spelling etc); an equivalence class of words
- **Token**: an instance of a word or term occurring in a document.
- **Type**: an equivalence class of tokens (same as “term” in most cases)

- Consider  $N=10^6$  documents, each with about 1000 tokens
- $10^9$  tokens at avg 6 Bytes per token  $\Rightarrow$  6GB
- Assume there are  $M=500,000$  distinct terms in the collection
- Size of incidence matrix is then  $500,000 \times 10^6$
- Half a trillion 0s and 1s

# Can't build the Term-Document incidence matrix

- Observation: the term-document matrix is very sparse
- Contains no more than one billion 1s.
- Better representation: only represent the things that do occur
- Also does not support more complex query operators such as proximity search
- We will move towards richer representations, beginning with the [inverted index](#).

# The inverted index

The inverted index consists of

- a **dictionary** of terms (also: lexicon, vocabulary)
- and a **postings list** for each term, i.e., a list that records which documents the term occurs in.

**Brutus** → 1 → 2 → 4 → 11 → 31 → 45 → 173 → 174

**Caesar** → 1 → 2 → 4 → 5 → 6 → 16 → 57 → 132 → 179

**Calpurnia** → 2 → 31 → 54 → 101

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## Our Boolean Query

Brutus AND Calpurnia

Locate the postings lists of both query terms and intersect them.

Brutus → 1 → 2 → 4 → 11 → 31 → 45 → 173 → 174

Calpurnia → 2 → 31 → 54 → 101

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Intersection 2 31

Note: this only works if postings lists are sorted

# Algorithm for intersection of two postings

```
INTERSECT (p1, p2)
1  answer ← <>
2  while p1 ≠ NIL and p2 ≠ NIL
3  do if docID(p1) = docID(p2)
4     then ADD (answer, docID(p1))
5     p1 ← next(p1)
6     p2 ← next(p2)
7  if docID(p1) < docID(p2)
8     then p1 ← next(p1)
9     else p2 ← next(p2)
10 return answer
```

Brutus → 1 → 2 → 4 → 11 → 31 → 45 → 173 → 174

Calpurnia → 2 → 31 → 54 → 101

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Intersection 2 31

# Complexity of the Intersection Algorithm

- Bounded by worst-case length of postings lists
- Thus “officially”  $O(N)$ , with  $N$  the number of documents in the document collection
- But in practice much, much better than linear scanning, which is asymptotically also  $O(N)$

# Query Optimisation: conjunctive terms

Organise order in which the postings lists are accessed so that least work needs to be done

Brutus AND Caesar AND Calpurnia

Process terms in increasing document frequency: execute as

(Calpurnia AND Brutus) AND Caesar

**Brutus** 8 → 1 → 2 → 4 → 11 → 31 → 45 → 173 → 174

**Caesar** 9 → 1 → 2 → 4 → 5 → 6 → 16 → 57 → 132 → 179

**Calpurnia** 4 → 2 → 31 → 54 → 101

(maddening OR crowd) AND (ignoble OR strife) AND (killed OR slain)

- Process the query in increasing order of the size of each disjunctive term
- Estimate this in turn (conservatively) by the sum of frequencies of its disjuncts

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The major steps in inverted index construction:

- Collect the documents to be indexed.
- Tokenize the text.
- Perform linguistic preprocessing of tokens.
- Index the documents that each term occurs in.

# Example: index creation by sorting

## Doc 1:

I did enact Julius  
Caesar: I was killed  
i' the Capitol; Brutus  
killed me.

⇒  
Tokenisation

Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

⇒  
Tokenisation

## Doc 2:

So let it be with  
Caesar. The noble  
Brutus hath told  
you Caesar was  
ambitious.

⇒  
Sorting

Term (sorted)	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	2
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	2
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	1
with	2

# Index creation; grouping step (“uniq”)

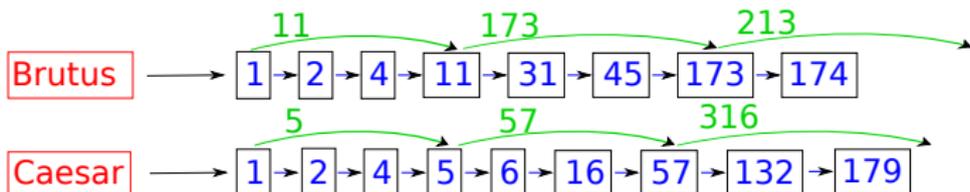
Term & doc. freq.		Postings list
ambitious 1	→	2
be 1	→	2
brutus 2	→	1 → 2
capitol 1	→	1
caesar 2	→	1 → 2
did 1	→	1
enact 1	→	1
hath 1	→	2
I 1	→	1
i' 1	→	1
it 1	→	2
julius 1	→	1
killed 1	→	1
let 1	→	2
me 1	→	1
noble 1	→	2
so 1	→	2
the 2	→	1 → 2
told 1	→	2
you 1	→	2
was 2	→	1 → 2
with 1	→	2

- Primary sort by term (dictionary)
- Secondary sort (within postings list) by document ID
- Document frequency (= length of postings list):
  - for more efficient Boolean searching (later today)
  - for term weighting (lecture 4)
- keep Dictionary in memory
- keep Postings List (much larger) on disk

# Data structures for Postings Lists

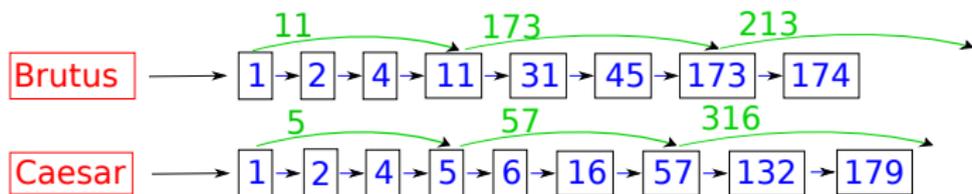
- Singly linked list
  - Allow cheap insertion of documents into postings lists (e.g., when recrawling)
  - Naturally extend to skip lists for faster access
- Variable length array
  - Better in terms of space requirements
  - Also better in terms of time requirements if memory caches are used, as they use contiguous memory
- Hybrid scheme: linked list of variable length array for each term.
  - write posting lists on disk as contiguous block without explicit pointers
  - minimises the size of postings lists and number of disk seeks

# Optimisation: Skip Lists



- Some postings lists can contain several million entries
- Check skip list if present to skip multiple entries
- $\sqrt{L}$  Skips can be placed evenly for a list of length  $L$ .

# Tradeoff Skip Lists



- Number of items skipped vs. frequency that skip can be taken
- More skips: each pointer skips only a few items, but we can frequently use it.
- Fewer skips: each skip pointer skips many items, but we can not use it very often.
- Skip pointers used to help a lot, but with today's fast CPUs, they don't help that much anymore.

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- To build an inverted index, we need to get from



Input: Friends, Romans, countrymen. So let it be with Caesar. . .

- Output: friend roman countryman so
    - Each token is a candidate for a postings entry.
    - What are valid tokens to emit?

- Up to now, we assumed that
  - We know what a document is.
  - We can “machine-read” each document
- More complex in reality

- We need do deal with format and language of each document
- Format could be excel, pdf, latex, word. . .
- What language is it in?
- What character set is it in?
- Each of these is a statistical classification problem
- Alternatively we can use heuristics

Text is not just a linear stream of logical “characters” ...

- Determine correct character encoding (Unicode UTF-8) – by ML or by metadata or heuristics.
- Compressions, binary representation (DOC)
- Treat XML characters separately (&amp)

# Format/Language: Complications

- A single index usually contains terms of several languages.
- Documents or their components can contain multiple languages/format, for instance a French email with a Spanish pdf attachment
- What is the document unit for indexing?
  - a file?
  - an email?
  - an email with 5 attachments?
  - an email thread?
- Answering the question “What is a document?” is not trivial.
- Smaller units raise precision, drop recall
- Also might have to deal with XML/hierarchies of HTML documents etc.

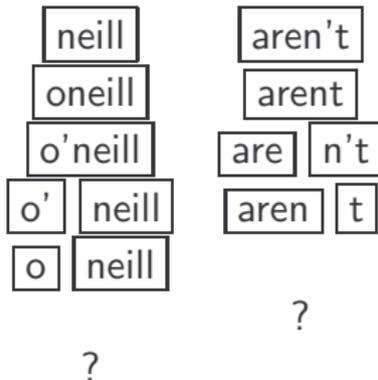
# Normalisation

- Need to normalise words in the indexed text as well as query terms to the same form
- Example: We want to match **U.S.A.** to **USA**
- We most commonly implicitly define **equivalence classes** of terms.
- Alternatively, we could do asymmetric expansion:

window → window, windows  
windows → Windows,  
windows, window  
Windows → Windows

- Either at query time, or at index time
- More powerful, but less efficient

Mr. O'Neill thinks that the boys' stories about Chile's capital aren't amusing.



# Tokenisation problems: One word or two? (or several)

- Hewlett-Packard
- State-of-the-art
- co-education
- the hold-him-back-and-drag-him-away maneuver
- data base
- San Francisco
- Los Angeles-based company
- cheap San Francisco-Los Angeles fares
- York University vs. New York University

20/3/91  
3/20/91  
Mar 20, 1991  
B-52  
100.2.86.144  
(800) 234-2333  
800.234.2333

- Older IR systems may not index numbers...
- ... but generally it's a useful feature.

莎拉波娃现在居住在美国东南部的佛罗里达。今年4月9日，莎拉波娃在美国第一大城市纽约度过了18岁生日。生日派对上，莎拉波娃露出了甜美的微笑。

- Need to perform word segmentation
- Use a lexicon or supervised machine-learning

## 和尚

- As one word, means “monk”
- As two words, means “and” and “still”

# Other cases of “no whitespace”: Compounding

Compounding in Dutch, German, Swedish

German

Lebensversicherungsgesellschaftsangestellter

leben+s+versicherung+s+gesellschaft+s+angestellter

# Other cases of “no whitespace” : Agglutination

“Agglutinative” languages do this not just for compounds:

Inuit

tusaatsiarunnangittualuujunga  
(= “I can’t hear very well”)

Finnish

epäjärjestelmällistytämättömyydellänsäkäänköhän  
(= “I wonder if – even with his/her quality of not having been made unsystematized”)

Turkish

Çekoslovakyalılaştıramadıklarımızdanmışçasına  
(= “as if you were one of those whom we could not make resemble the Czechoslovakian people”)

ノーベル平和賞を受賞したワンガリ・マータイさんが名誉会長を務めるMOTTAINAIキャンペーンの一環として、毎日新聞社とマガジンハウスは「私の、もったいない」を募集します。皆様が日ごろ「もったいない」と感じて実践していることや、それにまつわるエピソードを800字以内の文章にまとめ、簡単な写真、イラスト、図などを添えて10月20日までにお送りください。大賞受賞者には、50万円相当の旅行券とエコ製品2点の副賞が贈られます。

- Different scripts (alphabets) might be mixed in one language.
- Japanese has 4 scripts: kanja, katakana, hiragana, Romanji
- no spaces

- Direction of writing changes in some scripts (writing systems); e.g., Arabic.

استقلت الجزائر في سنة 1962 بعد 132 عاما من الاحتلال الفرنسي.

← → ← →

← START

‘Algeria achieved its independence in 1962 after 132 years of French occupation.’

- Rendering vs. conceptual order
- Bidirectionality is not a problem if Unicode encoding is chosen

- résumé vs. resume
- Universität
- Meaning-changing in some languages:

peña = cliff, pena = sorrow  
(Spanish)

- Main questions: will users apply it when querying?

- Reduce all letters to lower case
- Even though case can be semantically distinguishing

Fed vs. fed  
March vs. march  
Turkey vs. turkey  
US vs. us

- Best to reduce to lowercase because users will use lowercase regardless of correct capitalisation.

# Stop words

- Extremely common words which are of little value in helping select documents matching a user need

a, an, and, are, as, at, be, by, for, from, has, he, in, is, it, its, of, on, that, the, to, was, were, will, with

- Used to be standard in older IR systems.
- Need them to search for

to be or not to be  
prince of Denmark  
bamboo in water

- Length of practically used stoplists has shrunk over the years.
- Most web search engines do index stop words.

- Thesauri: semantic equivalence, car = automobile
- Soundex: phonetic equivalence, Muller = Mueller; [lecture 3](#)

- Reduce inflectional/variant forms to base form

am, are, is → **be**

car, car's, cars', cars → **car**

the boy's cars are different colours → **the boy car be different color**

- Lemmatisation implies doing “proper” reduction to dictionary headword form (the **lemma**)
- Inflectional morphology (cutting → **cut**)
- Derivational morphology (destruction → **destroy**)

- Stemming is a crude heuristic process that **chops off the ends of words** in the hope of achieving what “principled” lemmatisation attempts to do with a lot of linguistic knowledge.
- language dependent, but fast and space-efficient
- does not require a stem dictionary, only a suffix dictionary
- Often both inflectional and derivational

automate, automation, automatic → **automat**

- Root changes (deceive/deception, resume/resumption) aren't dealt with, but these are rare

- M. Porter, “An algorithm for suffix stripping”, Program 14(3):130-137, 1980
- Most common algorithm for stemming English
- Results suggest it is at least as good as other stemmers
- Syllable-like shapes + 5 phases of reductions
- Of the rules in a compound command, select the top one and exit that compound (this rule will have affected the longest suffix possible, due to the ordering of the rules).

# Stemming: Representation of a word

$[C] (VC)\{m\}[V]$

**C** : one or more adjacent consonants

**V** : one or more adjacent vowels

**[ ]** : optionality

**( )** : group operator

**{x}** : repetition x times

**m** : the “measure” of a word

shoe	$[sh]_C[oe]_V$	$m=0$
Mississippi	$[M]_C([i]_V[ss]_C)([i]_V[ss]_C)([i]_V[pp]_C)[i]_V$	$m=3$
ears	$([ea]_V[rs]_C)$	$m=1$

Notation: measure  $m$  is calculated on the word **excluding** the suffix of the rule under consideration

# Porter stemmer: selected rules

SSES → SS

IES → I

SS → SS

S →

caresses → caress

cares → care

(m>0) EED →

EE

feed → feed

agreed → agree

BUT: freed, succeed

(\*v\*) ED →

plastered → plaster

bled → bled

# Three stemmers: a comparison

Such an analysis can reveal features that are not easily visible from the variations in the individual genes and can lead to a picture of expression that is more biologically transparent and accessible to interpretation.

## Porter Stemmer

such an analysis can reveal features that are not easily visible from the variations in the individual genes and can lead to a picture of expression that is more biologically transparent and accessible to interpretation.

## Lovins Stemmer

such an analysis can reveal features that are not easily visible from the variations in the individual genes and can lead to a picture of expression that is more biologically transparent and accessible to interpretation.

## Paice Stemmer

such an analysis can reveal features that are not easily visible from the variations in the individual genes and can lead to a picture of expression that is more biologically transparent and accessible to interpretation.

# Does stemming improve effectiveness?

- In general, stemming increases effectiveness for some queries and decreases it for others.

## Example queries where stemming helps

**tartan sweaters** → sweater, sweaters

**sightseeing tour san francisco** → tour, tours

## Example queries where stemming hurts

**operational research** → “oper” = operates, operatives, operate, operation, operational, operative

**operating system** → operates, operatives, operate, operation, operational, operative

**operative dentistry** → operates, operatives, operate, operation, operational, operative

- We want to answer a query such as [cambridge university] – as a phrase.
- The Duke of Cambridge recently went for a term-long course to a famous university should not be a match
- About 10% of web queries are phrase queries.
- Consequence for inverted indexes: no longer sufficient to store docIDs in postings lists.
- Two ways of extending the inverted index:
  - biword index
  - positional index

- Index every consecutive pair of terms in the text as a phrase.

Friends, Romans, Countrymen

Generates two biwords:

- friends romans
  - romans countrymen
- Each of these biwords is now a vocabulary term.
  - Two-word phrases can now easily be answered.

## Longer phrase queries

- A long phrase like `cambridge university west campus` can be represented as the Boolean query

`cambridge university AND university west AND west campus`

- We need to do post-filtering of hits to identify subset that actually contains the 4-word phrase.

- Why are biword indexes rarely used?
- False positives, as noted above
- Index blowup due to very large term vocabulary

- Positional indexes are a more efficient alternative to biword indexes.
- Postings lists in a nonpositional index: each posting is just a docID
- Postings lists in a positional index: each posting is a docID and a list of positions (offsets)

# Positional indexes: Example

Query: "to<sub>1</sub> be<sub>2</sub> or<sub>3</sub> not<sub>4</sub> to<sub>5</sub> be<sub>6</sub>"

to, 993427:

< 1: < 7, 18, 33, 72, 86, 231>;  
2: <1, 17, 74, 222, 255>;  
4: <8, 16, 190, 429, 433>;  
5: <363, 367>;  
7: <13, 23, 191>;  
... ...>

be, 178239:

< 1: < 17, 25>;  
4: < 17, 191, 291, 430, 434>;  
5: <14, 19, 101>;  
... ...>

Document 4 is a match.

(As always: docid, term, doc freq; new: offsets)

# Proximity search

- We just saw how to use a positional index for phrase searches.
- We can also use it for proximity search.

## employment /4 place

- Find all documents that contain **employment** and **place** within 4 words of each other.
- HIT: **Employment** agencies that **place** healthcare workers are seeing growth.
- NO HIT: **Employment** agencies that have learned to adapt now **place** healthcare workers.

- Use the positional index
- Simplest algorithm: look at cross-product of positions of (i) “employment” in document and (ii) “place” in document
- Very inefficient for frequent words, especially stop words
- Note that we want to return the actual matching positions, not just a list of documents.
- This is important for dynamic summaries etc.

# Proximity intersection

```
PositionalIntersect(p1, p2, k)
1 answer ← <>
2 while p1 ≠ nil and p2 ≠ nil
3 do if docID(p1) = docID(p2)
4     then l ← <>
5         pp1 ← positions(p1)
6         pp2 ← positions(p2)
7         while pp1 ≠ nil
8             do while pp2 ≠ nil
9                 do if |pos(pp1) - pos(pp2)| ≤ k
10                    then Add(l, pos(pp2))
11                       else if pos(pp2) > pos(pp1)
12                          then break
13                          pp2 ← next(pp2)
14                    while l ≠ <> and |l [0] - pos(pp1)| > k
15                       do Delete(l [0])
16                       for each ps in l
17                           do Add(answer, docID(p1), pos(pp1), psi)
18                           pp1 ← next(pp1)
19                       p1 ← next(p1)
20                       p2 ← next(p2)
21                else if docID(p1) < docID(p2)
22                    then p1 ← next(p1)
23                else p2 ← next(p2)
24 return answer
```

# Combination scheme

- Biword indexes and positional indexes can be profitably combined.
- Many biwords are extremely frequent: Michael Jackson, Britney Spears etc
- For these biwords, increased speed compared to positional postings intersection is substantial.
- Combination scheme: Include frequent biwords as vocabulary terms in the index. Do all other phrases by positional intersection.
- Williams et al. (2004) evaluate a more sophisticated mixed indexing scheme. Faster than a positional index, at a cost of 26% more space for index.
- For web search engines, positional queries are much more expensive than regular Boolean queries.

- Shakespeare's collected works are not large enough to demonstrate scalable index construction algorithms.
- Instead, we will use the [Reuters RCV1](#) collection.
- English newswire articles published in a 12 month period (1995/6)

$N$	documents	800,000
$M$	terms (= word types)	400,000
$T$	non-positional postings	100,000,000

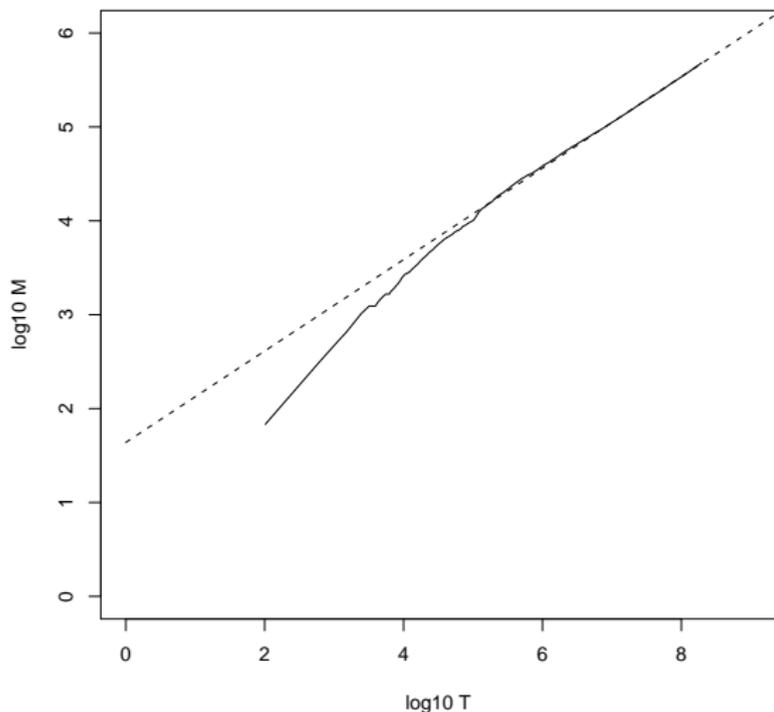
# Effect of preprocessing for Reuters

size of	word types (terms)	non-positional postings	positional postings (word tokens)
	dictionary	non-positional index	positional index
	size $\Delta$ cml	size $\Delta$ cml	size $\Delta$ cml
unfiltered	484,494	109,971,179	197,879,290
no numbers	473,723 -2 -2	100,680,242 -8 -8	179,158,204 -9 -9
case folding	391,523 -17 -19	96,969,056 -3 -12	179,158,204 -0 -9
30 stopw's	391,493 -0 -19	83,390,443 -14 -24	121,857,825 -31 -38
150 stopw's	391,373 -0 -19	67,001,847 -30 -39	94,516,599 -47 -52
stemming	322,383 -17 -33	63,812,300 -4 -42	94,516,599 -0 -52

# How big is the term vocabulary?

- That is, how many distinct words are there?
- Can we assume there is an upper bound?
- Not really: At least  $70^{20} \approx 10^{37}$  different words of length 20.
- The vocabulary will keep growing with collection size.
- Heaps' law:  $M = kT^b$
- $M$  is the size of the vocabulary,  $T$  is the number of tokens in the collection.
- Typical values for the parameters  $k$  and  $b$  are:  $30 \leq k \leq 100$  and  $b \approx 0.5$ .
- Heaps' law is linear in log-log space.
  - It is the simplest possible relationship between collection size and vocabulary size in log-log space.
  - Empirical law

# Heaps' law for Reuters



Vocabulary size  $M$  as a function of collection size  $T$  (number of tokens) for Reuters-RCV1. For these data, the dashed line  $\log_{10} M = 0.49 * \log_{10} T + 1.64$  is the best least squares fit. Thus,  $M = 10^{1.64} T^{0.49}$  and  $k = 10^{1.64} \approx 44$  and  $b = 0.49$ .

- Good, as we just saw in the graph.
- Example: for the first 1,000,020 tokens Heaps' law predicts 38,323 terms:

$$44 \times 1,000,020^{0.49} \approx 38,323$$

- The actual number is 38,365 terms, very close to the prediction.
- Empirical observation: fit is good in general.

- Understanding of the basic unit of classical information retrieval systems: **words** and **documents**: What is a document, what is a term?
- Tokenization: how to get from raw text to terms (or tokens)
- More complex indexes for phrases

- MRS Chapter 2.2
- MRS Chapter 2.4