

# Heapsort example

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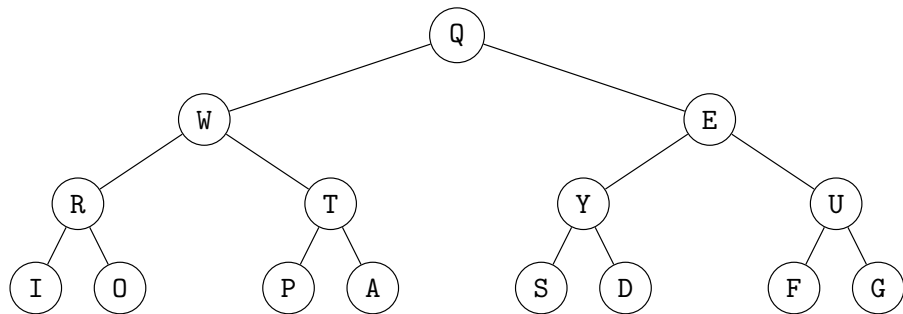
February 1, 2015

# Input

Initial state of the heap

We are given an array that needs to be sorted:

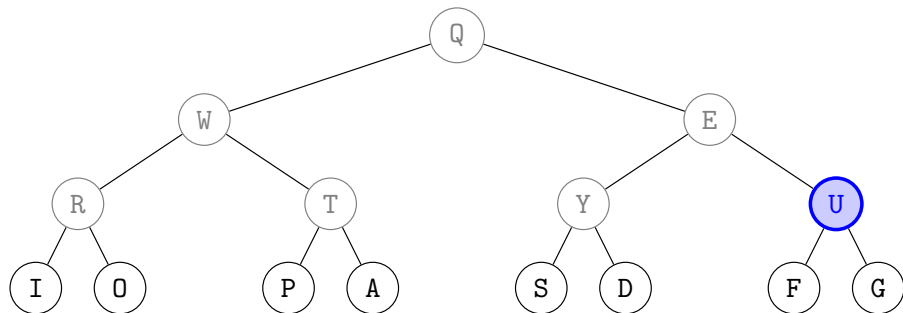
QWERTYUIOPASDFG



# Making the array satisfy the max-heap property

heapify(6)

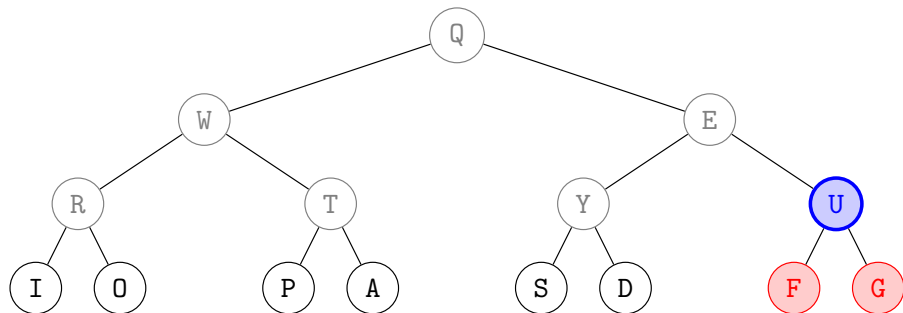
QWERTY [U] IOPASDFG



# Making the array satisfy the max-heap property

heapify(6)

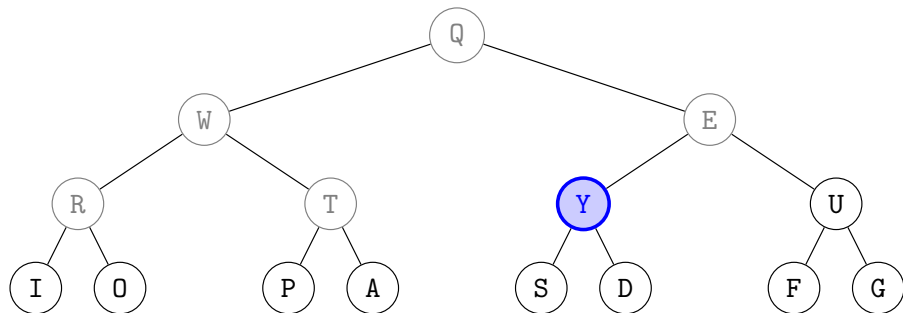
QWERTY [U] IOPASDFG



# Making the array satisfy the max-heap property

heapify(5)

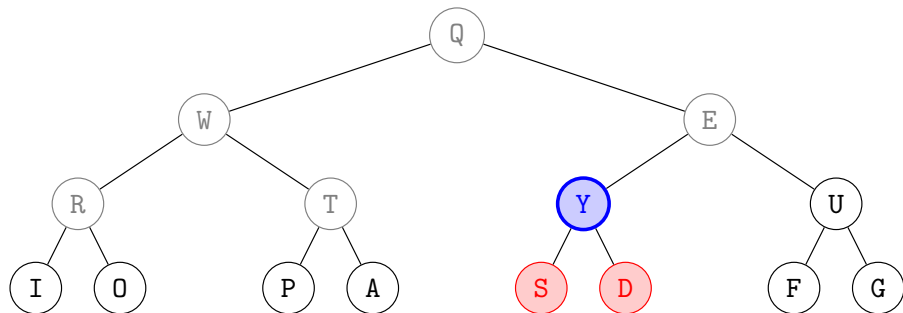
QWERT [Y] UIOPASDFG



# Making the array satisfy the max-heap property

heapify(5)

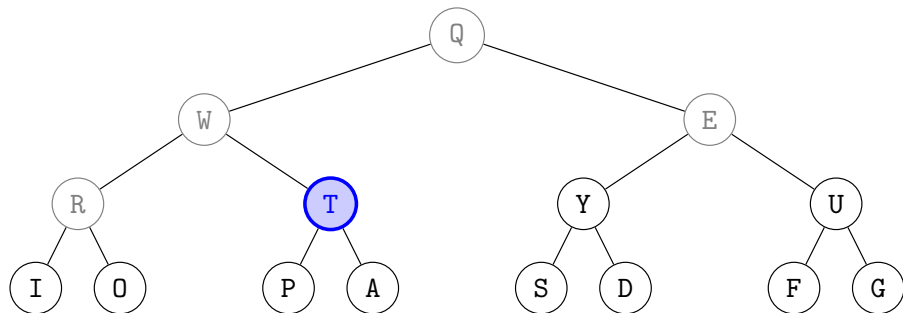
QWERT [Y] UIOPASDFG



# Making the array satisfy the max-heap property

heapify(4)

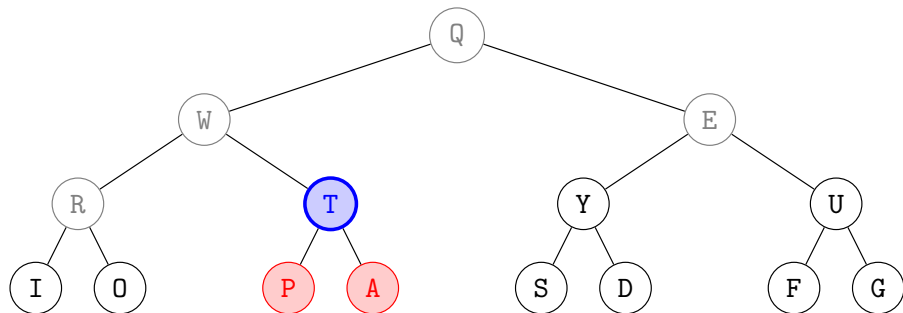
QWER [T] YUIOPASDFG



# Making the array satisfy the max-heap property

heapify(4)

QWER [T] YUIOPASDFG

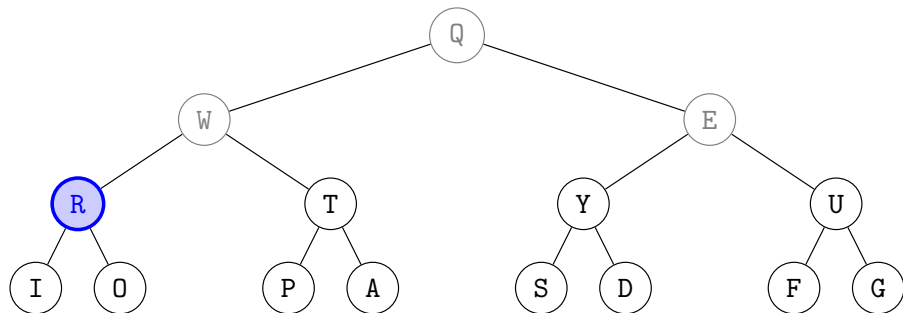




# Making the array satisfy the max-heap property

heapify(3)

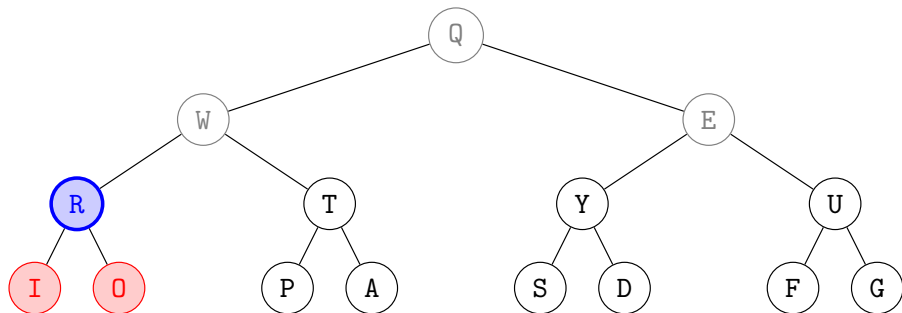
QWE [R] TYUIOPASDFG



# Making the array satisfy the max-heap property

heapify(3)

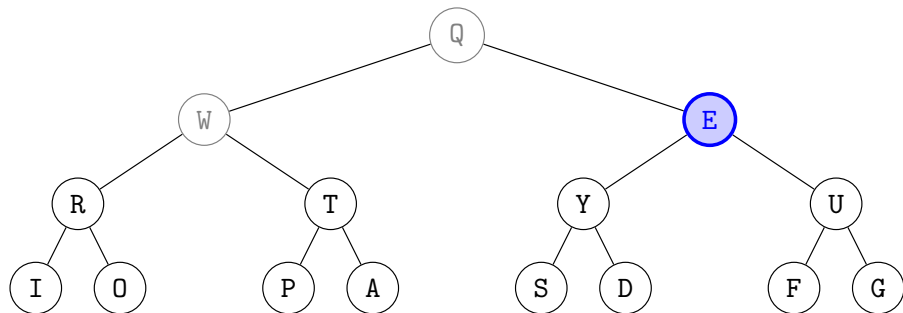
QWE [R] TYUIOPASDFG



# Making the array satisfy the max-heap property

heapify(2)

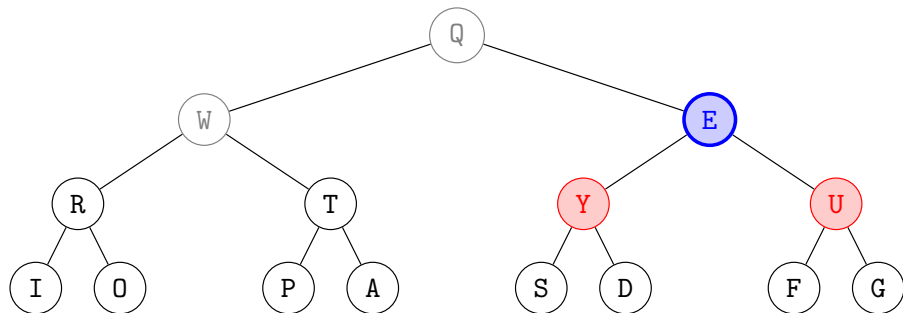
QW[E]RTYUIOPASDFG



# Making the array satisfy the max-heap property

heapify(2)

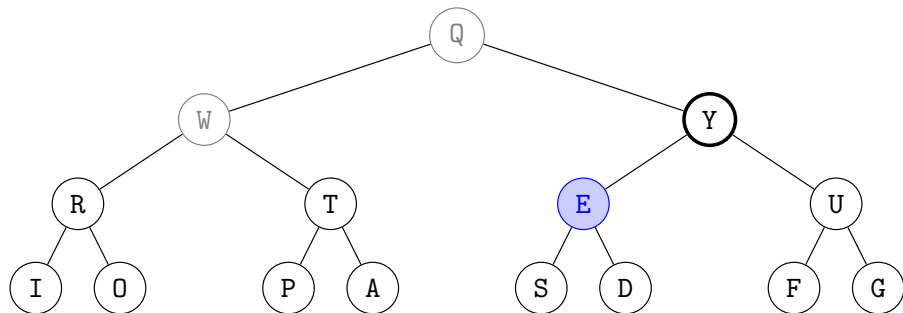
QW[E]RTYUIOPASDFG



# Making the array satisfy the max-heap property

heapify(2)

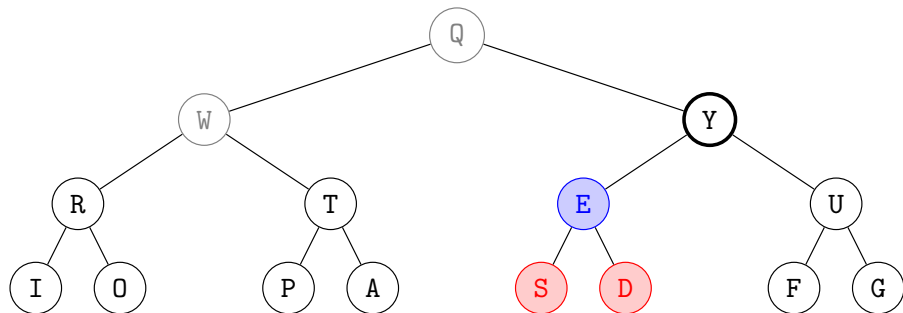
QW[Y]RTEUIOPASDFG



# Making the array satisfy the max-heap property

heapify(2)

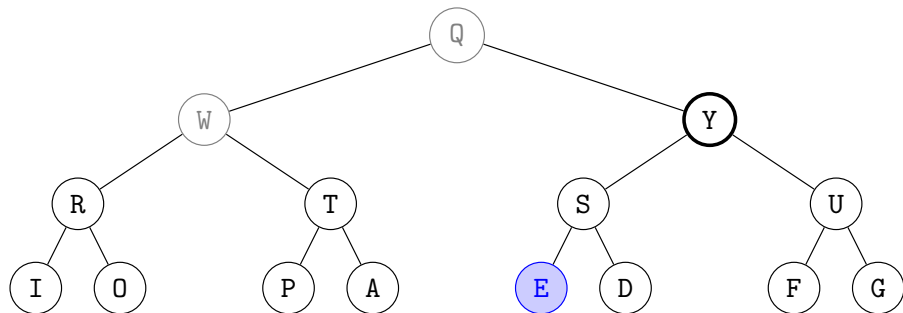
QW[Y]RTEUIOPASDFG



# Making the array satisfy the max-heap property

heapify(2)

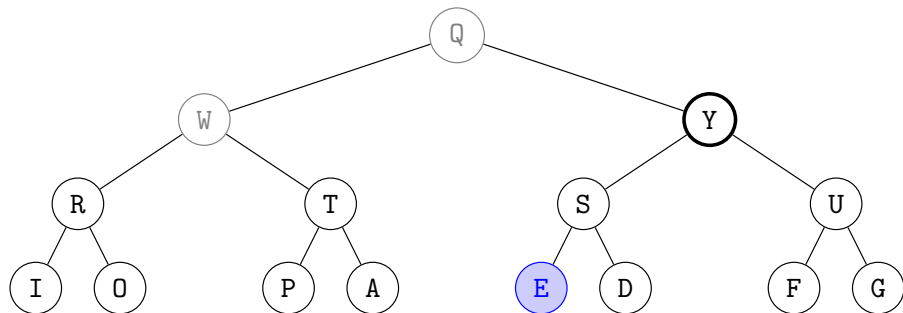
QW[Y]RTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(2)

QW[Y]RTSUIOPAE~~DFG~~

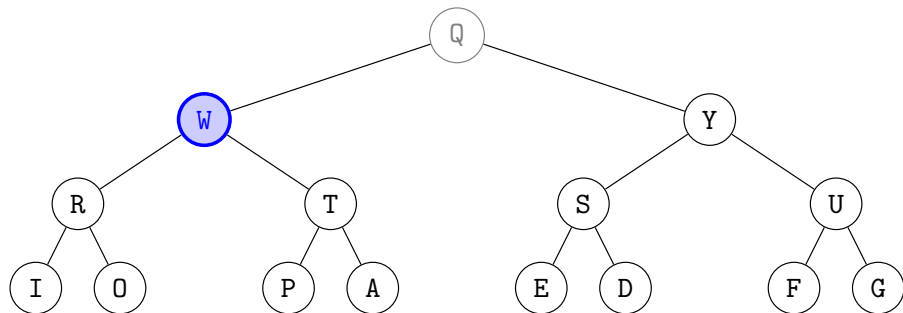




# Making the array satisfy the max-heap property

heapify(1)

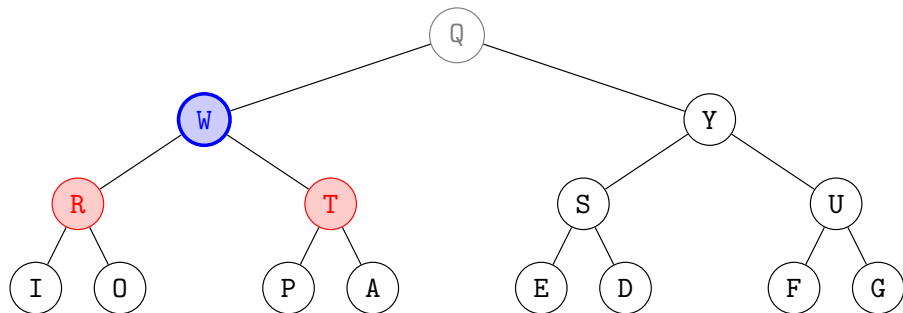
Q [W] YRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(1)

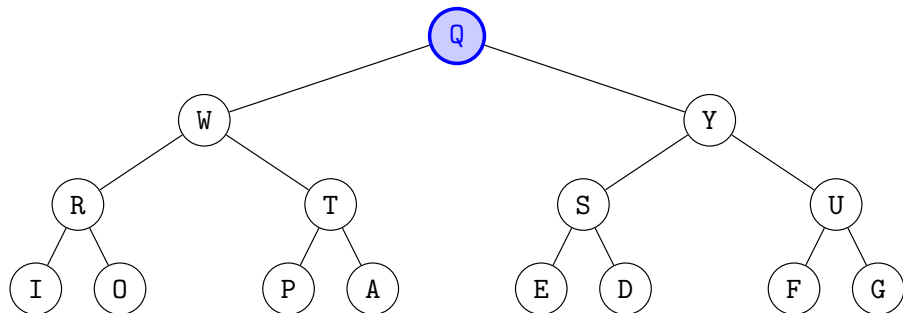
Q [W] YRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

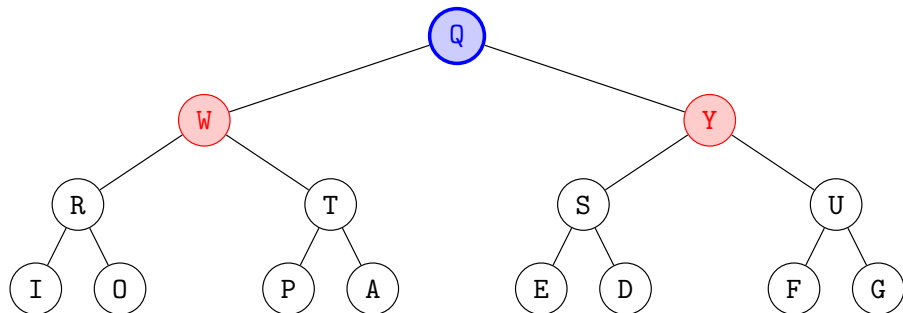
[Q] WYRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

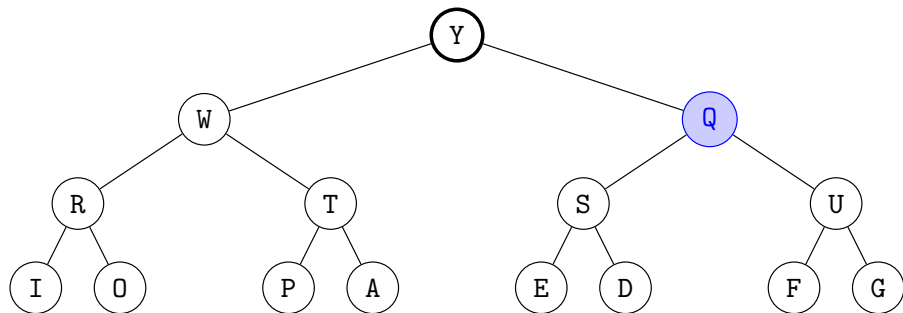
[Q]WYRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

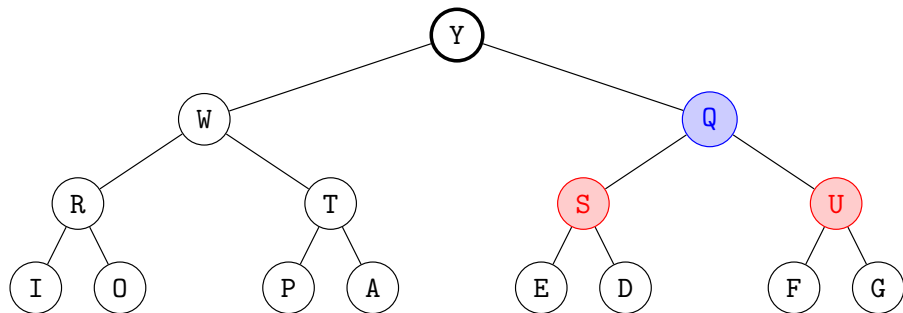
[Y]WQRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

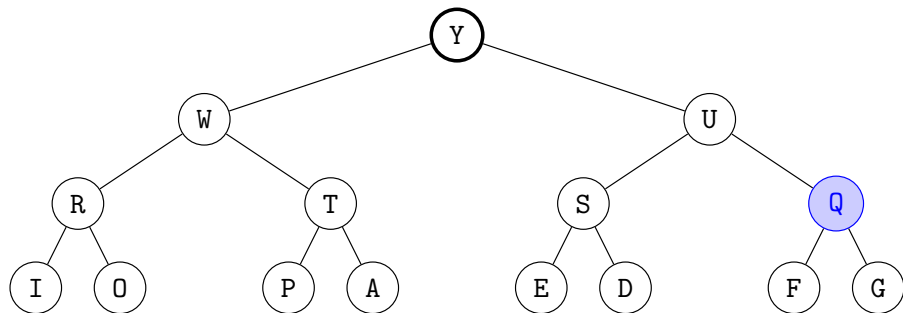
[Y]WQRTSUIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

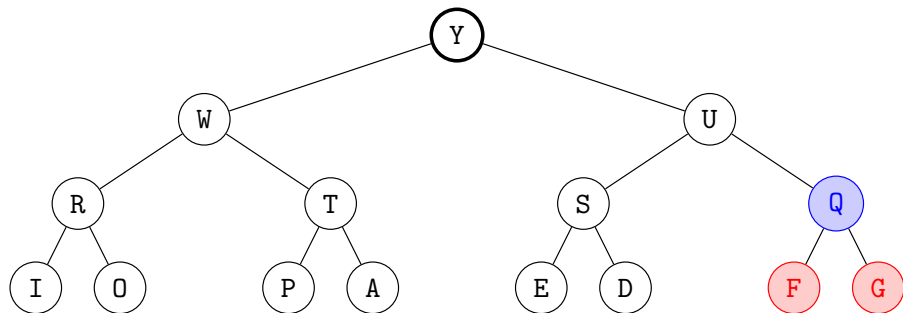
[Y]WURTSQIOPAEDFG



# Making the array satisfy the max-heap property

heapify(0)

[Y] WURTSQIOPAEDFG



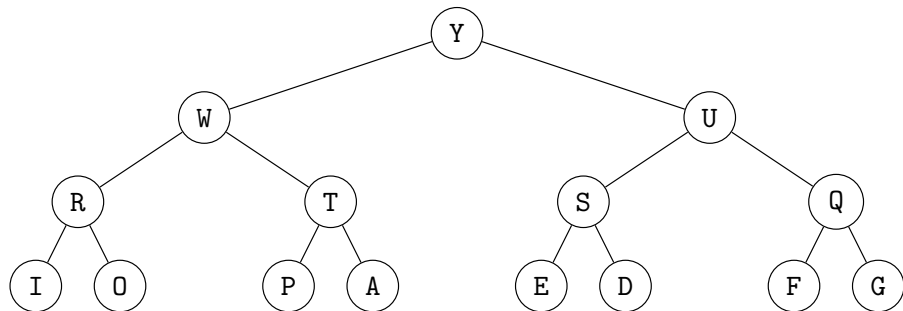


# End of step 1

We have a heap!

Now the array satisfies the binary max-heap property:

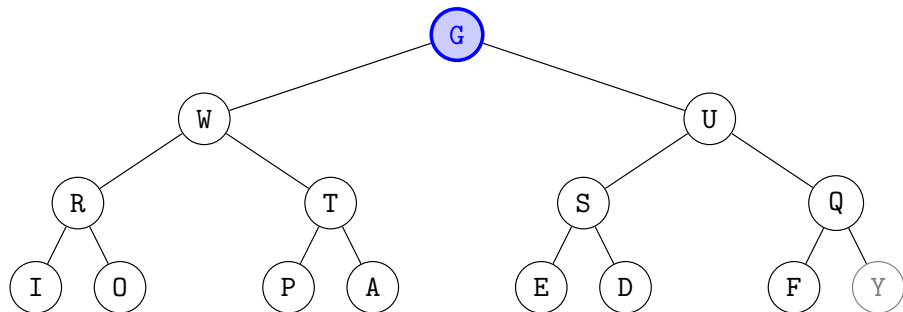
YWURTSQIOPAEDFG



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

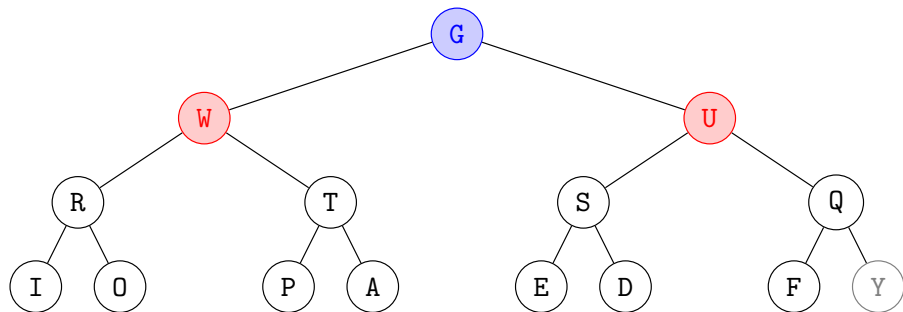
[G] WURTSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

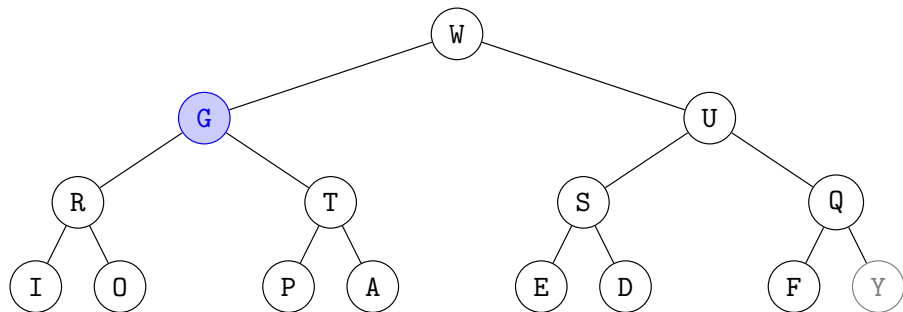
GWURTSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

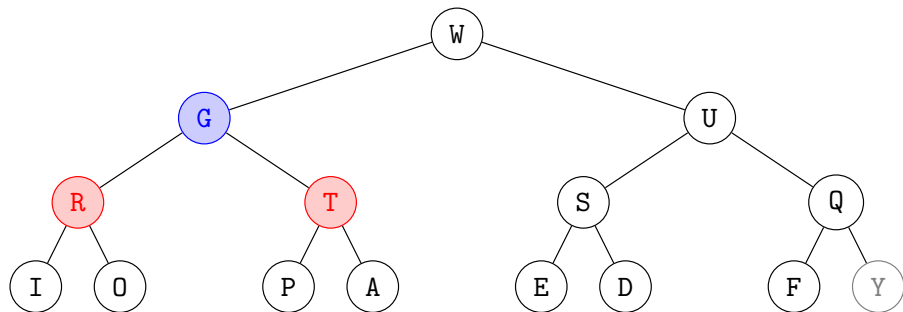
WGURTSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

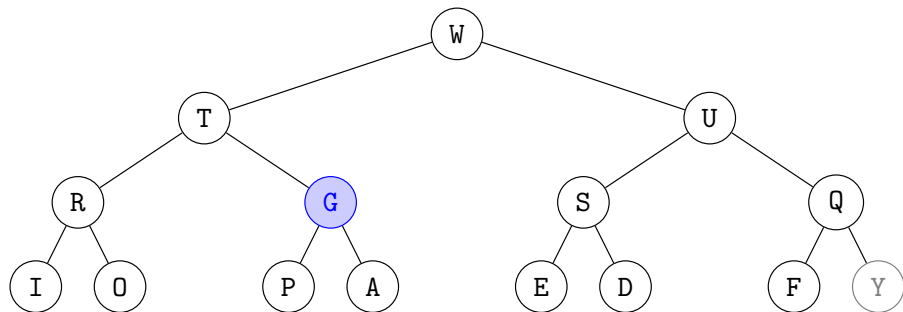
WGURTSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

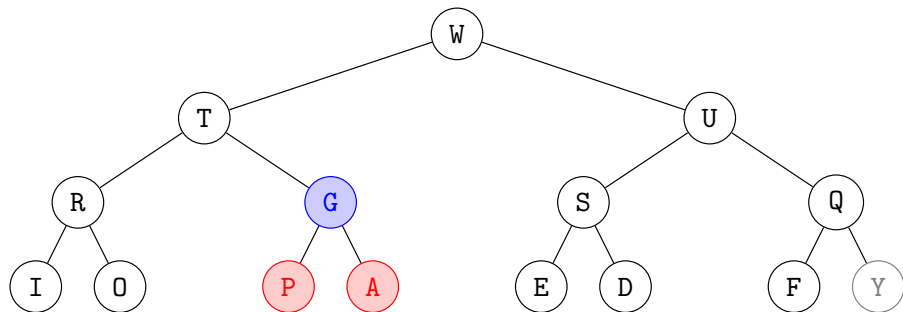
WTURGSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

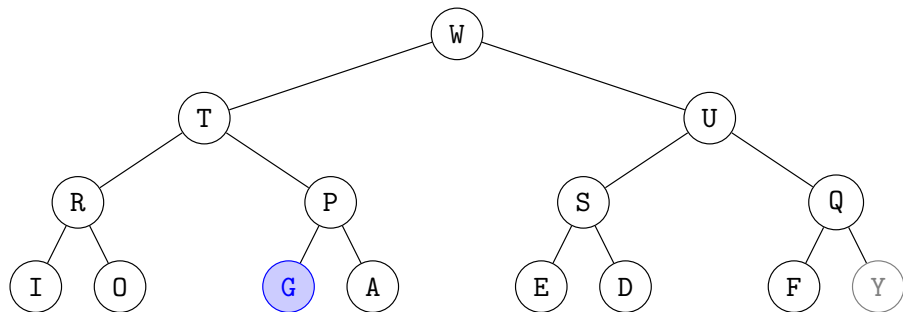
WTURGSQIOPAEDFY



# Removal of elements from the heap

We have the 14<sup>th</sup> element!

WTURPSQIOGAEDFY

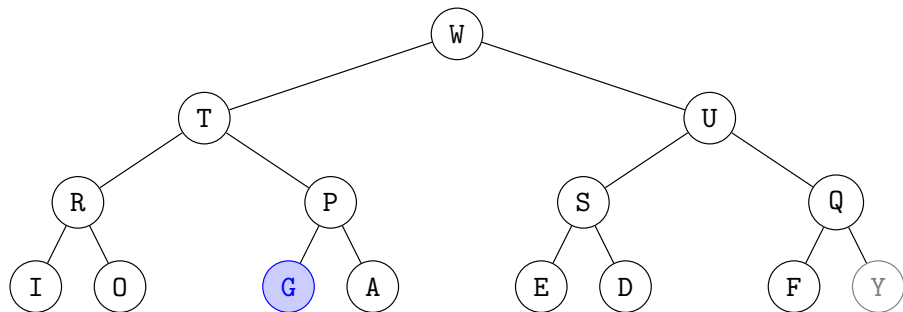




# Removal of elements from the heap

We have the 14<sup>th</sup> element!

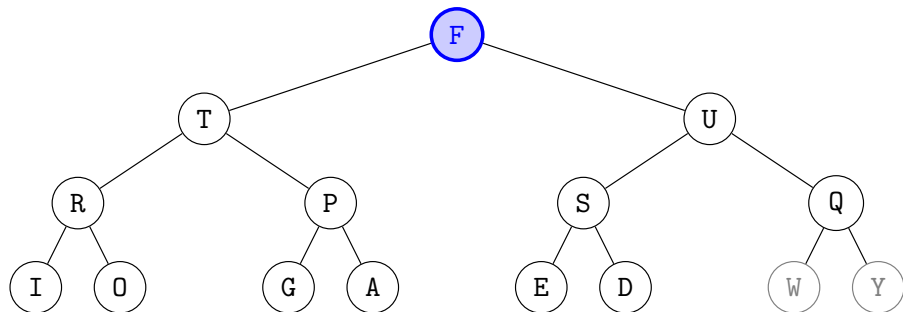
WTURPSQIOGAEDFY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

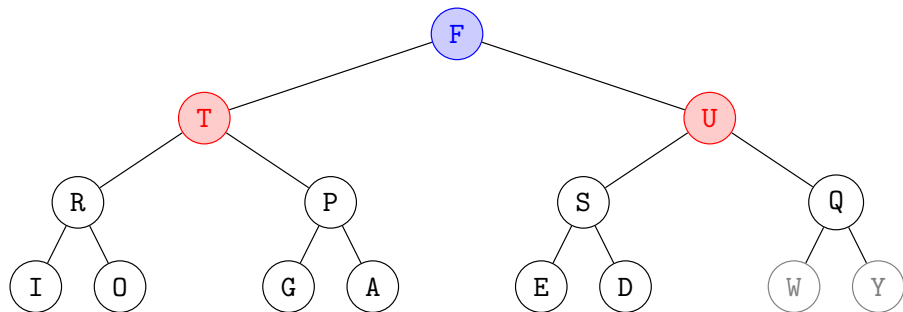
[F] TURPSQIOGAEDWY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

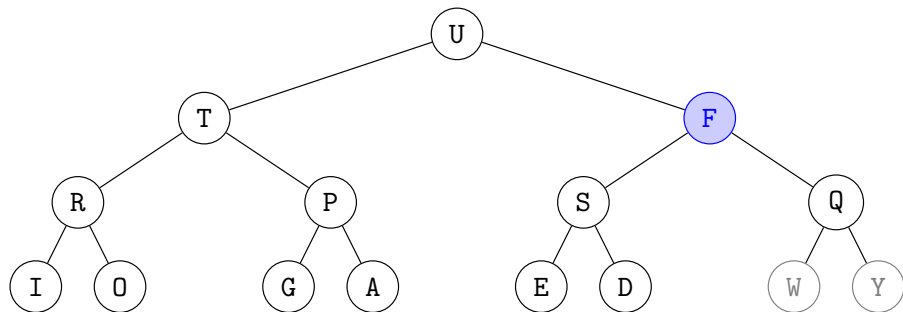
FTURPSQIOGAEDWY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

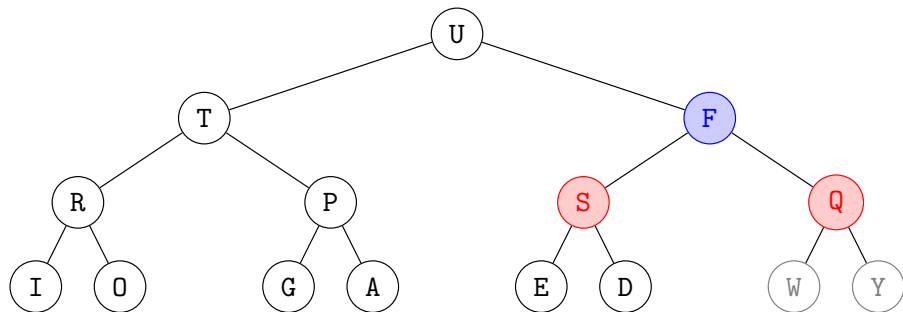
UTFRPSQIOGAEDWY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

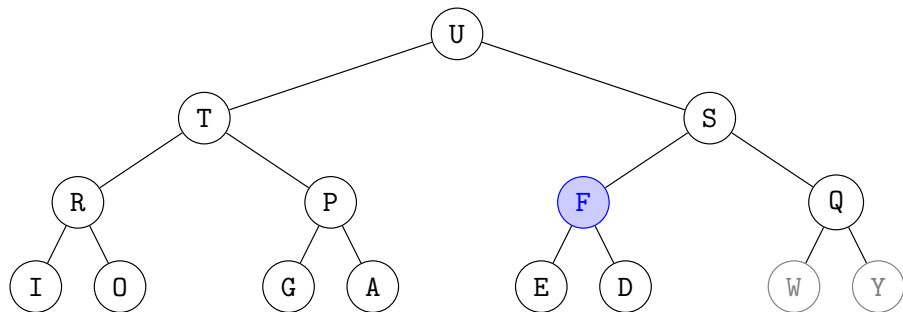
UTFRPSQIOGAEDWY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

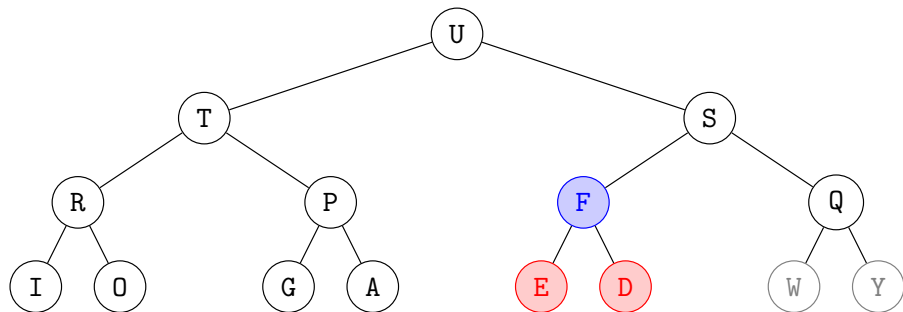
UTSRPFQIOGAEDWY



# Removal of elements from the heap

We have the 13<sup>th</sup> element!

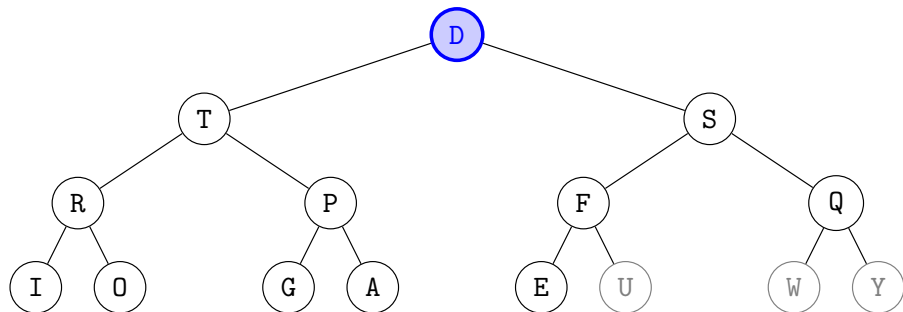
UTSRPFQIOGAEDWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

[D] TSRPFQIOGAEUWY

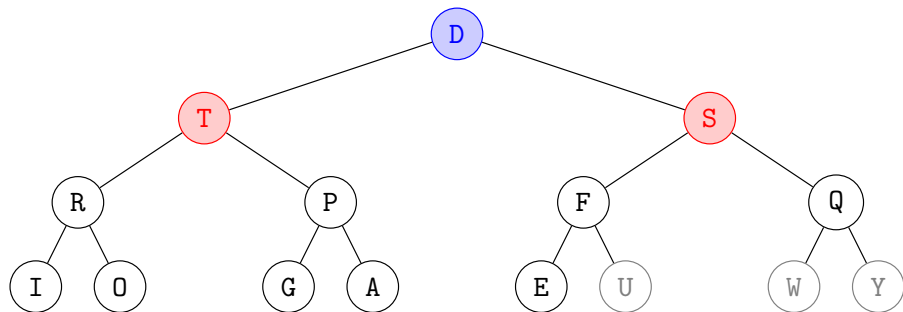




# Removal of elements from the heap

We have the 12<sup>th</sup> element!

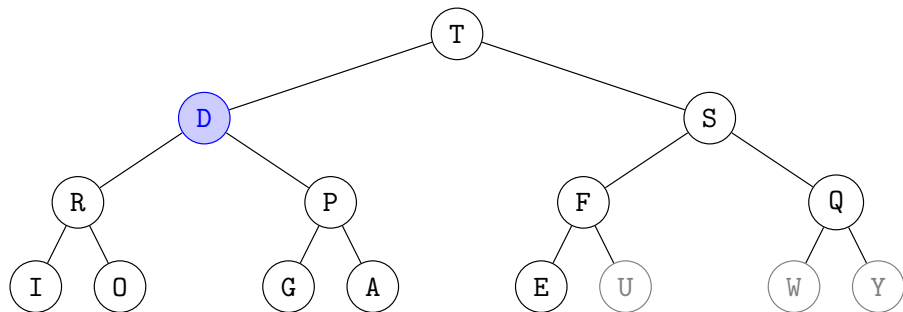
DTSRPFQIOGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

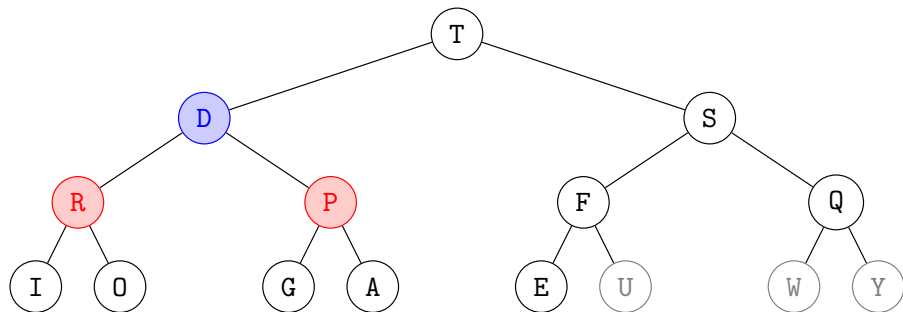
TDSRPFQIOGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

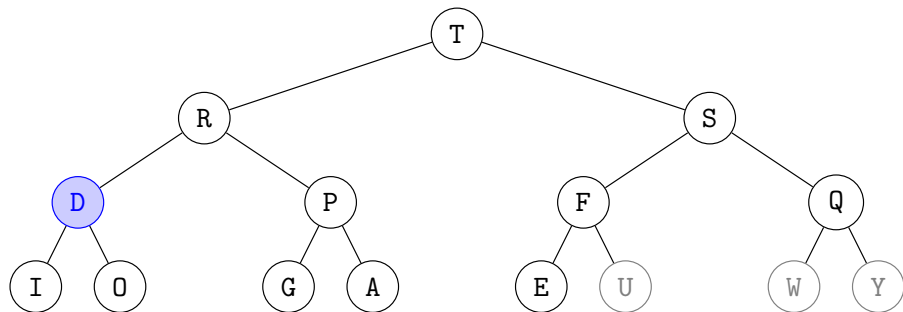
TDSRPFQIOGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

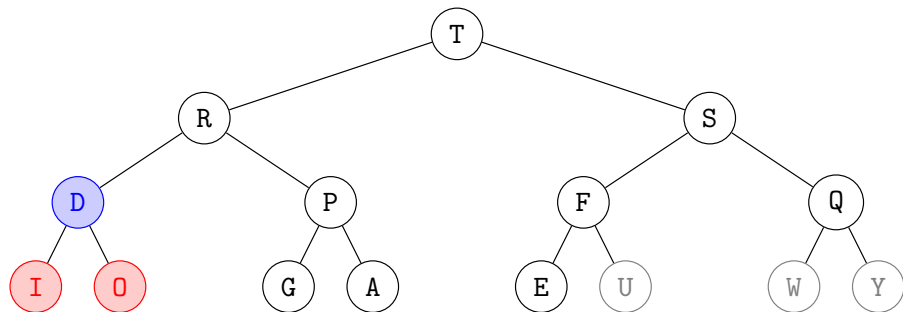
TRSDPFQIOGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

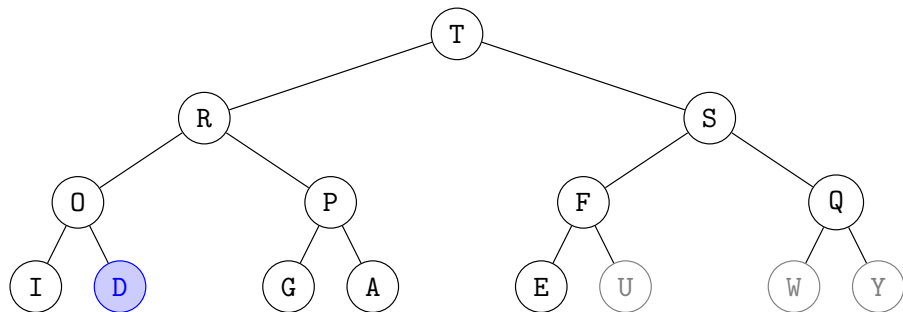
TRSDPFQIOGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

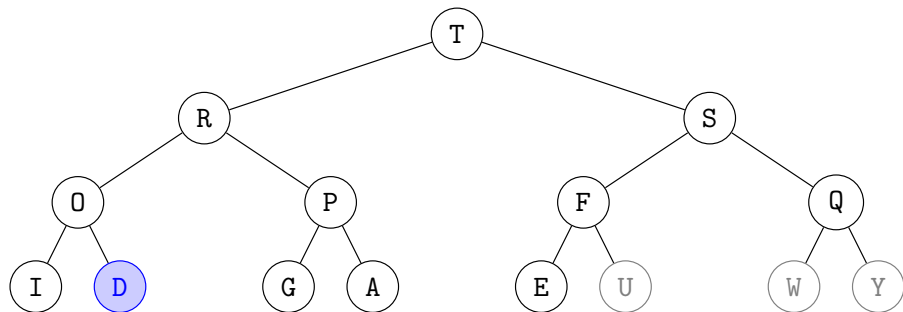
TRSOPFQIDGAEUWY



# Removal of elements from the heap

We have the 12<sup>th</sup> element!

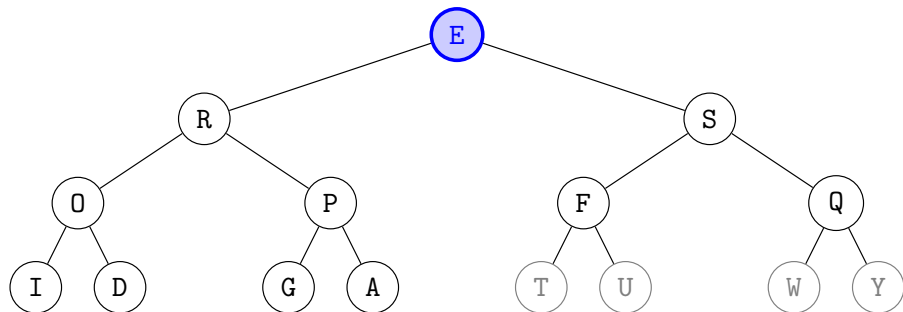
TRSOPFQIDGAEUWY



# Removal of elements from the heap

We have the 11<sup>th</sup> element!

[E] RSOPFQIDGATUWY

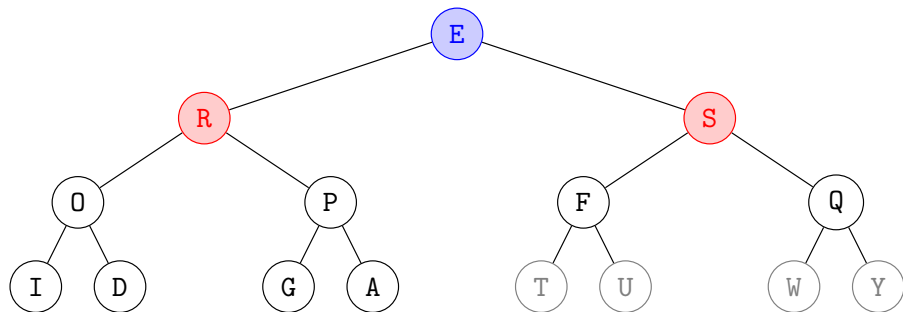




# Removal of elements from the heap

We have the 11<sup>th</sup> element!

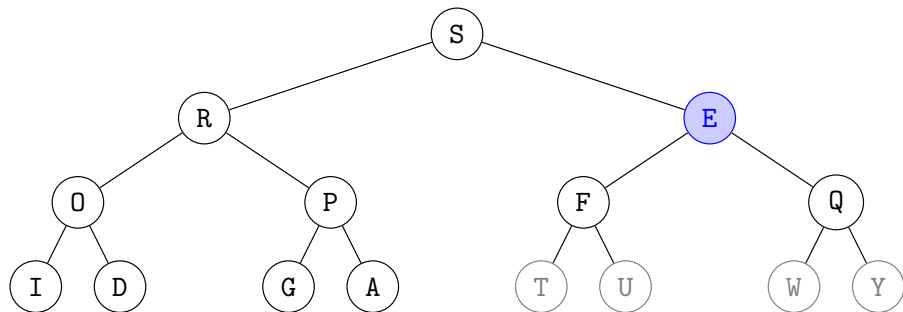
ERSOPFQIDGATUWY



# Removal of elements from the heap

We have the 11<sup>th</sup> element!

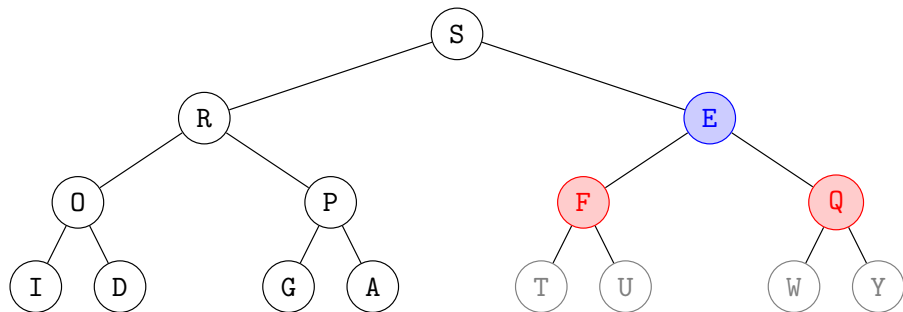
SREOPFQIDGATUWY



# Removal of elements from the heap

We have the 11<sup>th</sup> element!

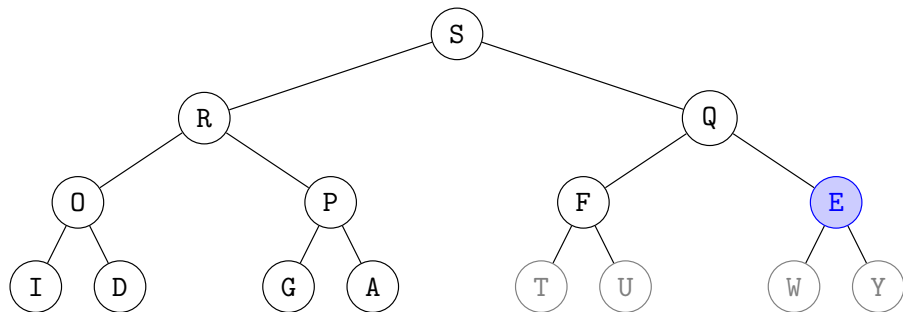
SREOPFQIDGATUWY



# Removal of elements from the heap

We have the 11<sup>th</sup> element!

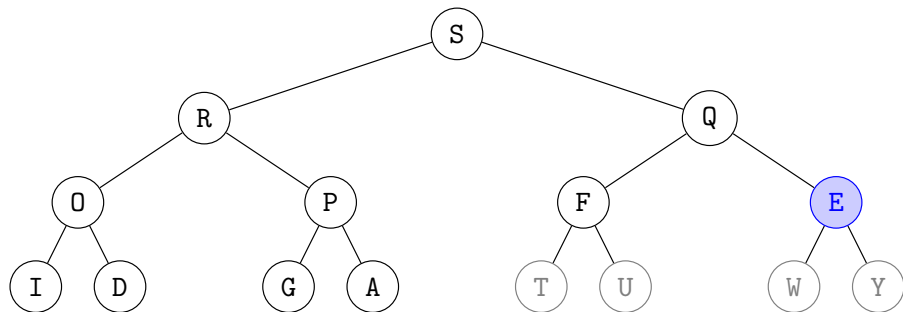
SRQOPF**E**IDGATUWY



# Removal of elements from the heap

We have the 11<sup>th</sup> element!

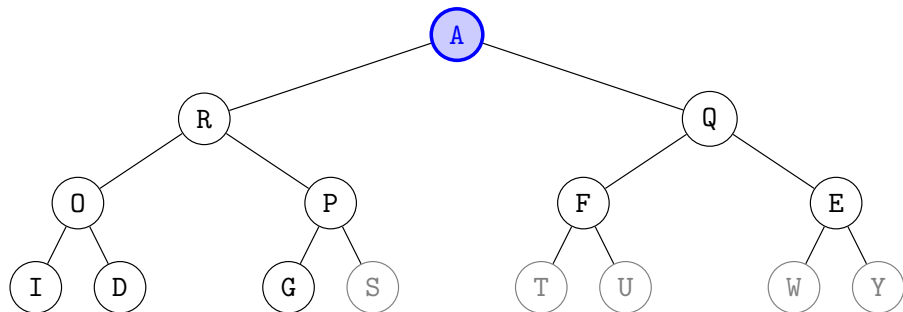
SRQOPF**E**IDGATUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

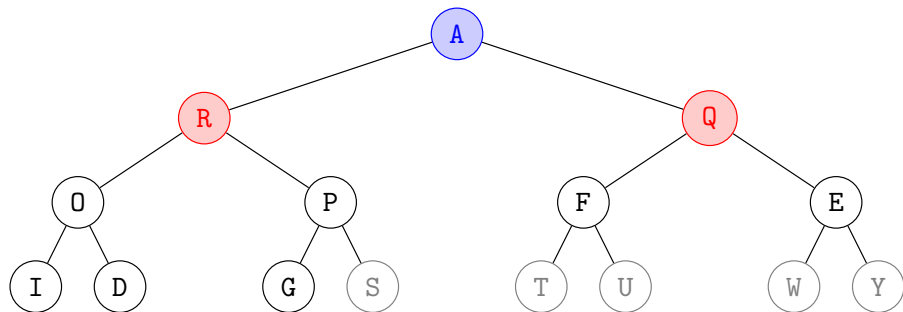
[A] RQOPFEIDGSTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

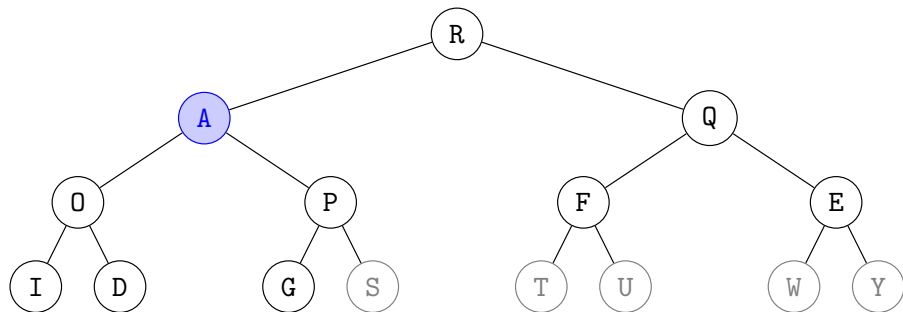
ARQOPFEIDGSTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

RAQOPFEIDGSTUWY

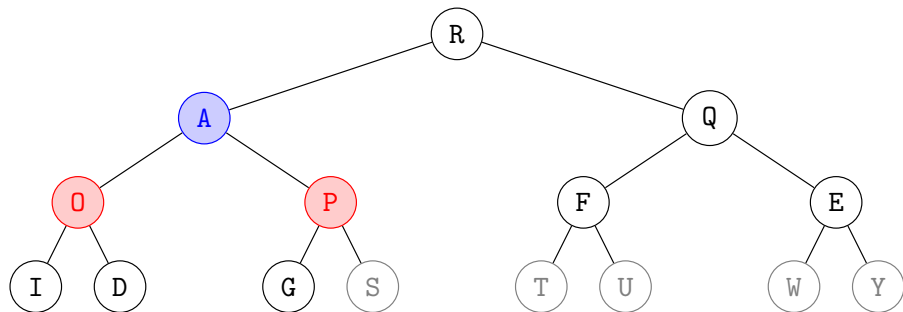




# Removal of elements from the heap

We have the 10<sup>th</sup> element!

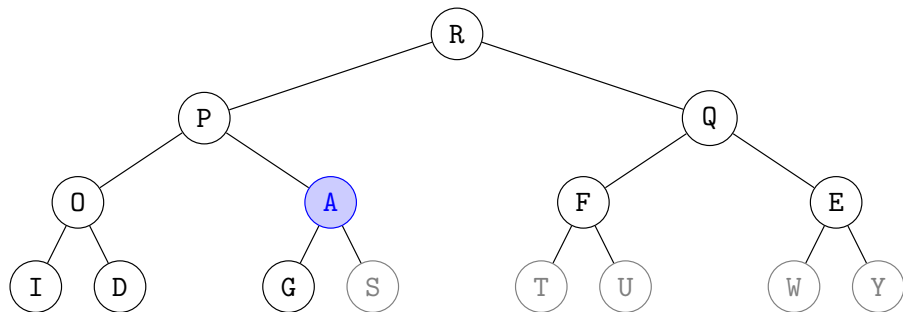
RAQOPFEIDGSTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

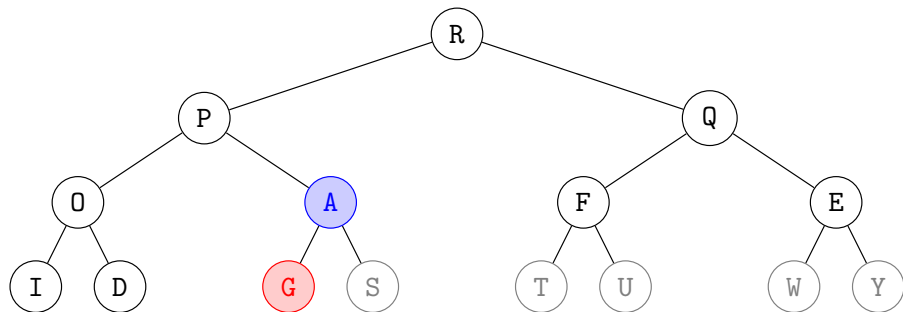
RPQQA**A**FEIDGSTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

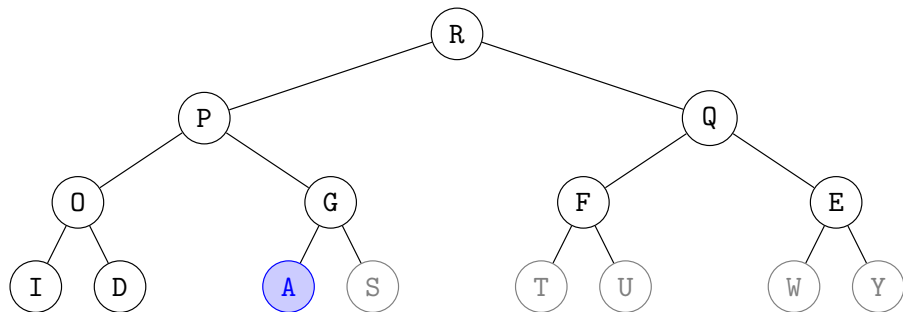
RPQOAFEIDGSTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

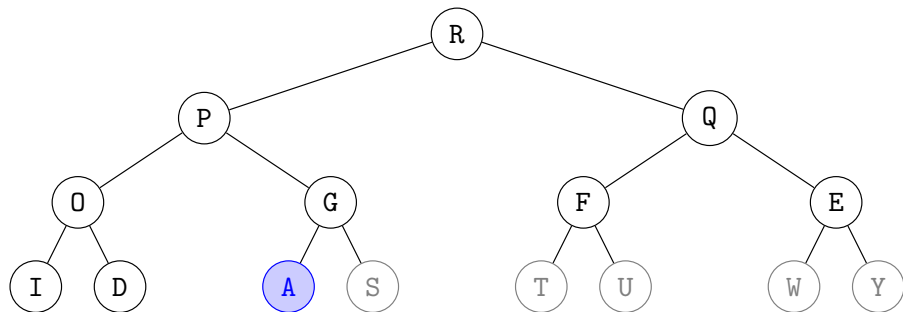
RPQOGFEIDASTUWY



# Removal of elements from the heap

We have the 10<sup>th</sup> element!

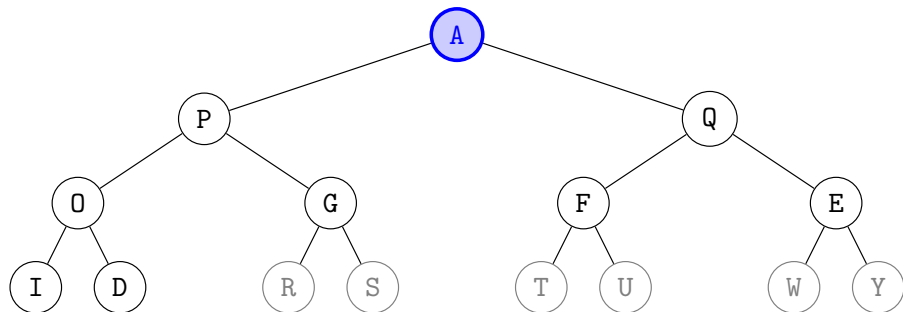
RPQOGFEIDASTUWY



# Removal of elements from the heap

We have the 9<sup>th</sup> element!

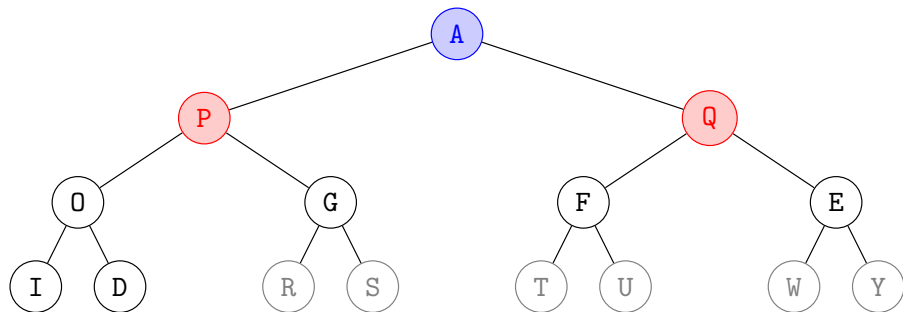
[A] PQOGFEIDRSTUWY



# Removal of elements from the heap

We have the 9<sup>th</sup> element!

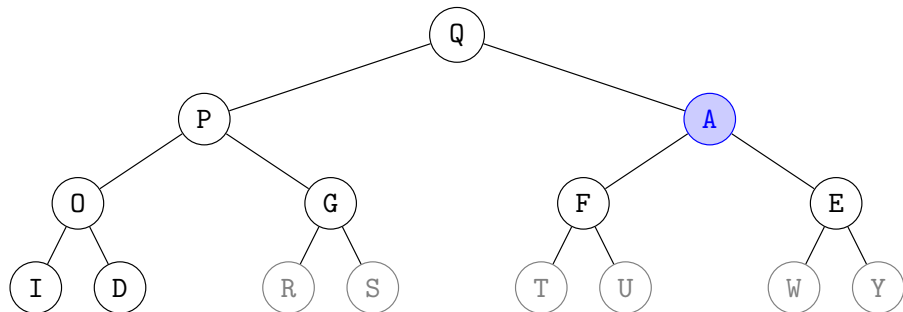
APQOGFEIDRSTUWY



# Removal of elements from the heap

We have the 9<sup>th</sup> element!

Q**A**O G F E I D R S T U W Y

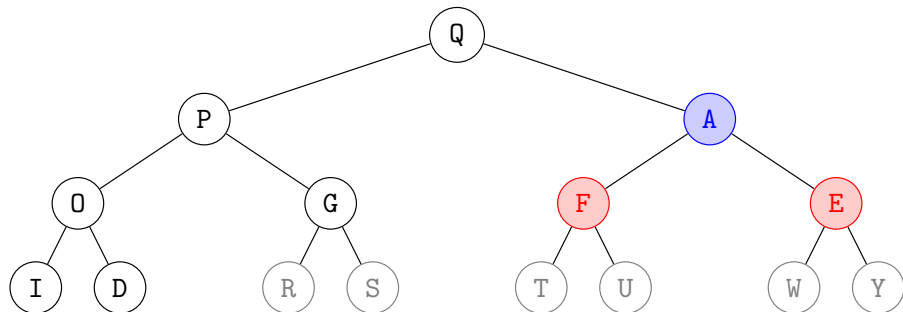




# Removal of elements from the heap

We have the 9<sup>th</sup> element!

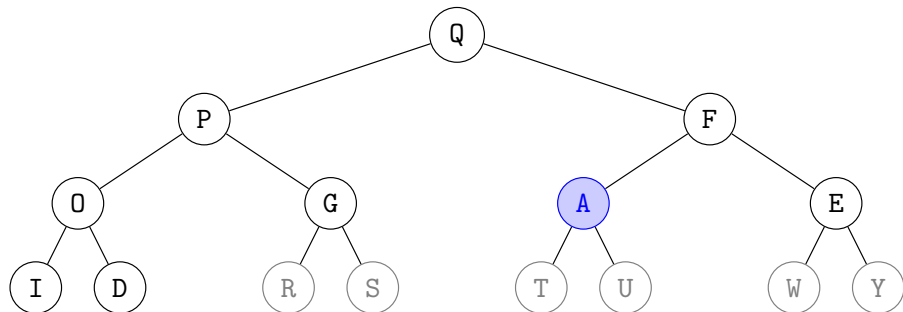
Q**P**AOG**F**EIDRSTUWY



# Removal of elements from the heap

We have the 9<sup>th</sup> element!

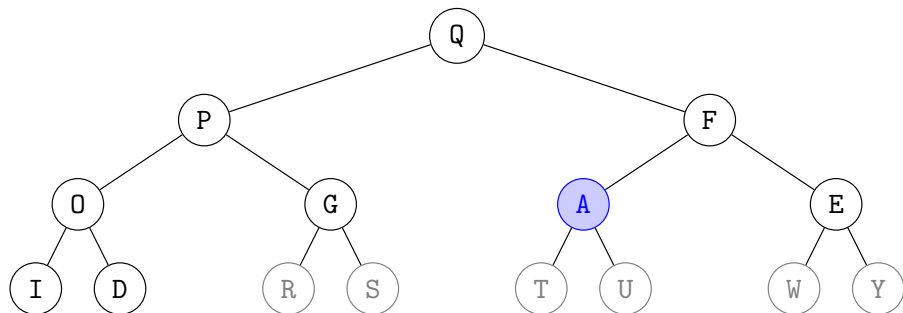
QPFOG**A**EIDRSTUWY



# Removal of elements from the heap

We have the 9<sup>th</sup> element!

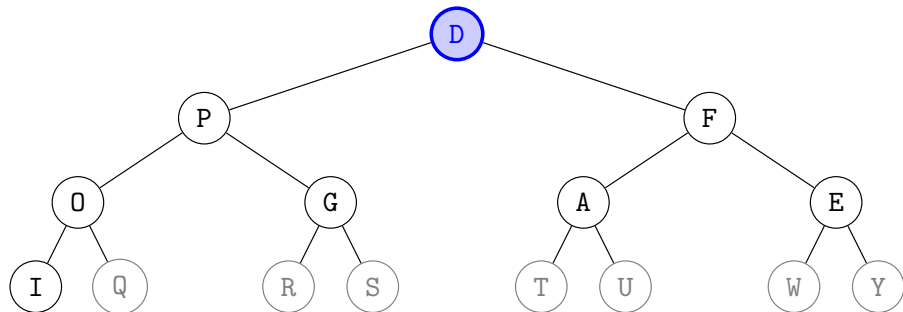
QPFOG**A**EIDRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

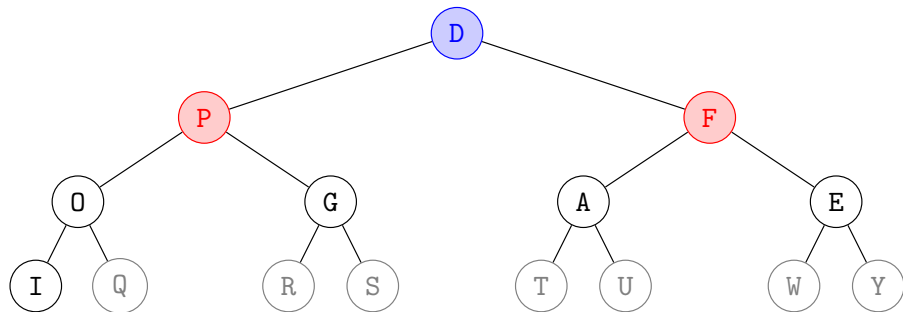
[D] PFOGAEIQRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

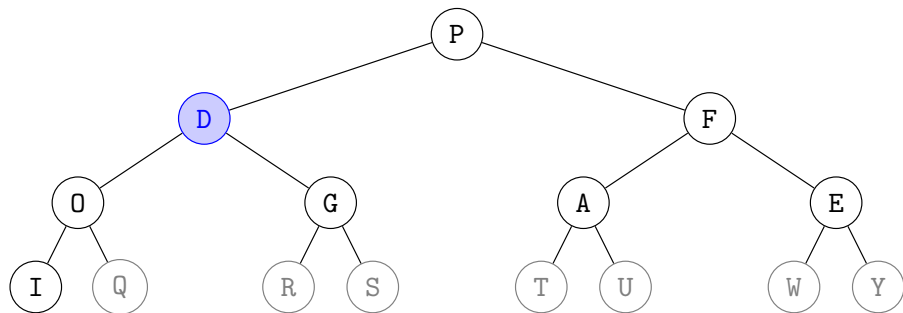
DPFOGAEIQRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

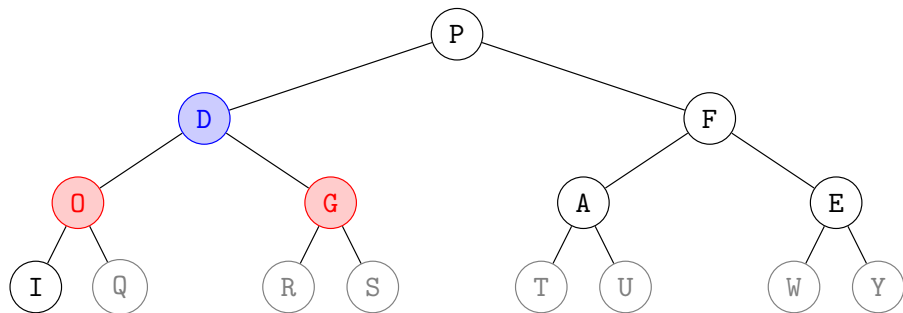
P D F O G A E I Q R S T U W Y



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

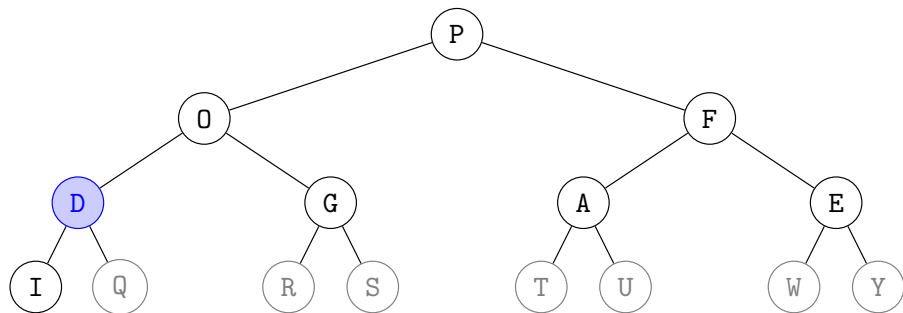
PDFOGAEIQRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

POFDGAEIQRSTUWY

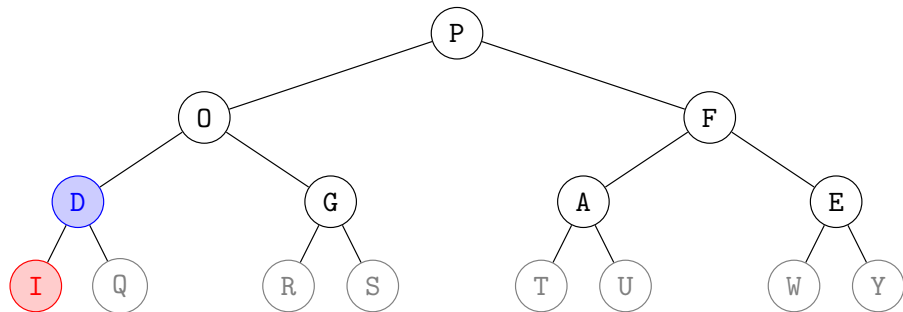




# Removal of elements from the heap

We have the 8<sup>th</sup> element!

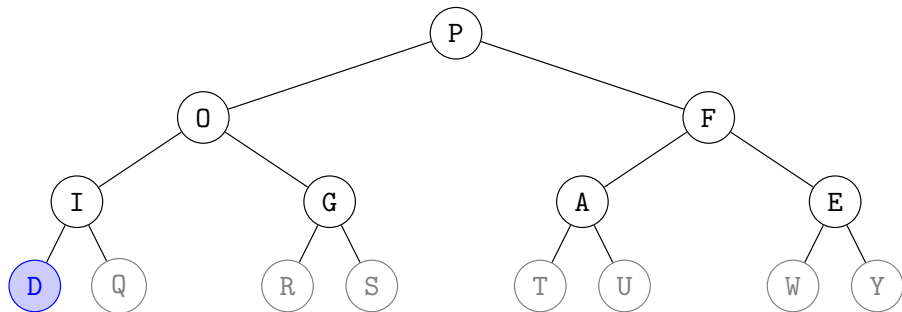
POFDGAEIQRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

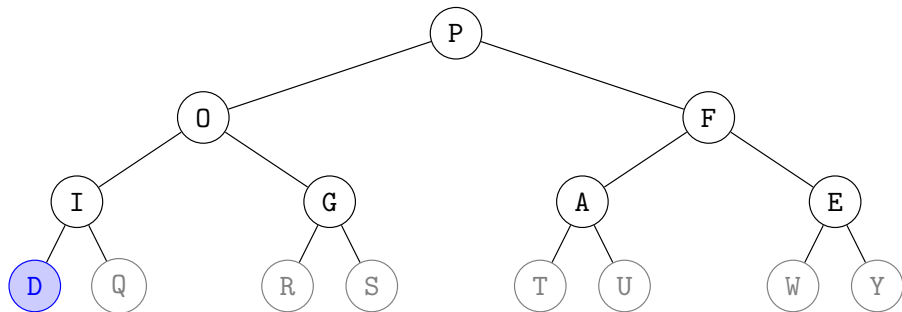
POFIGAEDQRSTUWY



# Removal of elements from the heap

We have the 8<sup>th</sup> element!

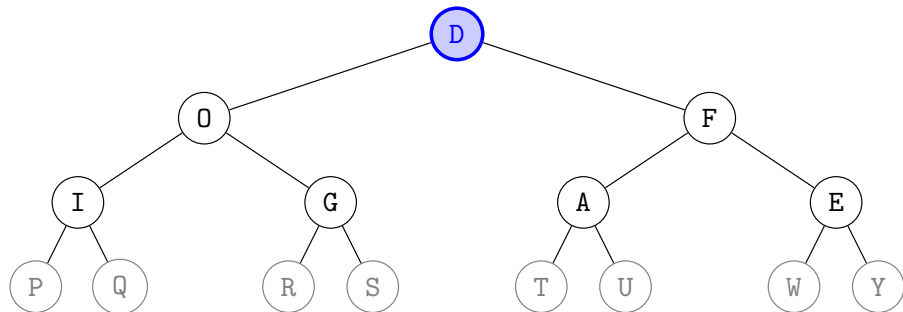
POFIGAEDQRSTUWY



# Removal of elements from the heap

We have the 7<sup>th</sup> element!

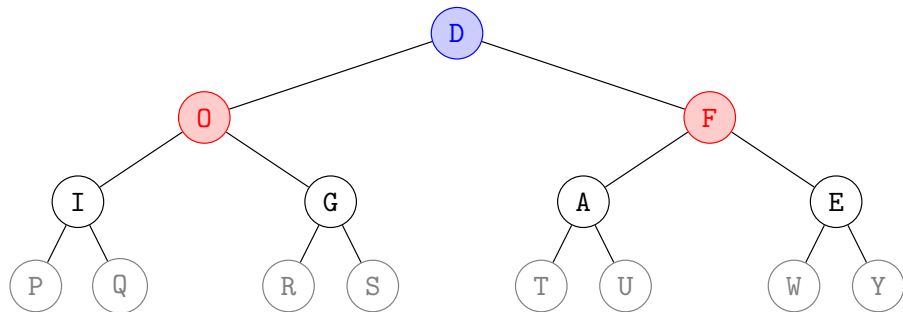
[D] OFIGAEPQRSTUVWXYZ



# Removal of elements from the heap

We have the 7<sup>th</sup> element!

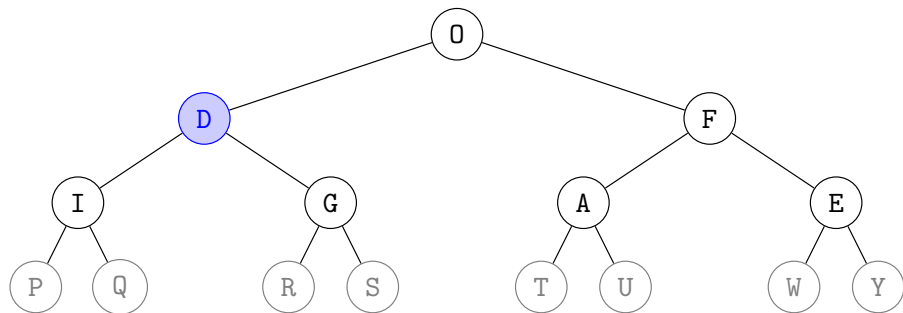
DOFIGAEPQRSTUWY



# Removal of elements from the heap

We have the 7<sup>th</sup> element!

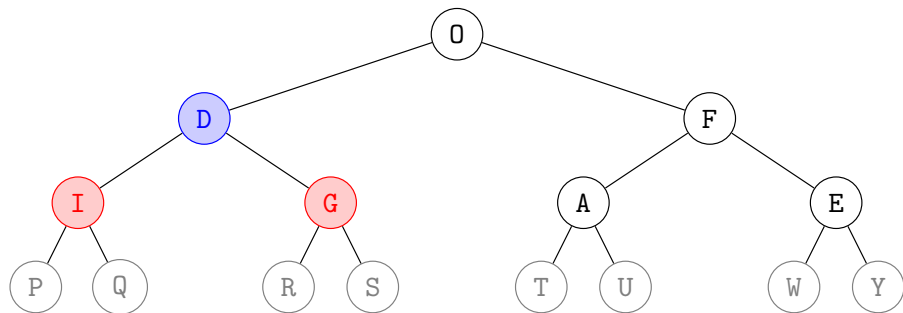
ODFIGAEPQRSTUVWXYZ



# Removal of elements from the heap

We have the 7<sup>th</sup> element!

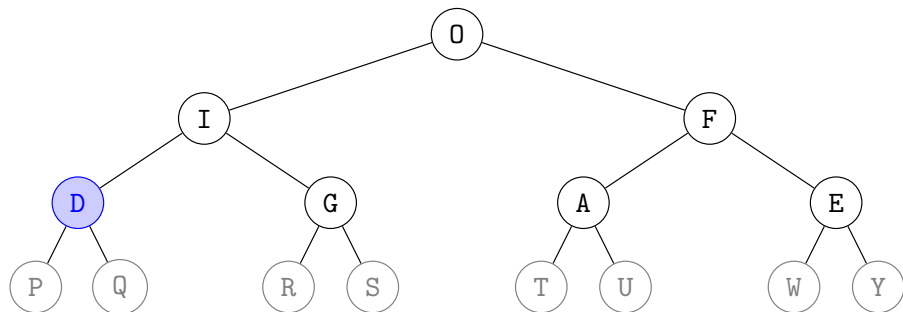
ODFIGAEPQRSTUVWXYZ



# Removal of elements from the heap

We have the 7<sup>th</sup> element!

OIFDGAEPQRSTUWY

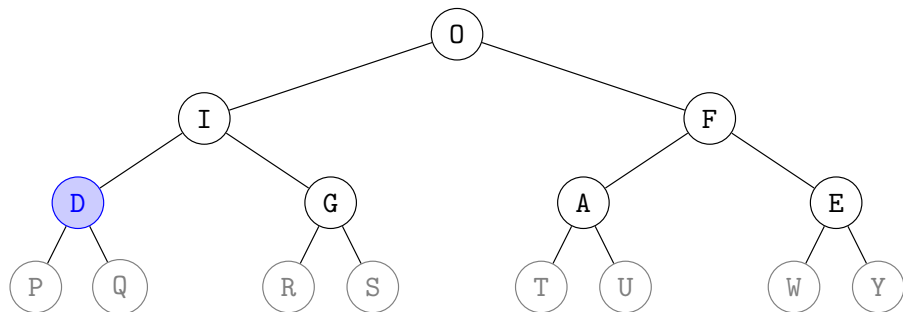




# Removal of elements from the heap

We have the 7<sup>th</sup> element!

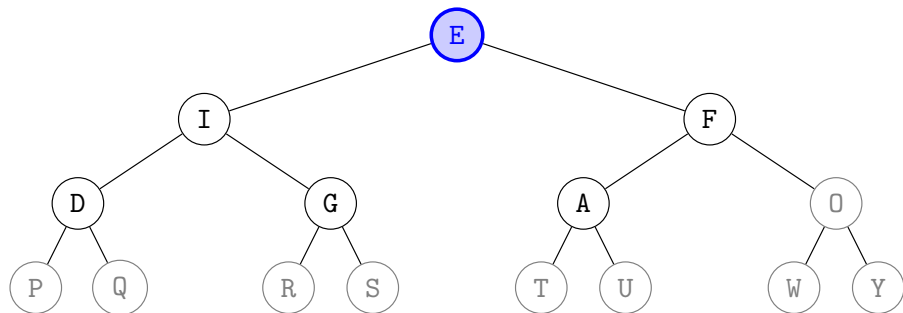
O I F D G A E P Q R S T U W Y



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

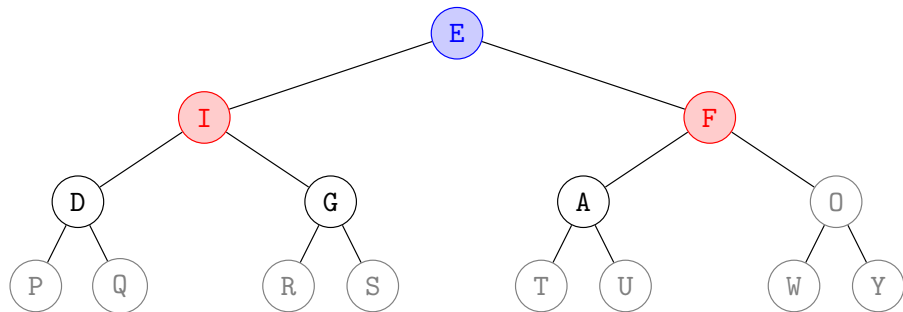
[E] IFDGAOPQRSTUWY



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

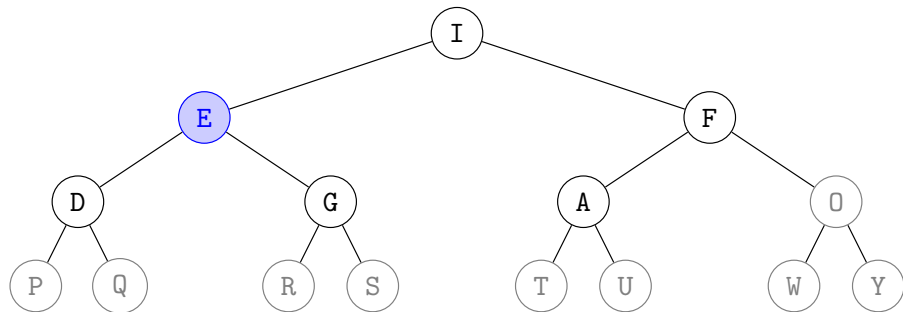
EIFDGAOPQRSTUWY



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

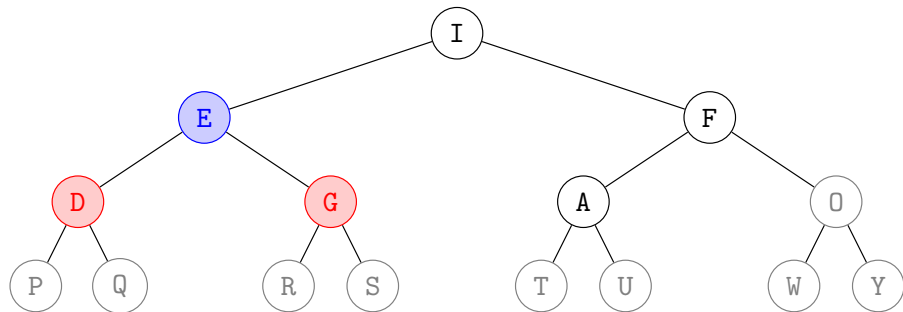
IEFDGAOPQRSTUWY



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

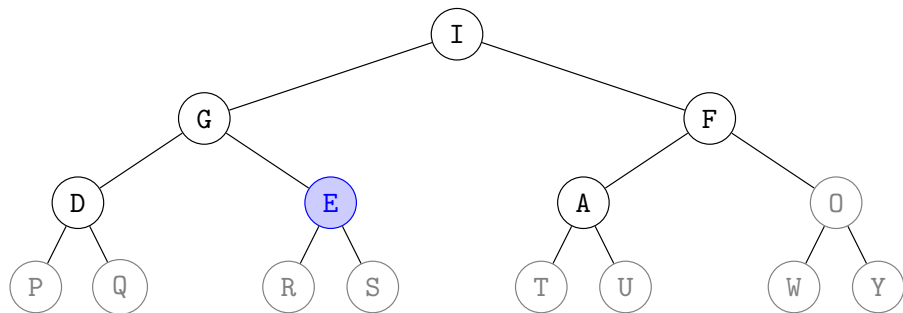
IEFDGAOPQRSTUWY



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

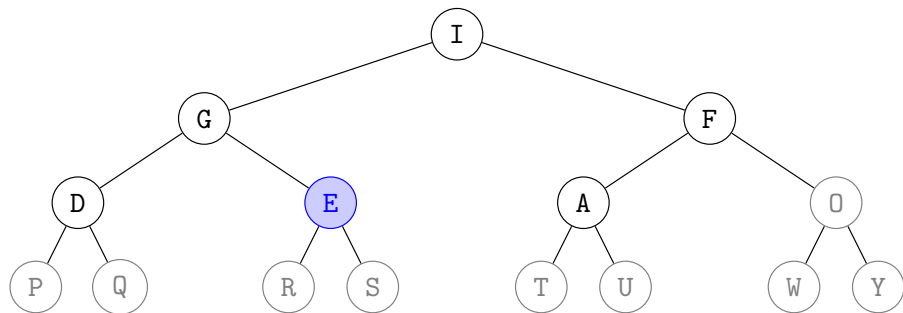
IGFDEAOPQRSTUWY



# Removal of elements from the heap

We have the 6<sup>th</sup> element!

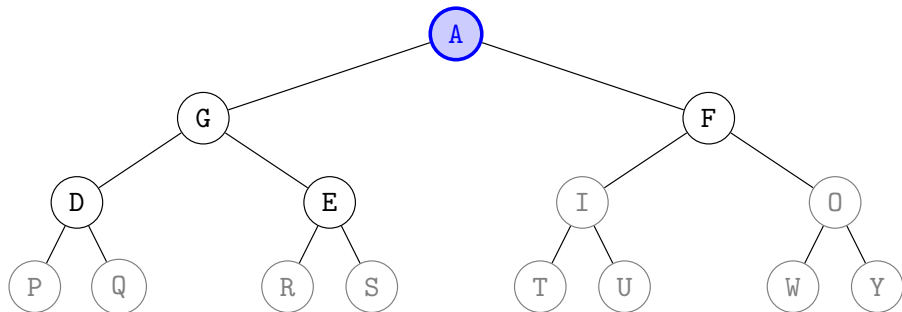
IGFDEAOPQRSTUWY



# Removal of elements from the heap

We have the 5<sup>th</sup> element!

[A] GFDEIOPQRSTUWY

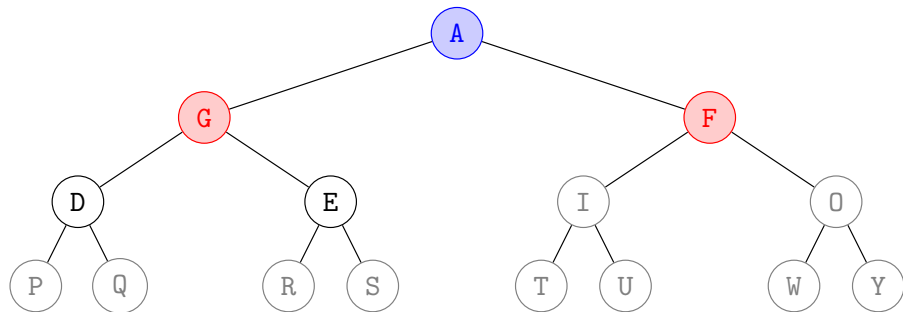




# Removal of elements from the heap

We have the 5<sup>th</sup> element!

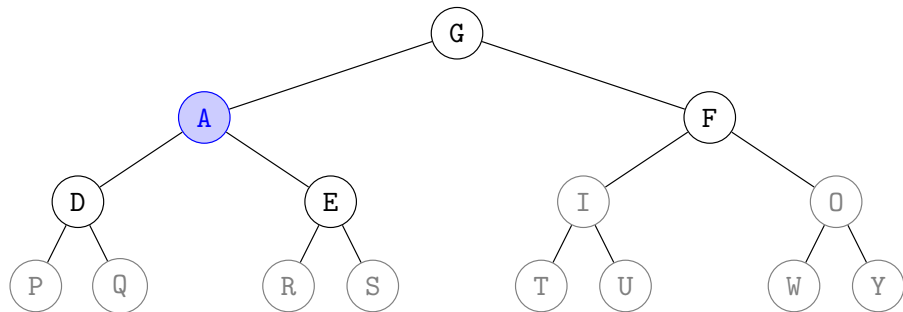
AGFDEIOPQRSTUWY



# Removal of elements from the heap

We have the 5<sup>th</sup> element!

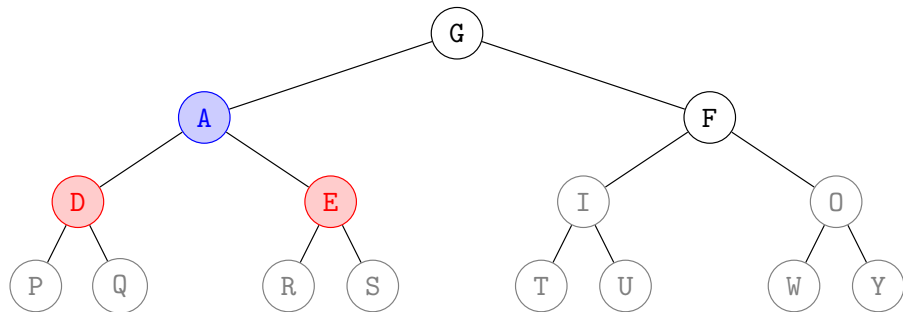
GAFDEIOPQRSTUWY



# Removal of elements from the heap

We have the 5<sup>th</sup> element!

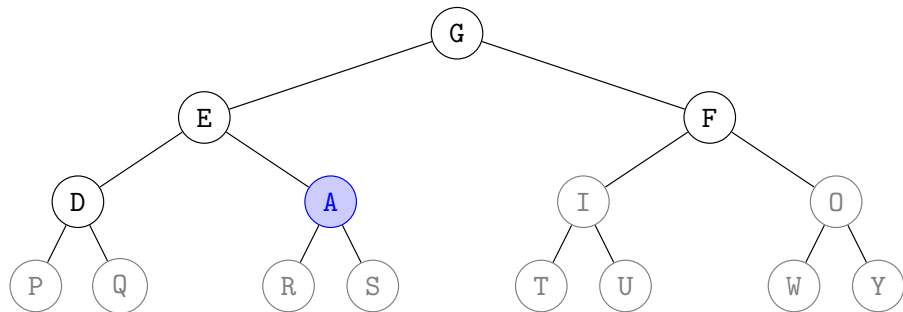
GAFDEIOPQRSTUWY



# Removal of elements from the heap

We have the 5<sup>th</sup> element!

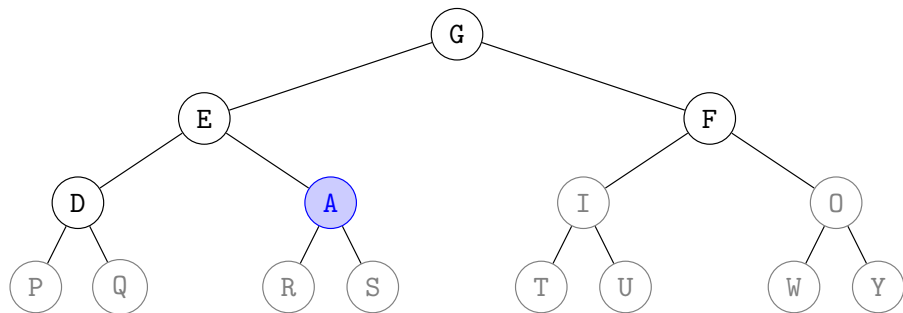
GEFD**A**IOPQRSTUVWXYZ



# Removal of elements from the heap

We have the 5<sup>th</sup> element!

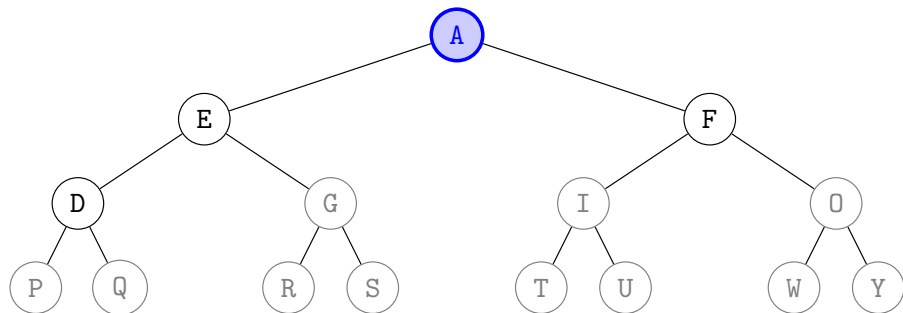
GEFD**A**IOPQRSTUVWXYZ



# Removal of elements from the heap

We have the 4<sup>th</sup> element!

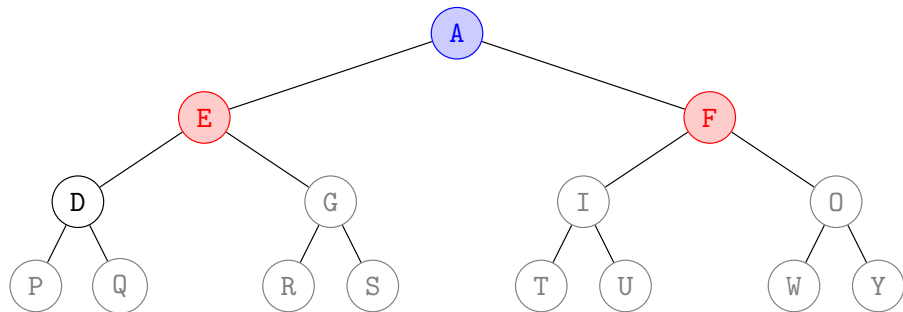
[A] EFDGIOPQRSTUWY



# Removal of elements from the heap

We have the 4<sup>th</sup> element!

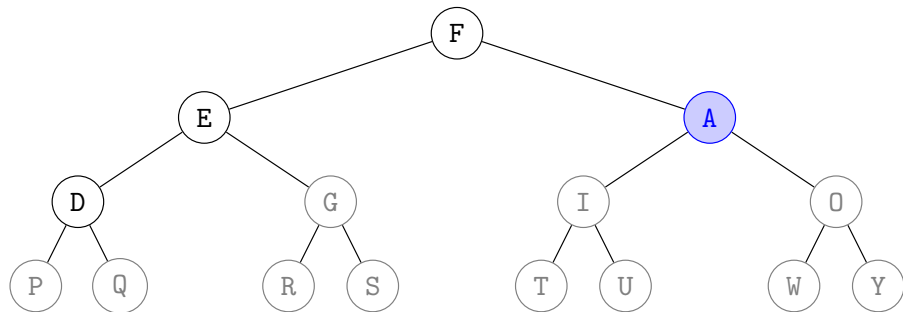
A**E**FDGIOPQRSTUWY



# Removal of elements from the heap

We have the 4<sup>th</sup> element!

FEADGIOPQRSTUWY

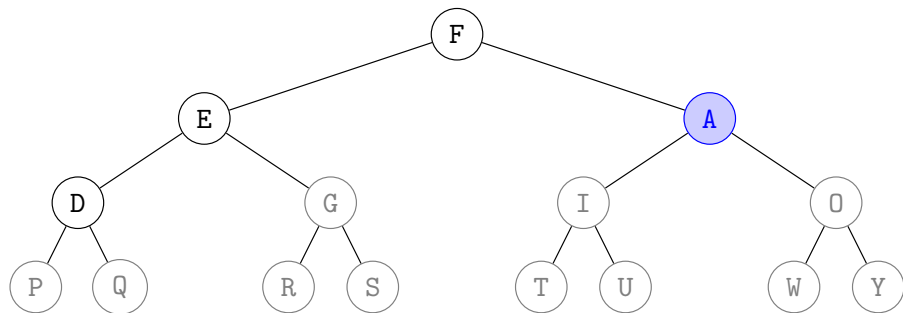




# Removal of elements from the heap

We have the 4<sup>th</sup> element!

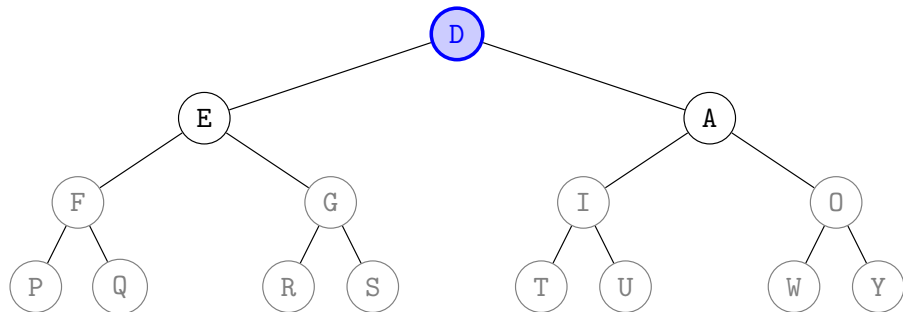
FEADGIOPQRSTUWY



# Removal of elements from the heap

We have the 3<sup>rd</sup> element!

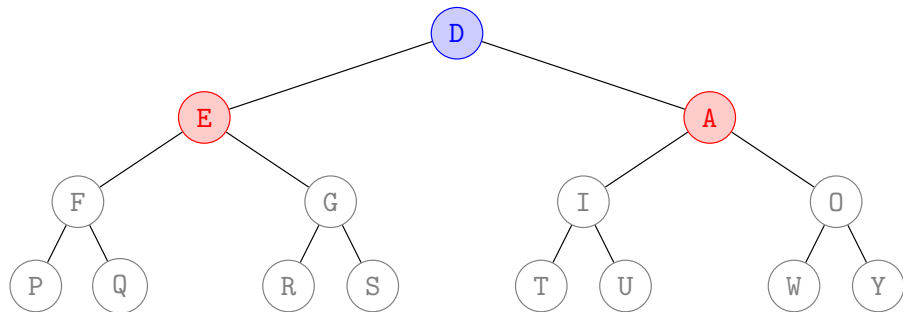
[D] EAFGIOPQRSTUWY



# Removal of elements from the heap

We have the 3<sup>rd</sup> element!

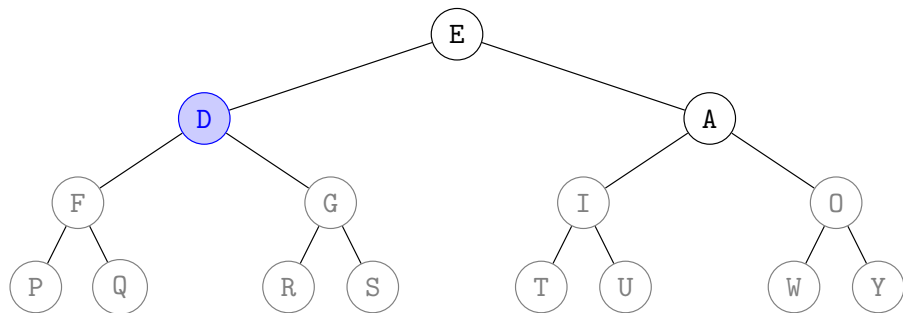
DEAFGIOPQRSTUWY



# Removal of elements from the heap

We have the 3<sup>rd</sup> element!

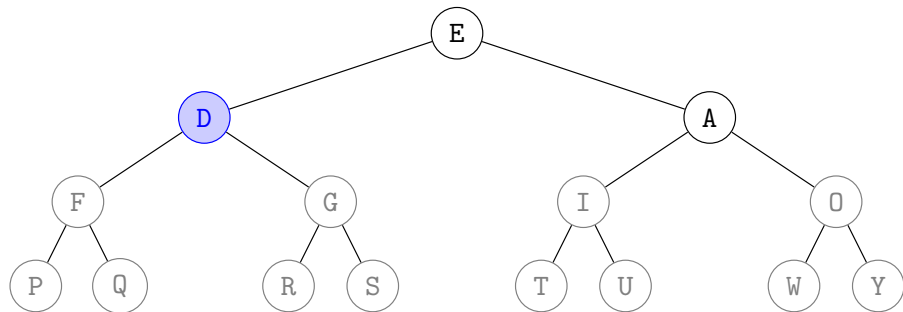
EDAFGIOPQRSTUWY



# Removal of elements from the heap

We have the 3<sup>rd</sup> element!

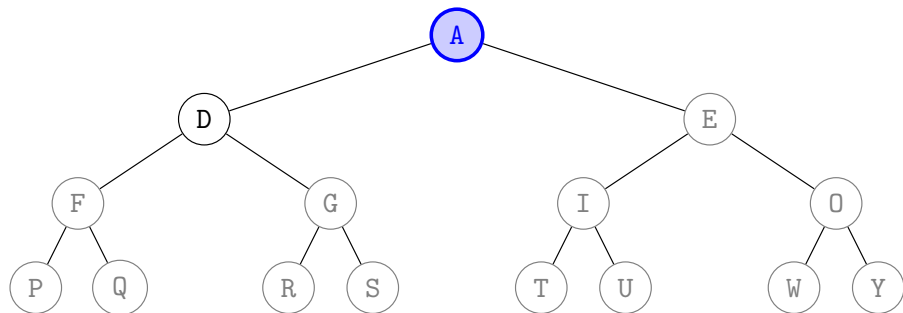
EDAFGIOPQRSTUWY



# Removal of elements from the heap

We have the 2<sup>nd</sup> element!

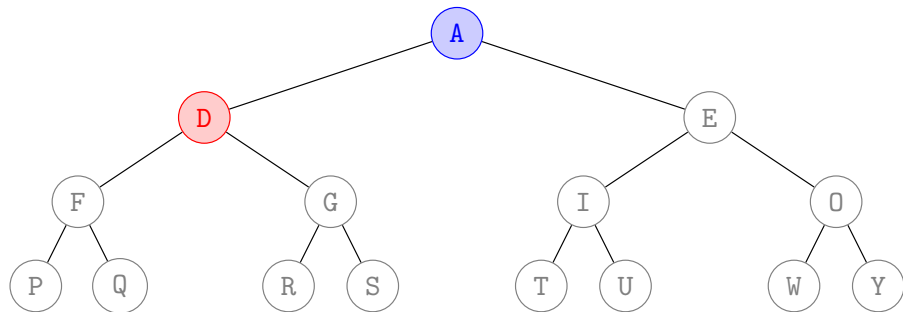
[A]DEFGIOPQRSTUWY



# Removal of elements from the heap

We have the 2<sup>nd</sup> element!

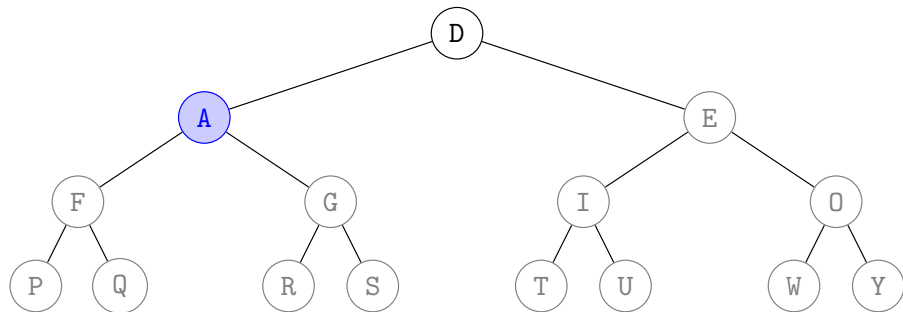
ADEFGIOPQRSTUWY



# Removal of elements from the heap

We have the 2<sup>nd</sup> element!

DAEFGIOPQRSTUWY

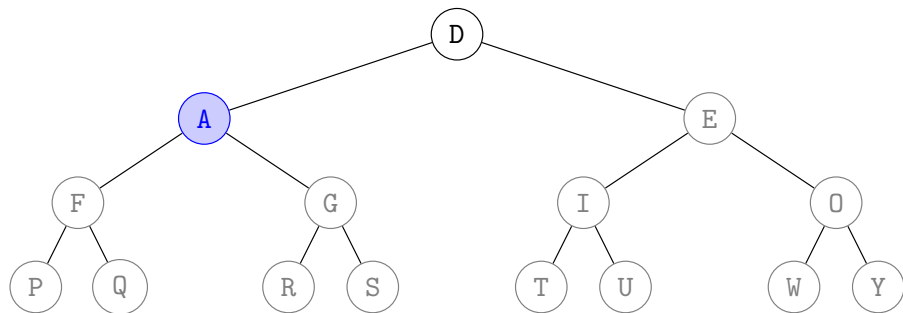




# Removal of elements from the heap

We have the 2<sup>nd</sup> element!

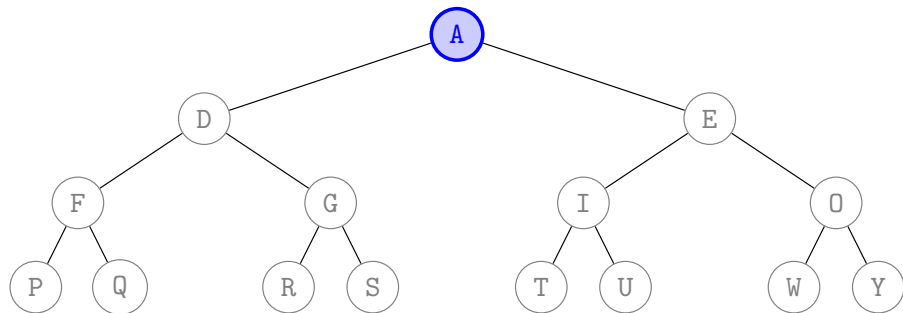
DAEFGIOPQRSTUWY



# Removal of elements from the heap

We have the 1<sup>st</sup> element!

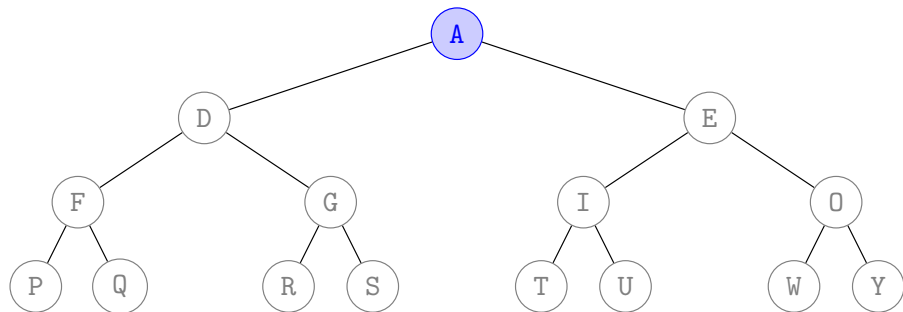
[A] DEFGIOPQRSTUWY



# Removal of elements from the heap

We have the 1<sup>st</sup> element!

ADEFGIOPQRSTUWY



# We are done!

What we get in the end is sorted:

ADEFGIOPQRSTUVWXYZ