Heapsort example

Andrej Ivaskovic

University of Cambridge

February 1, 2015
We are given an array that needs to be sorted:

QWERTYUIOPASDFG
Making the array satisfy the max-heap property

heapify(6)
Making the array satisfy the max-heap property

heapify(6)
Making the array satisfy the max-heap property

heapify(5)
Making the array satisfy the max-heap property

heapify(5)
Making the array satisfy the max-heap property

heapify(4)
Making the array satisfy the max-heap property

heapify(4)
Making the array satisfy the max-heap property

heapify(3)
Making the array satisfy the max-heap property

heapify(3)
Making the array satisfy the max-heap property

heapify(2)
Making the array satisfy the max-heap property

heapify(2)

QW[ENRTYUIOPASDFG

```
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```
Making the array satisfy the max-heap property

heapify(2)
Making the array satisfy the max-heap property

heapify(2)
Making the array satisfy the max-heap property

heapify(2)

QW[Y]RTSUIOPAEDEFG
Making the array satisfy the max-heap property

heapify(2)
Making the array satisfy the max-heap property

heapify(1)

Q[Q][W]YRTSUIOPAEDFG

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Making the array satisfy the max-heap property

heapify(1)
Making the array satisfy the max-heap property

heapify(0)
Making the array satisfy the max-heap property

heapify(0)
Making the array satisfy the max-heap property

heapify(0)
Making the array satisfy the max-heap property

heapify(0)
Making the array satisfy the max-heap property

heapify(0)
Making the array satisfy the max-heap property

heapify(0)
We have a heap!

Now the array satisfies the binary max-heap property:

YWURTSQIOPAEDFG
Removal of elements from the heap

We have the 14\textsuperscript{th} element!
Removal of elements from the heap

We have the 14\textsuperscript{th} element!

GWURTSQIOPAEDFY
Removal of elements from the heap

We have the 14\textsuperscript{th} element!
Removal of elements from the heap

We have the 14\textsuperscript{th} element!

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Removal of elements from the heap

We have the 14\textsuperscript{th} element!

\begin{itemize}
  \item \texttt{WTURGSQIOPAEDFY}
\end{itemize}
Removal of elements from the heap

We have the 14\textsuperscript{th} element!
Removal of elements from the heap

We have the 14\textsuperscript{th} element!

WTURPSQIOGAEDFY
Removal of elements from the heap

We have the 14\textsuperscript{th} element!
Removal of elements from the heap

We have the 13th element!
Removal of elements from the heap

We have the 13th element!
Removal of elements from the heap

We have the 13th element!
Removal of elements from the heap

We have the 13\textsuperscript{th} element!

UTFRP S Q IOGAEDWY

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Removal of elements from the heap

We have the 13th element!
Removal of elements from the heap

We have the 13\textsuperscript{th} element!
Removal of elements from the heap

We have the 12th element!
Removal of elements from the heap

We have the 12\textsuperscript{th} element!
Removal of elements from the heap

We have the 12th element!
Removal of elements from the heap

We have the 12\textsuperscript{th} element!
Removal of elements from the heap
We have the 12th element!
Removal of elements from the heap

We have the 12\textsuperscript{th} element!
Removal of elements from the heap

We have the 12th element!
Removal of elements from the heap

We have the 12\textsuperscript{th} element!
Removal of elements from the heap

We have the 11\textsuperscript{th} element!
Removal of elements from the heap

We have the 11th element!

ERSOPFQIDGATUWY

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Removal of elements from the heap

We have the 11\textsuperscript{th} element!
Removal of elements from the heap

We have the 11\textsuperscript{th} element!

\[\text{SREOPFQIDGATUWY}\]
Removal of elements from the heap

We have the 11\textsuperscript{th} element!

\begin{center}
\begin{tikzpicture}
  \node (S) {S}
    child {node (R) {R}
      child {node (O) {O}
        child {node (I) {I}}
        child {node (D) {D}}
      }
      child {node (P) {P}
        child {node (G) {G}}
        child {node (A) {A}}
      }
    }
    child {node (Q) {Q}
      child {node (F) {F}
        child {node (T) {T}}
        child {node {U}}
      }
      child {node {E}
        child {node {W}}
        child {node {Y}}
      }
    }
\end{tikzpicture}
\end{center}
Removal of elements from the heap

We have the $11^{\text{th}}$ element!
Removal of elements from the heap

We have the 10\textsuperscript{th} element!
Removal of elements from the heap

We have the 10th element!

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Removal of elements from the heap

We have the 10th element!
Removal of elements from the heap
We have the 10\textsuperscript{th} element!
Removal of elements from the heap

We have the 10\textsuperscript{th} element!
Removal of elements from the heap

We have the 10\textsuperscript{th} element!

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Removal of elements from the heap

We have the 10th element!
Removal of elements from the heap

We have the 10\textsuperscript{th} element!
Removal of elements from the heap

We have the 9\textsuperscript{th} element!

\[ [A]PQOGFEIDRSUWY \]
Removal of elements from the heap

We have the 9th element!
Removal of elements from the heap

We have the 9th element!
Removal of elements from the heap

We have the 9\textsuperscript{th} element!
Removal of elements from the heap

We have the 9\textsuperscript{th} element!
Removal of elements from the heap

We have the 9th element!

QPFOGAEIDRSTUWY
Removal of elements from the heap

We have the 8\textsuperscript{th} element!

[D]PFOGAEIQRSTUWY
Removal of elements from the heap

We have the 8\textsuperscript{th} element!

DPFOGAEIQRSTUWY
Removal of elements from the heap
We have the 8\textsuperscript{th} element!

\begin{center}
\includegraphics[width=\textwidth]{heap_tree.png}
\end{center}

PDFOGAEIQRSTUWY
Removal of elements from the heap

We have the 8\textsuperscript{th} element!

PDFOGAEIQRSTUWY

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Removal of elements from the heap

We have the 8\textsuperscript{th} element!
Removal of elements from the heap

We have the 8th element!

POFDGAEIQRSTUWY
Removal of elements from the heap

We have the 8th element!
Removal of elements from the heap

We have the 8th element!

POFIGAEDQRSTUWY

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Removal of elements from the heap

We have the 7\textsuperscript{th} element!
Removal of elements from the heap

We have the 7\textsuperscript{th} element!
Removal of elements from the heap

We have the 7th element!
Removal of elements from the heap

We have the 7\textsuperscript{th} element!
Removal of elements from the heap

We have the 7\textsuperscript{th} element!

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Removal of elements from the heap

We have the 7th element!
Removal of elements from the heap

We have the 6th element!
Removal of elements from the heap

We have the 6\textsuperscript{th} element!
Removal of elements from the heap

We have the 6\textsuperscript{th} element!

IEFDGAOPQRSTUWY
Removal of elements from the heap

We have the 6\textsuperscript{th} element!
Removal of elements from the heap

We have the 6\textsuperscript{th} element!

IGFDEAOPQRSTUWY

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Removal of elements from the heap

We have the 6\textsuperscript{th} element!
Removal of elements from the heap

We have the 5th element!
Removal of elements from the heap

We have the 5\textsuperscript{th} element!
Removal of elements from the heap

We have the 5\textsuperscript{th} element!
Removal of elements from the heap

We have the 5\textsuperscript{th} element!

GAFDEIOPQRSTUWY

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Removal of elements from the heap

We have the 5th element!

GEFDAIOPQRSTUWY
Removal of elements from the heap

We have the 5th element!
Removal of elements from the heap

We have the 4\textsuperscript{th} element!
Removal of elements from the heap

We have the 4th element!

AEFDGIOPQRSTUWY
Removal of elements from the heap

We have the 4\textsuperscript{th} element!

FEADGIOPQRSTUWY

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Removal of elements from the heap

We have the 4\textsuperscript{th} element!
Removal of elements from the heap

We have the 3rd element!

[DEAFGIOPQRSTUWY]
Removal of elements from the heap

We have the 3\textsuperscript{rd} element!
Removal of elements from the heap

We have the 3\textsuperscript{rd} element!

EDAFGIOPQRSTUWY
Removal of elements from the heap

We have the 3\textsuperscript{rd} element!
Removal of elements from the heap

We have the 2\textsuperscript{nd} element!

\[A\]DEFGIOPQRSTUWY
Removal of elements from the heap

We have the 2\textsuperscript{nd} element!

\texttt{ADEFGIOPQRSTUWY}

![Heap diagram]

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Removal of elements from the heap

We have the 2\textsuperscript{nd} element!

DAEFGIOPQRSTUWY
Removal of elements from the heap

We have the 2\textsuperscript{nd} element!
Removal of elements from the heap

We have the 1\textsuperscript{st} element!
Removal of elements from the heap

We have the 1\textsuperscript{st} element!
We are done!

What we get in the end is sorted:

ADEFGIOPQRSTUWY