

Social and Technological Network Analysis

Lecture 5: Structure of the Web, Search and Power Laws

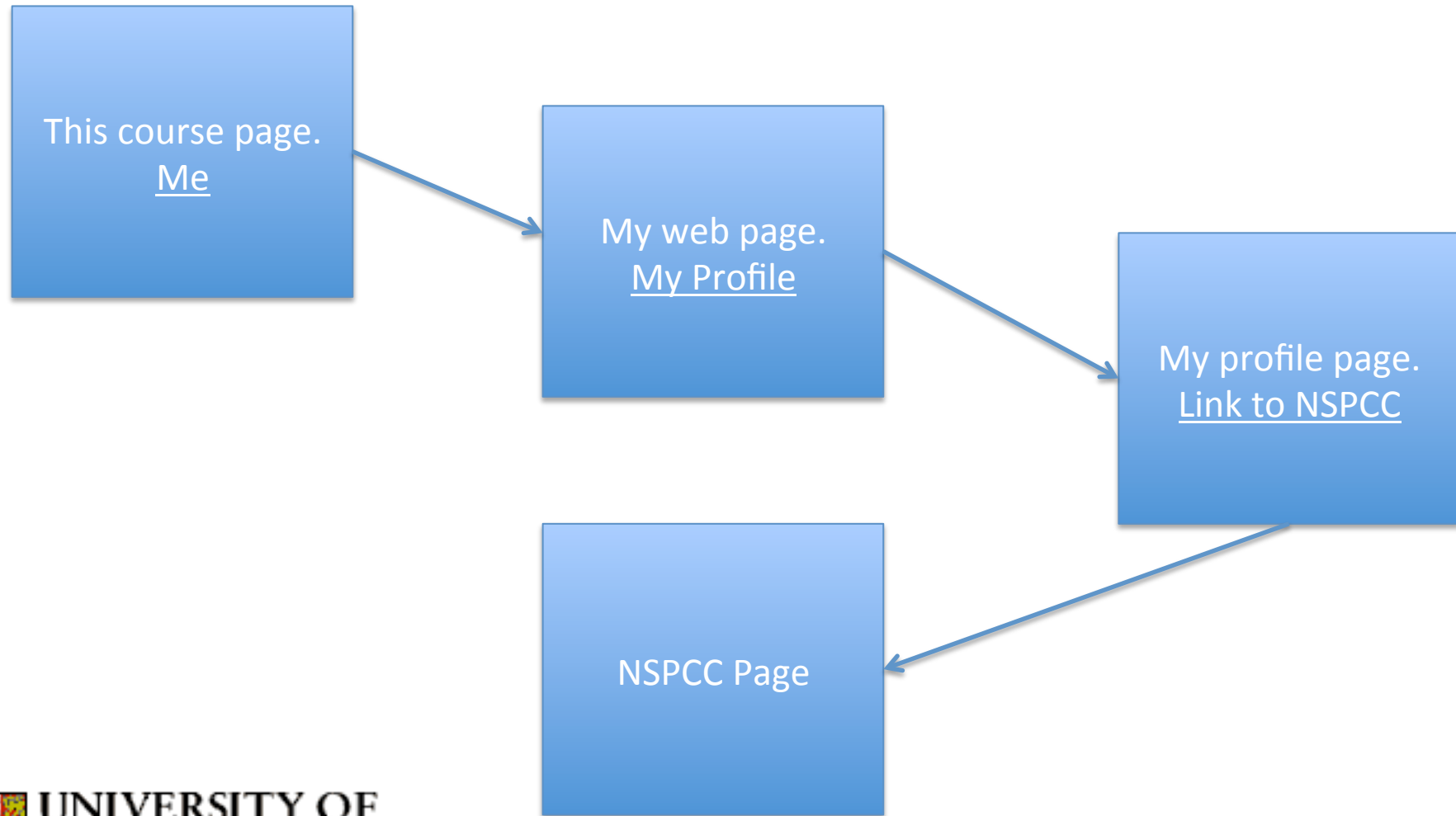
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In This Lecture



-
- We describe power law networks and their properties and show examples of networks which are power law in nature, including the web.
 - We present the preferential attachment model which allows the generation of power law networks.
 - We study prediction of power laws
 - We introduce search and PageRank

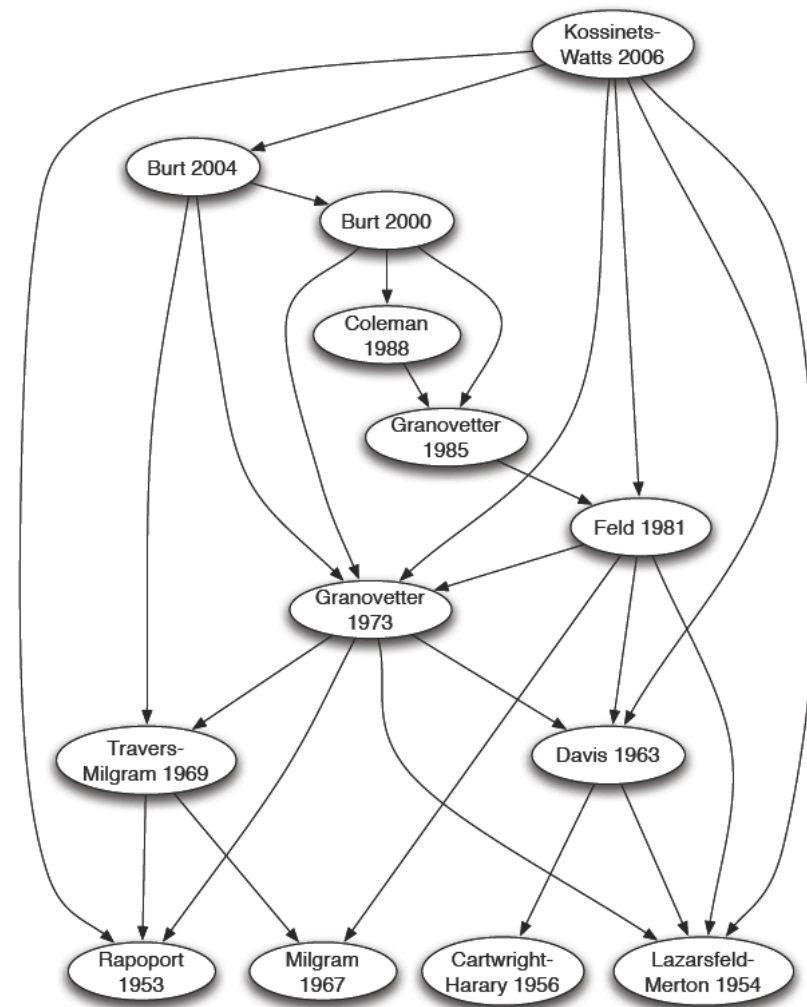
The Web is a Graph...



Precursor of hypertexts



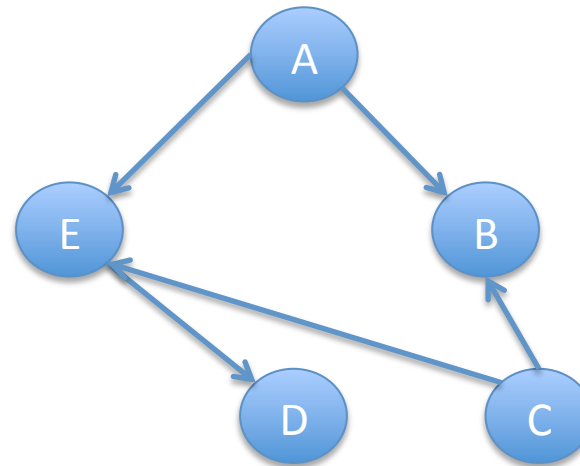
- Citation networks of books and articles.
- Difference: links point only backwards in time



Web is a Directed Graph



- **Path:** A path from A to B exists if there is a sequence of nodes beginning with A and ending with B such that each consecutive pair of nodes is connected by an edge pointing in the forward direction.



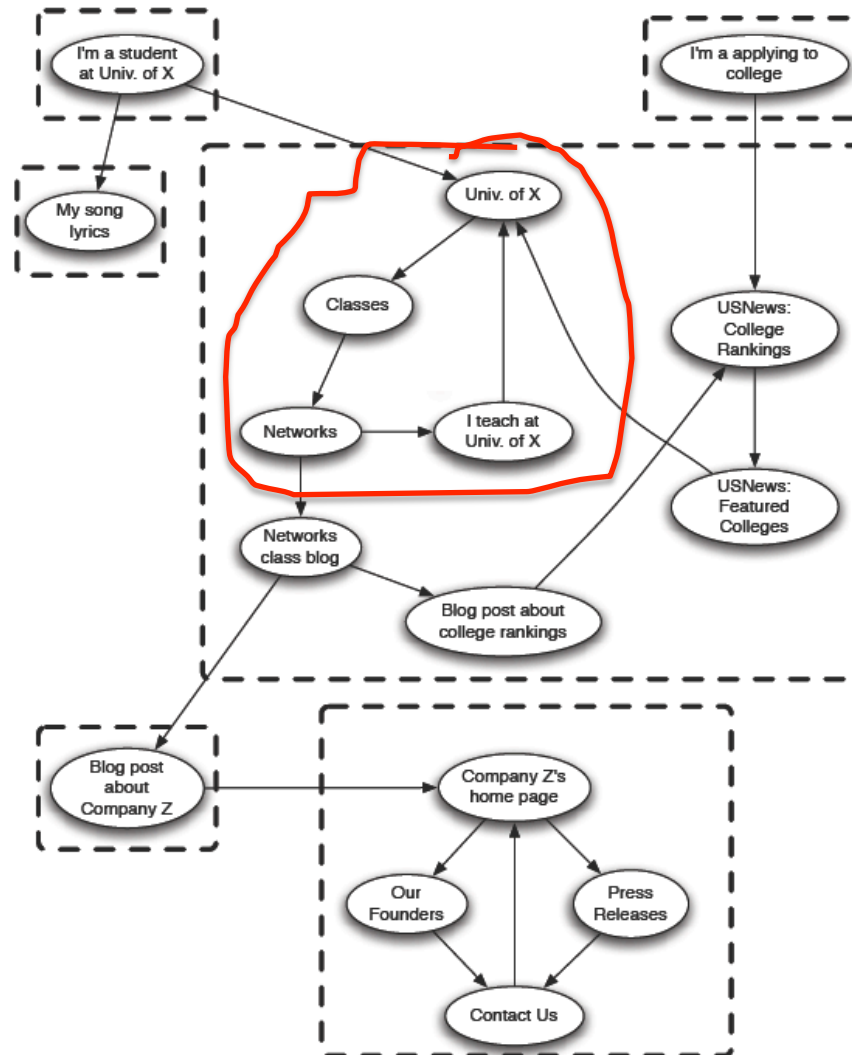
Strongly Connected Component



- A strongly connected component (SCC) in a directed graph is a subset of nodes such that:
 - i) Every pair in the subset has a path to each other
 - ii) The subset is not part of some larger subset with property i)
- Weakly connected component (WCC) is the connected component in **the undirected** graph derived from the directed graph.
 - Two nodes can be in the same WCC even if there no directed path between them.



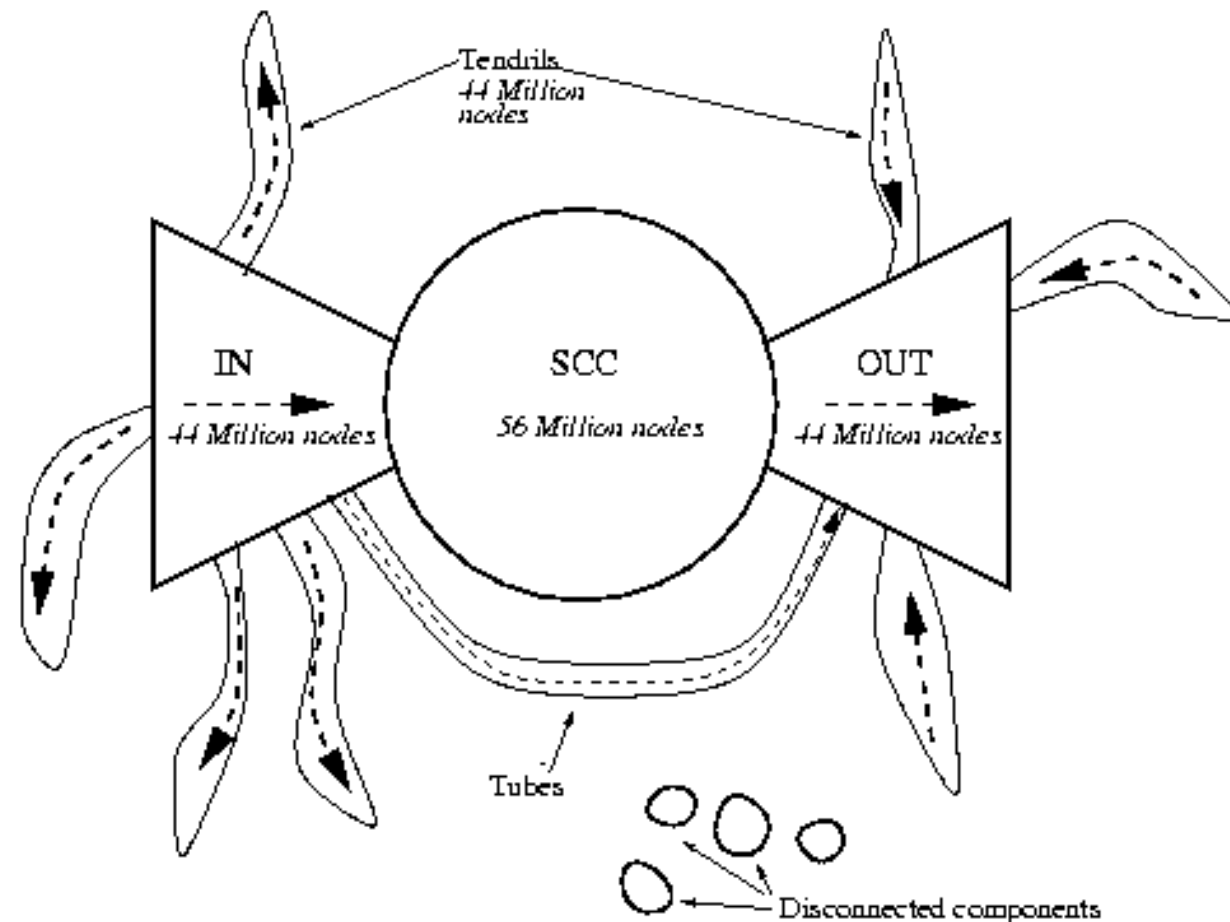
SCC example





The Web

- Broder'00
- Data from Altavista (200 million pages)
- 186M nodes in the WCC (90% of links)



Popularity of Web Pages

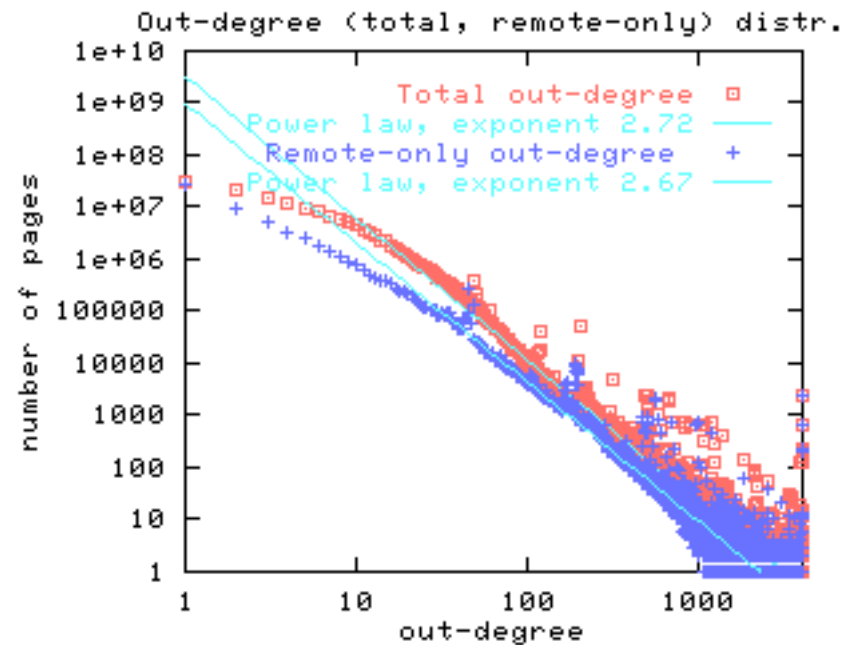
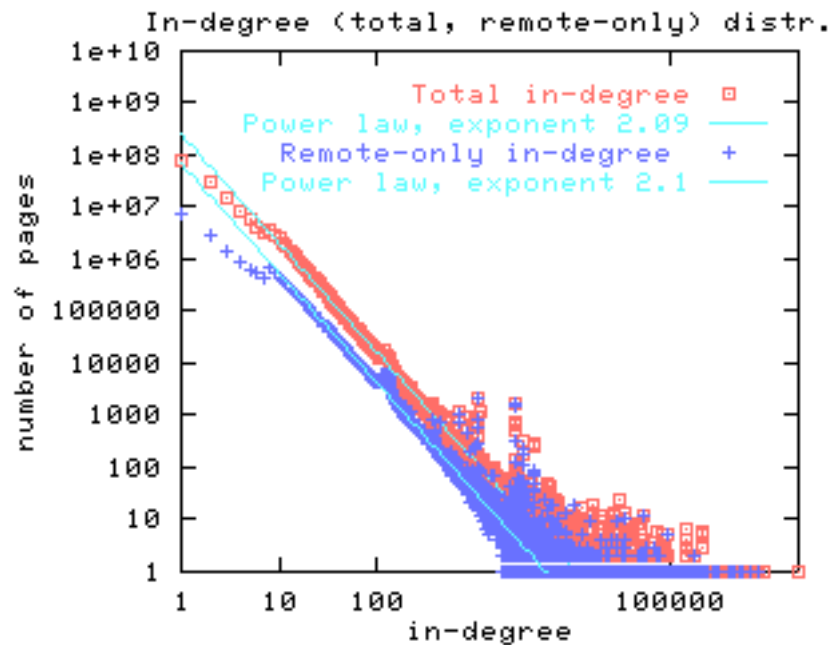


- How do we expect the popularity of web pages to be distributed?
 - What fraction of web pages have k in-links?
 - If each page decides independently at random whether to link to any given other page then the n of in-links of a page is the sum of independent random quantities \rightarrow normal distribution
 - In this case, the number pages with k in-links decreases exponentially in k
 - Is this true for the Web?

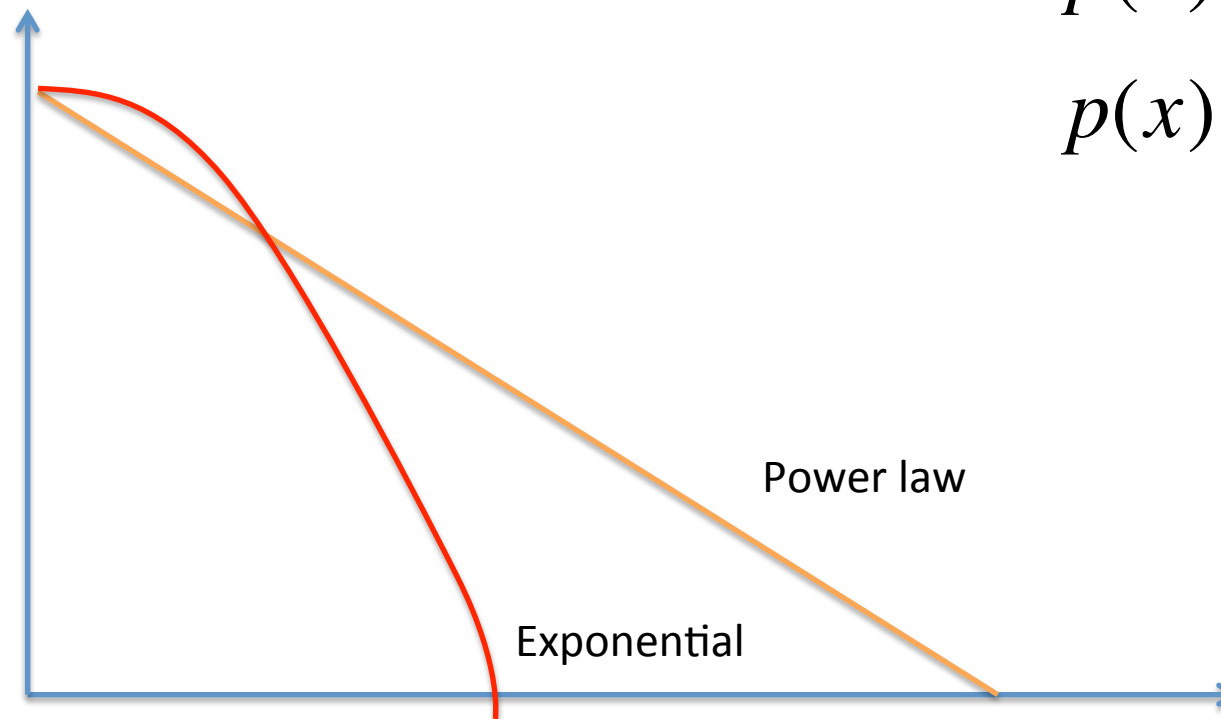
Degree distribution for the Web



- Finding: degree distr. proportional to $\sim 1/k^2$
- $1/k^2$ decreases much more slowly than a normal distribution



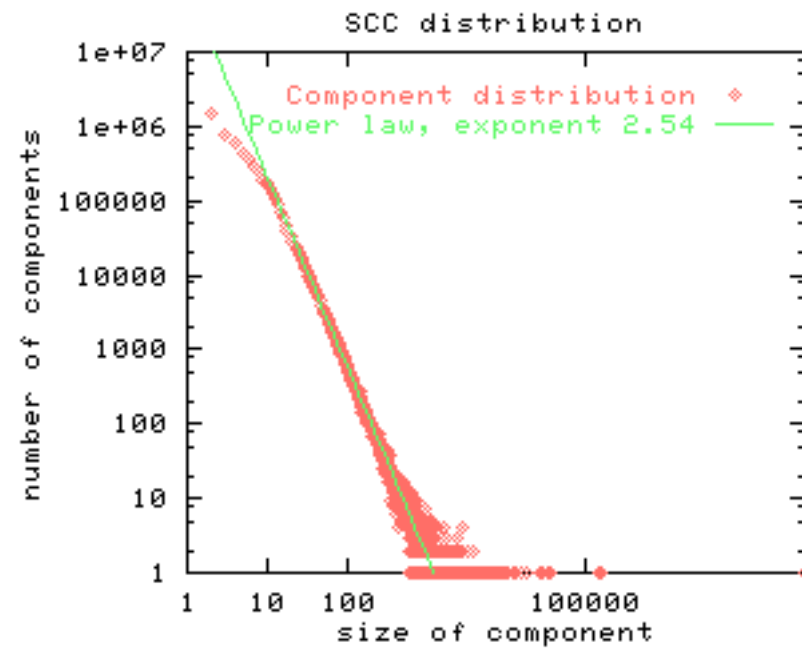
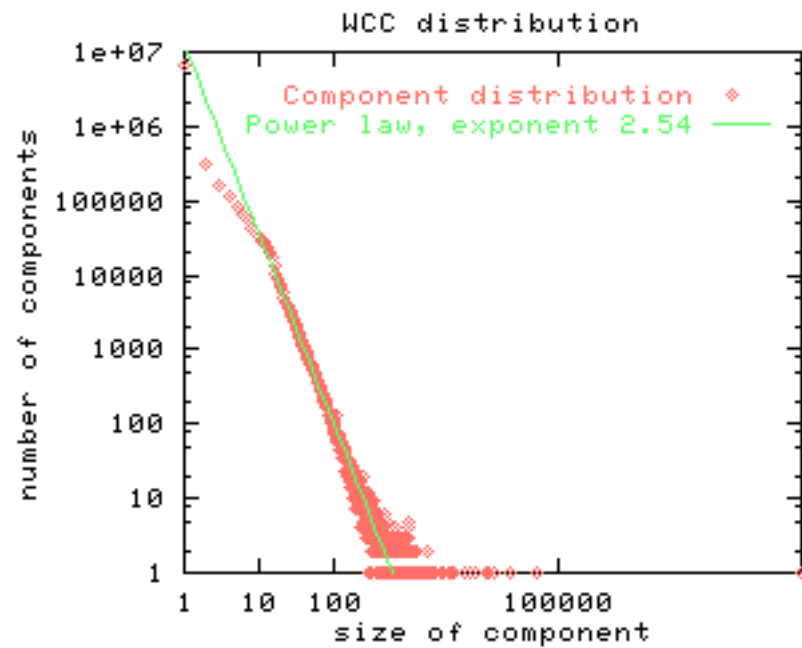
Power Law vs Exponential



$$p(x) = x^{-\alpha}$$

$$p(x) = e^{-\lambda x}$$

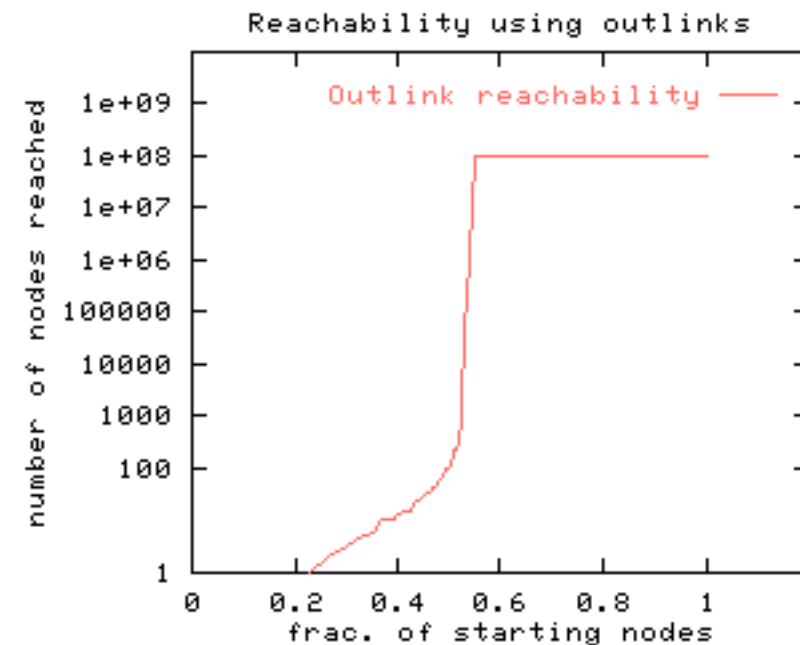
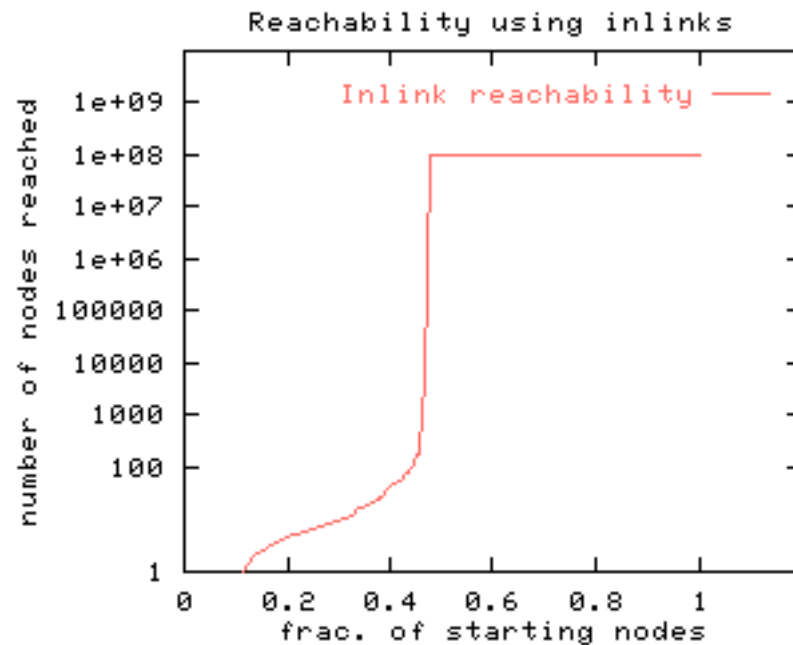
Distribution of WCC and SCC



Reachability



- Followed links backwards and forward



Diameter of the Web



- 75% of the time there is no directed path between two random nodes
- Average distance of existing paths: 16
- Average distance of undirected paths: 6.83

- Diameter in the SCC is at least 28

Power Laws aka Scale Free Networks



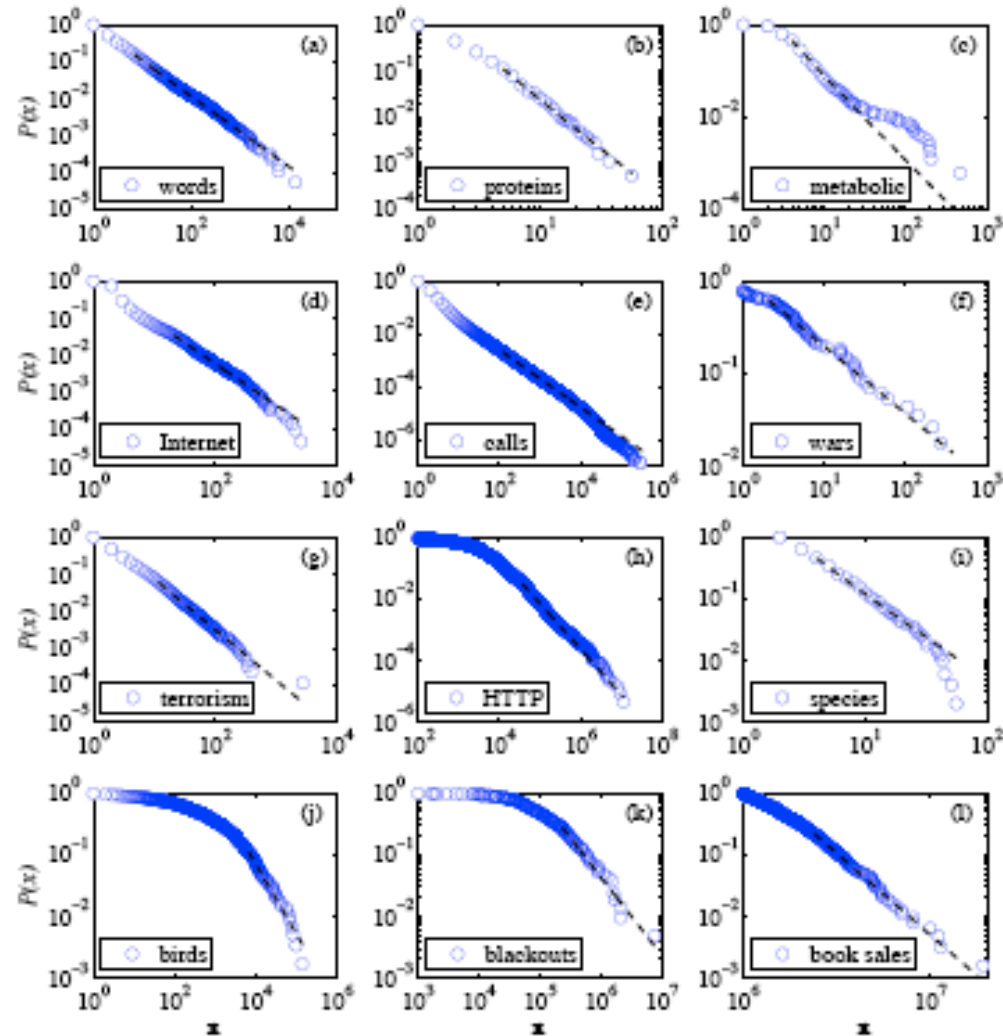
- We have seen that the degree distribution followed a straight line in log-log

$$\ln p_k = -\alpha \ln k + c$$

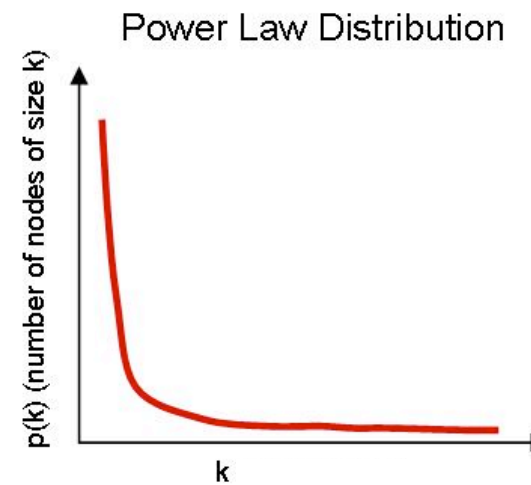
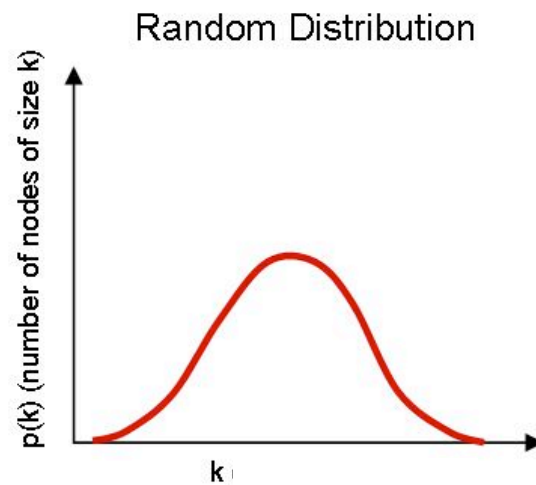
$$p_k = Ck^{-\alpha}$$

- α defines the slope of the curve
- α is typically between 2 and 3.

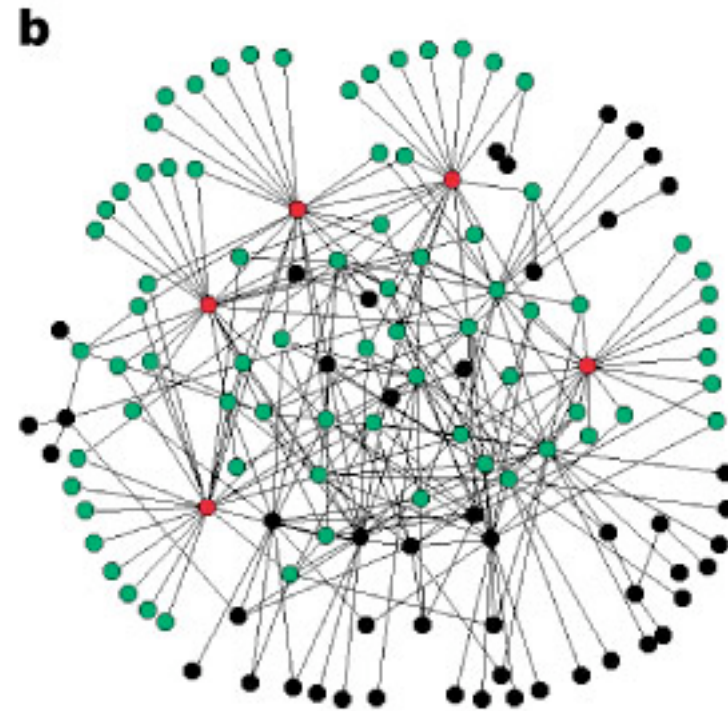
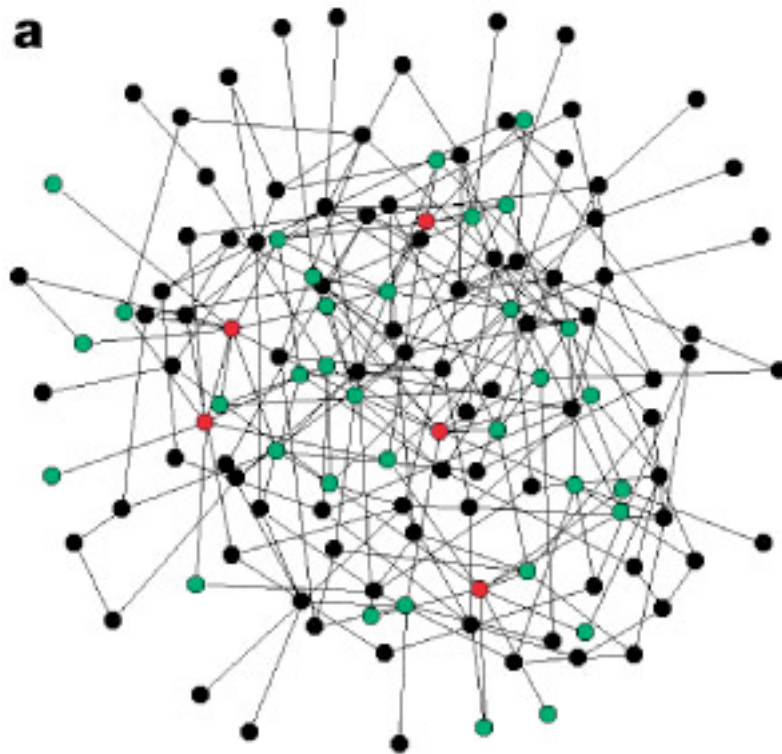
Power Laws in various domains



What does it mean?



Random vs Power Law Networks



What's a good model for scale free networks

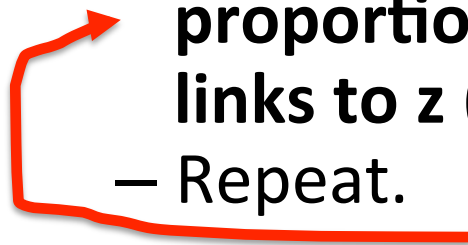


- Let's use the web network as example:
- Pages are created in order (1,2,3..)
- Page j created and it links to an earlier page in the following way:
 - With prob. p , j chooses page i at random and links it;
 - With prob. $1-p$, j chooses page i and links to the page i points to.
 - Repeat.
- The middle step is essentially a copy of the node i behaviour...

Preferential attachment



- Pages are created in order (1,2,3..)
- Page j created and it links to an earlier page in the following way:
 - With prob. p , j chooses page i at random and links it;
 - **With prob. $1-p$, j chooses a page z with prob. proportional to z 's current number of in-links and links to z (ie proportional to degree).**
 - Repeat.



Rich-get-richer model

**If we run this for many pages the fraction of pages with k in-links will be distributed approximately according to a power law $1/k^c$
 c depends on p**

Intuition



- With probability $1-p$ page j chooses a page l with probability proportional to l 's number of inlinks and creates a link to l .
- This mechanism predicts that the growth happens so that
 - A page's popularity growth at a rate proportional to its current value.
 - The rich get richer effect amplifies the larger values

Preferential Attachment



- What have we shown?
- There is a “copying” behaviour happening in these networks where nodes seem to emulate other nodes.
- This is shown true for selection of books, songs, web pages, movies etc.

How predictable is the rich-get-richer process?

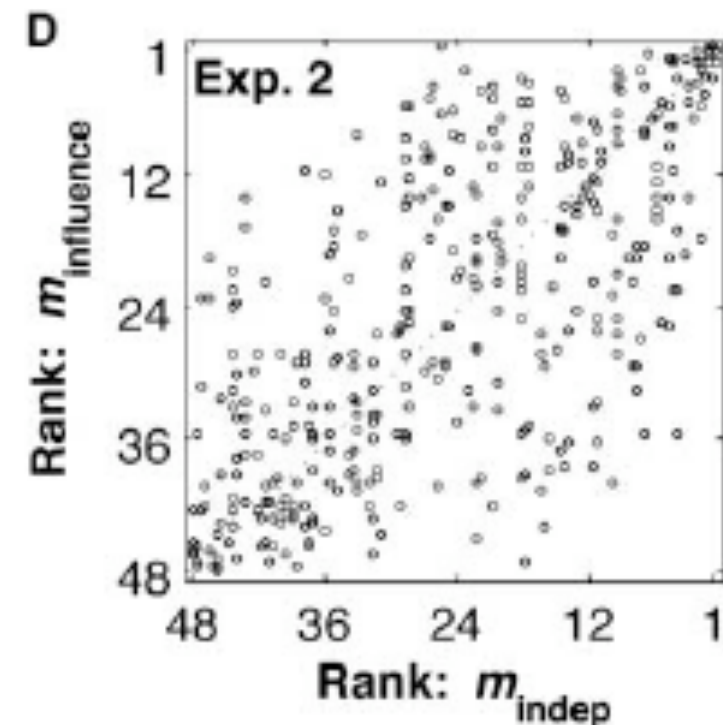
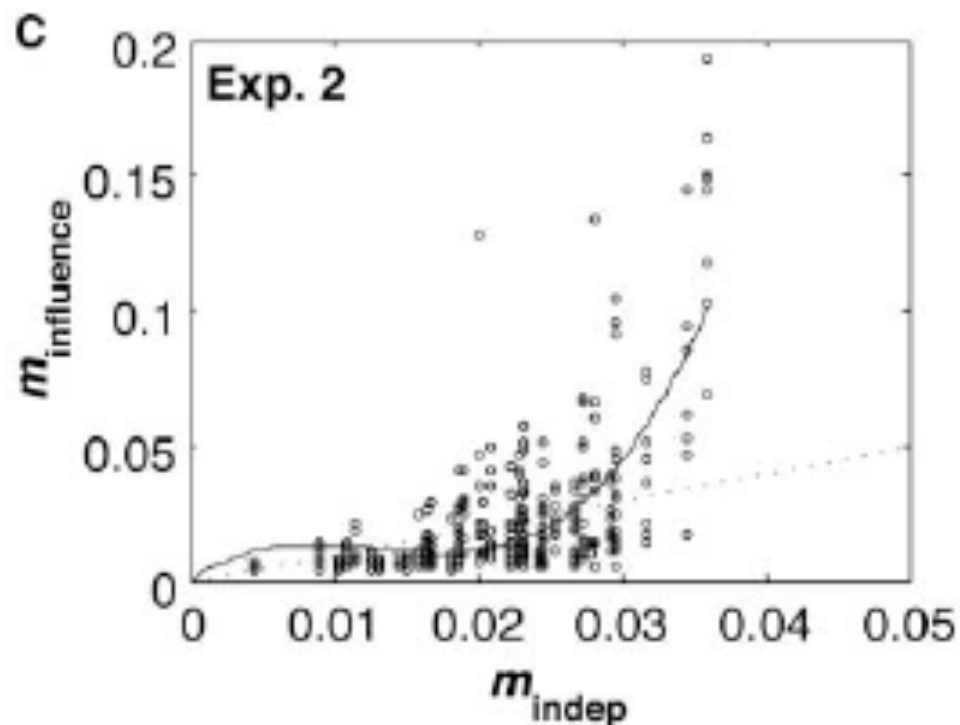


- Is the popularity of items in the power law predictable?
- Would a popular book still be popular if we go back in time and start the process again?
- Experiments show it would not...



Unpredictability [Salganik et al 06]

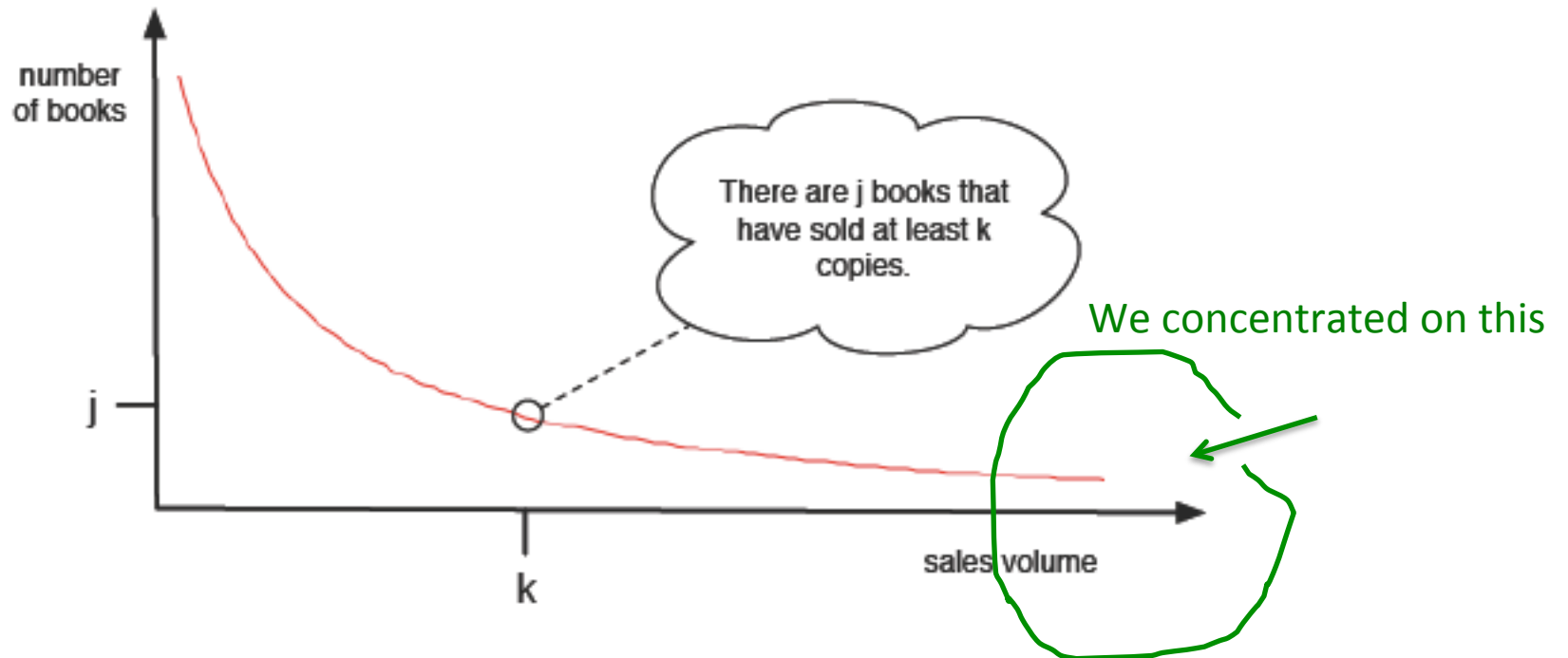
- 48 songs, 14,000 participants, 8 servers



View of the curve



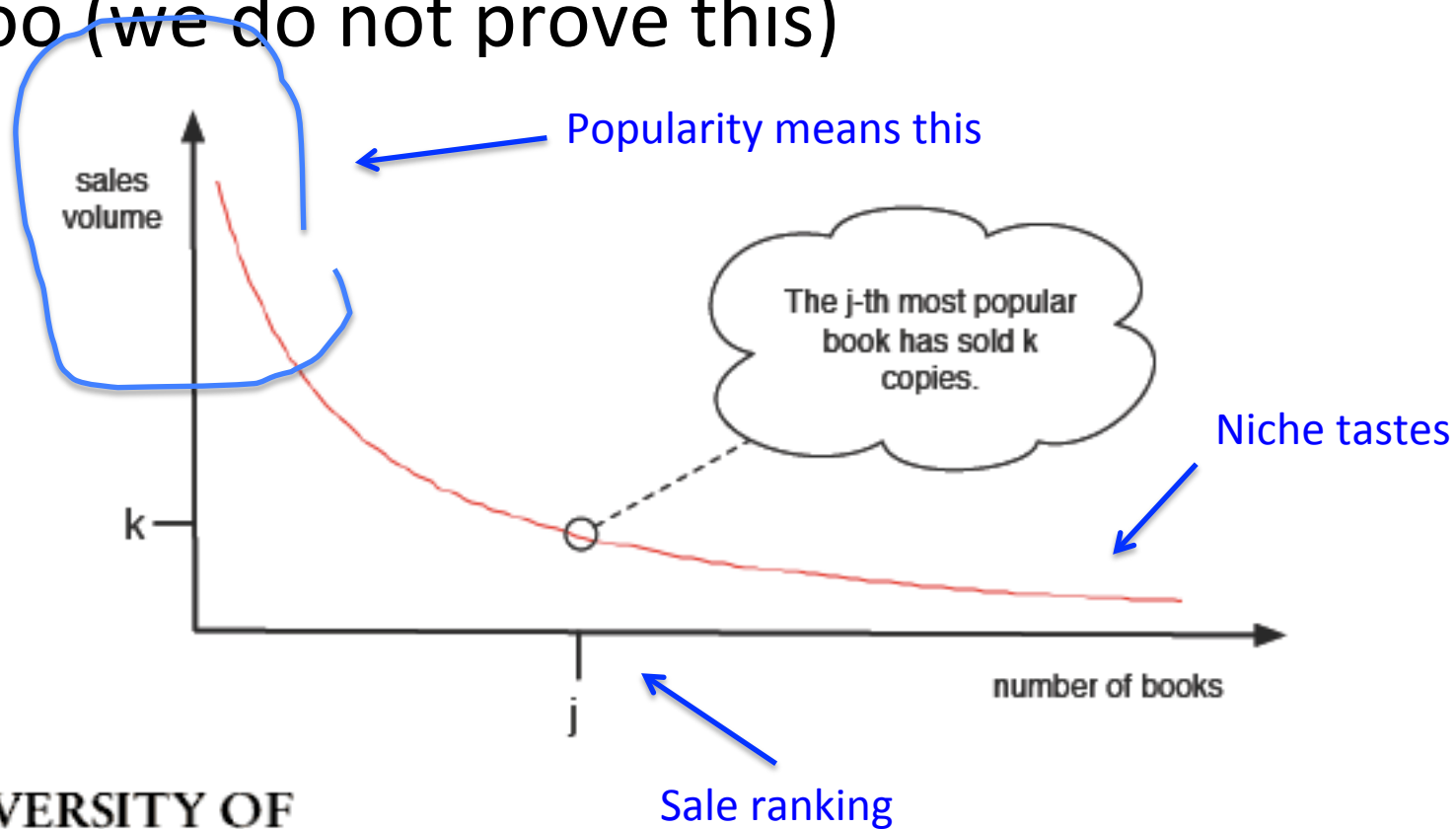
- The way we have seen the curve so far...



Let's transform the function



- If the initial function is a power law, this one is too (we do not prove this)



Search



- Information retrieval problem: synonyms (jump/leap), polysemy (Leopard), etc
- Now with the web: diversity in authoring introduces issues of common criteria for ranking documents
- The web offers abundance of information: whom do we trust as source?
- Still one issue: static content versus real time
 - World trade center query on 11/9/01
 - Twitter helps solving these issues these days

Automate the Search



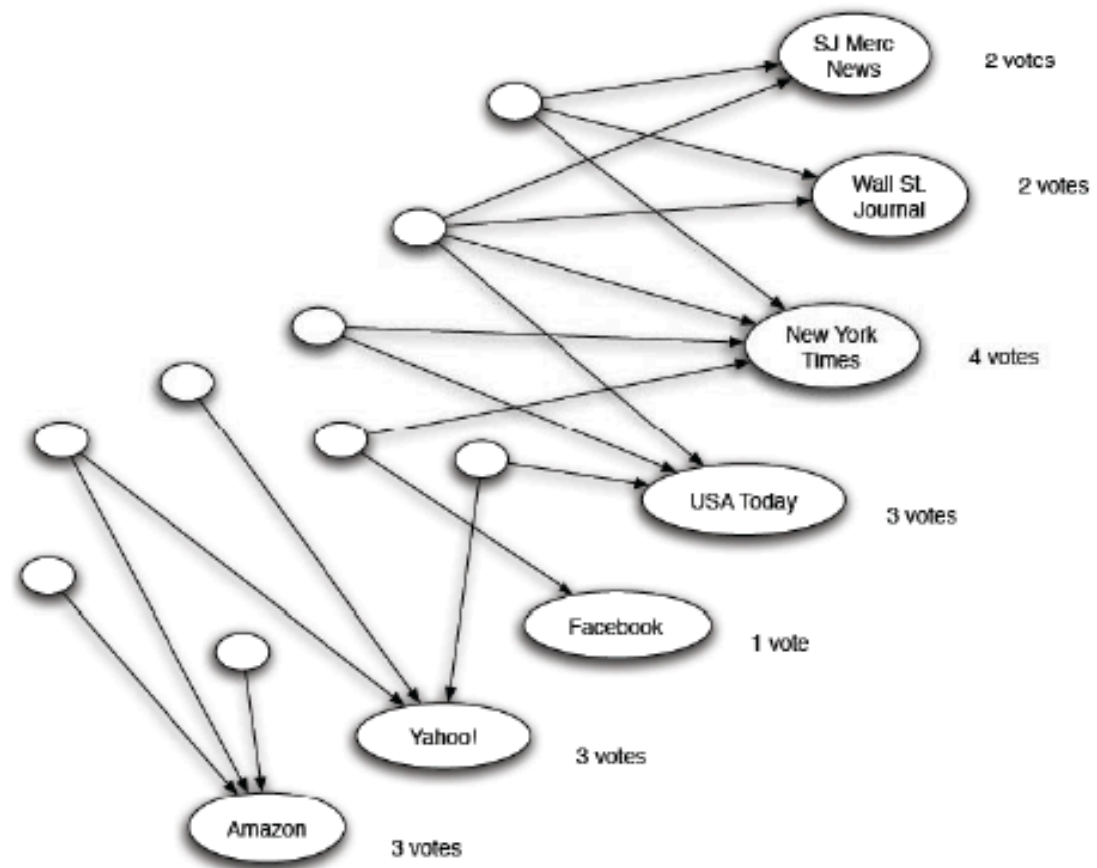
- When searching “Computer Laboratory” on Google the first link is for the department’s page.
- How does Google know this is the best answer?
- We could collect a large sample of pages relevant to “computer laboratory” and collect their votes through their links.
- The pages receiving more in-links are ranked first.
- But if we use **the network structure** more deeply we can improve results.

Example: Query “newspaper”

Authorities



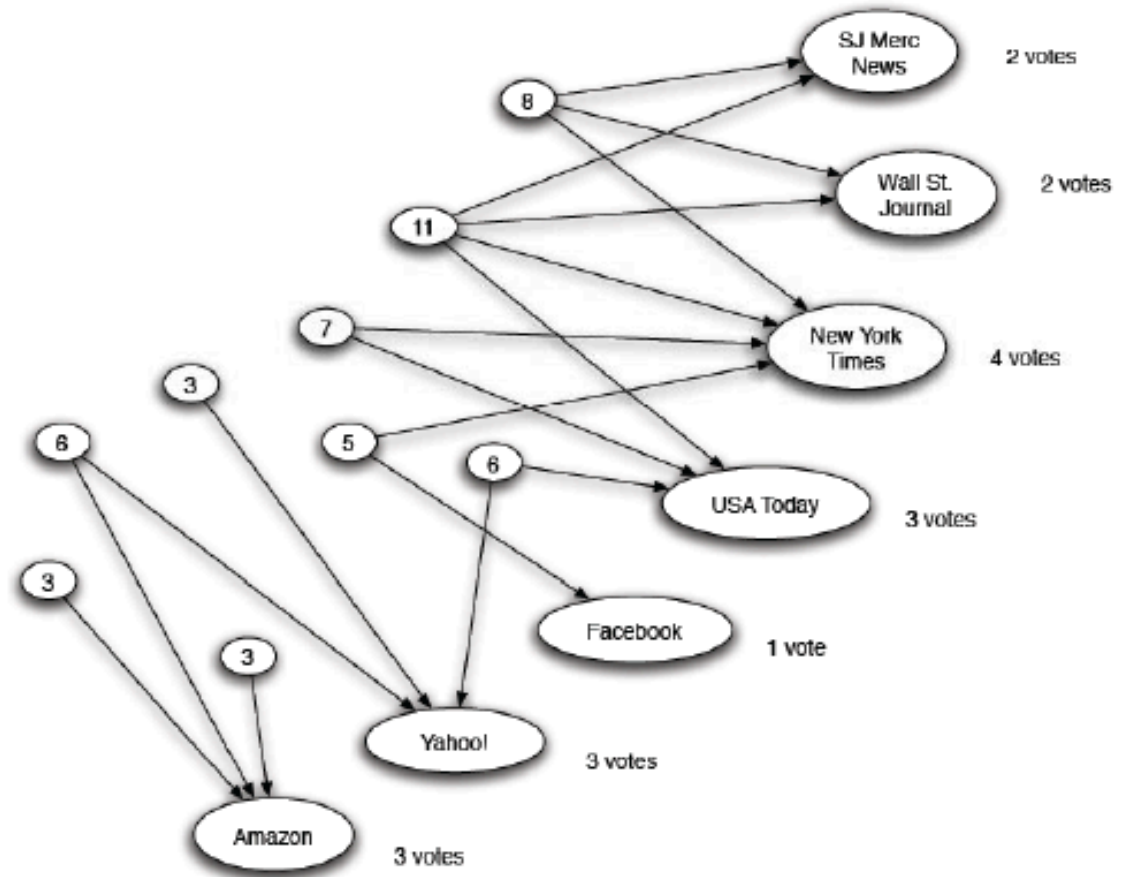
- Links are seen as votes.
- **Authorities** are established: the highly endorsed pages



A Refinement: Hubs



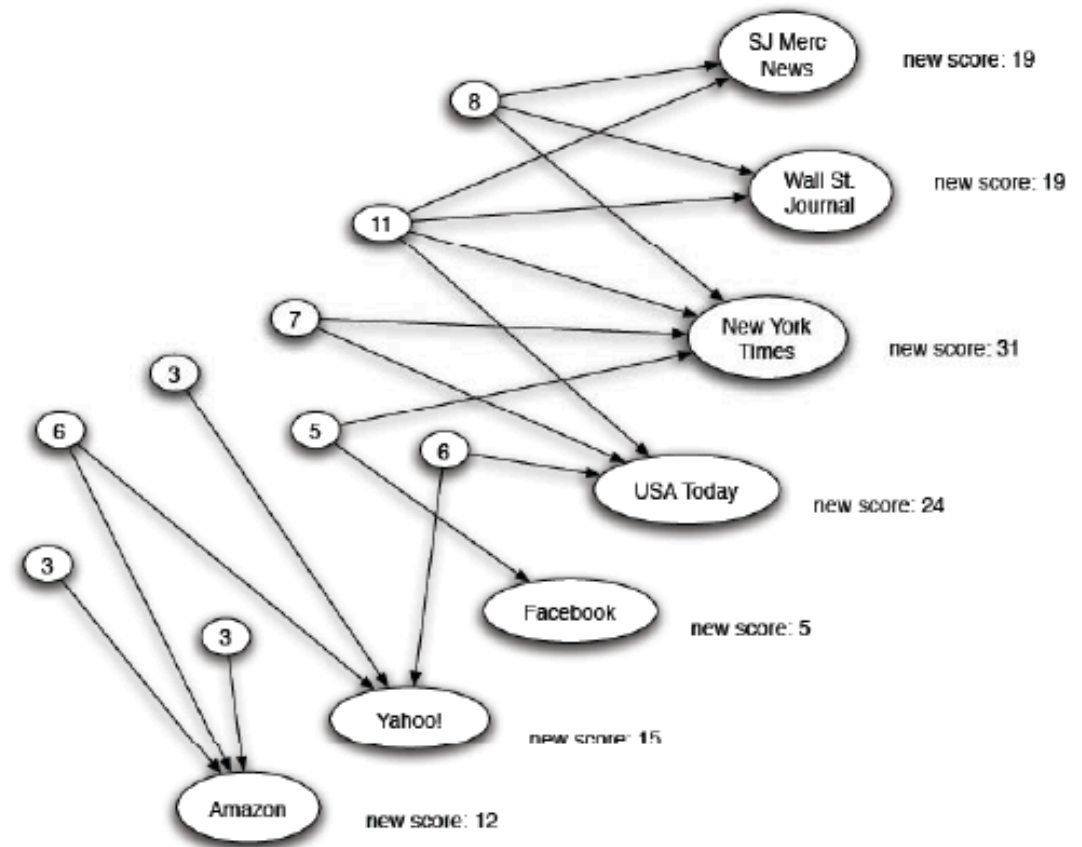
- Numbers are reported back on the source page and aggregate.
- Hubs are high value lists



Principle of Repeated Improvement



- And we are now reweighting the authorities
- When do we stop?



Repeating and Normalizing



- The process can be repeated
- Normalization:
 - Each authority score is divided by the sum of all authority scores
 - Each hub score is divided by the sum of all hub scores

More Formally: does the process converge?



- Each page has an authority a_i and a hub h_i score
- Initially $a_i = h_i = 1$

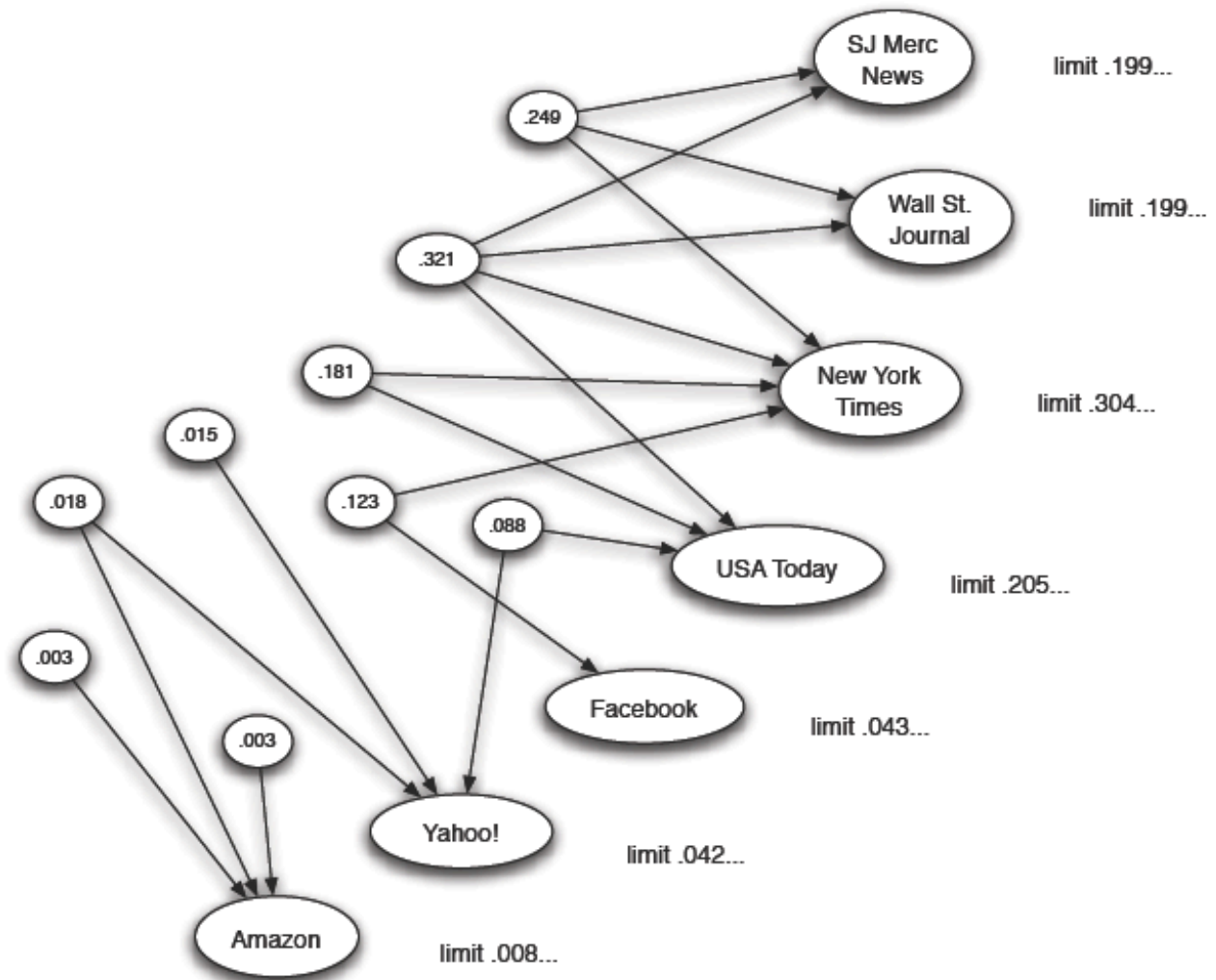
- At each step
$$a_i = \sum_{j \rightarrow i} h_j$$

$$h_j = \sum_{j \rightarrow i} a_i$$

- Normalize
$$\sum a_i = 1$$

$$\sum h_j = 1$$

The process converges



PageRank



- We have seen hubs and authorities
 - Hubs can “collect” links to important authorities who do not point to each others
 - There are other models: better for the web, where one prominent can endorse another.
- The **PageRank** model is based on transferrable importance.



PageRank Concepts

- Pages pass endorsements on outgoing links as fractions which depend on out-degree
- Initial PageRank value of each node in a network of n nodes: $1/n$.
- Choose a number of steps k .
- **[Basic] Update rule:** each page divides its pagerank equally over the outgoing links and passes an equal share to the pointed pages. Each page's new rank is the sum of received pageranks.

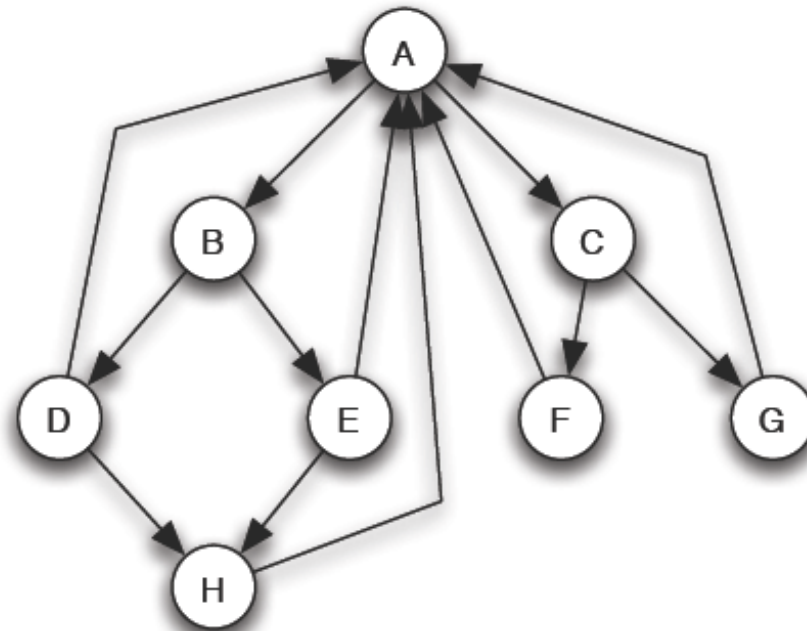
Example



- All pages start with PageRank= 1/8

Step	A	B	C	D	E	F	G	H
1	1/2	1/16	1/16	1/16	1/16	1/16	1/16	1/8
2	3/16	1/4	1/4	1/32	1/32	1/32	1/32	1/16

A becomes important and
B,C benefit too at step 2

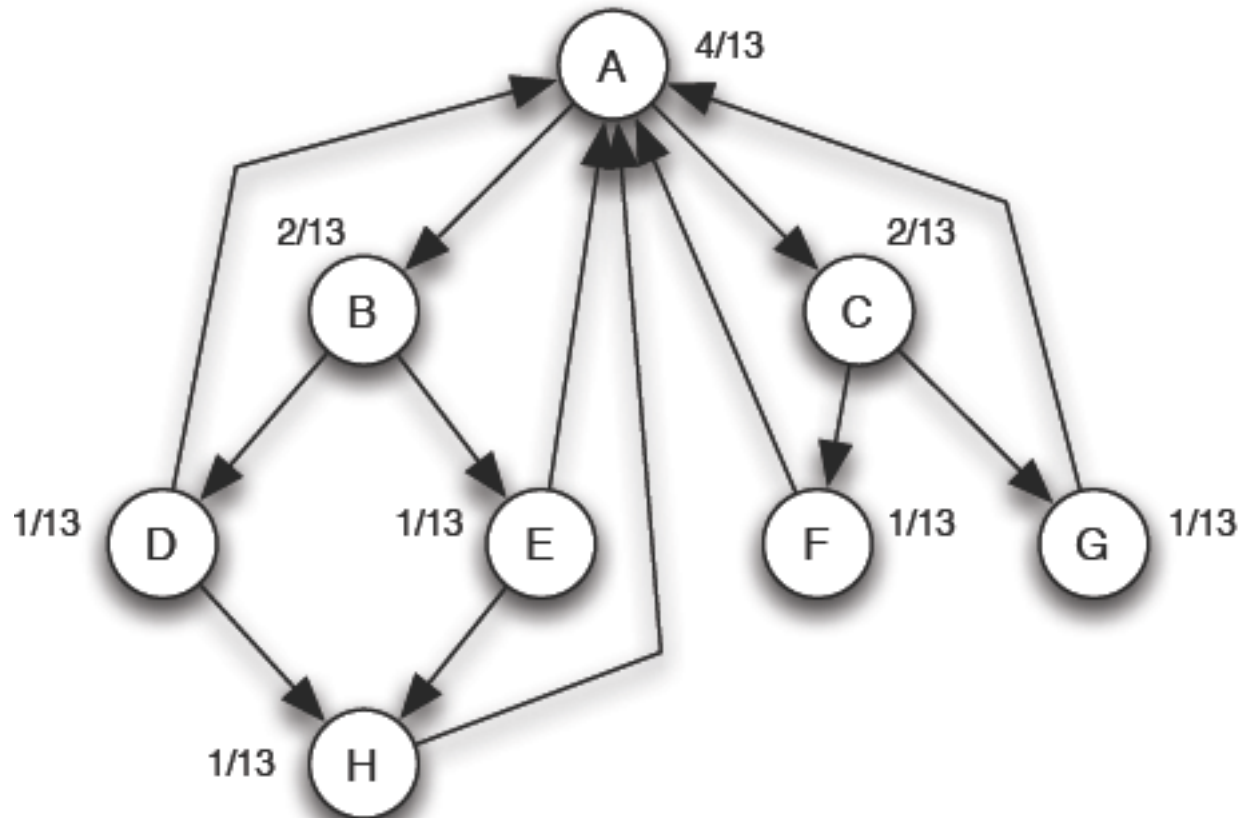


Convergence



- Except for some special cases, PageRank values of all nodes converge to limiting values when the number of steps goes to infinity.
- The convergence case is one where the PageRank of each page does not change anymore, i.e., they regenerate themselves.

Example of Equilibrium

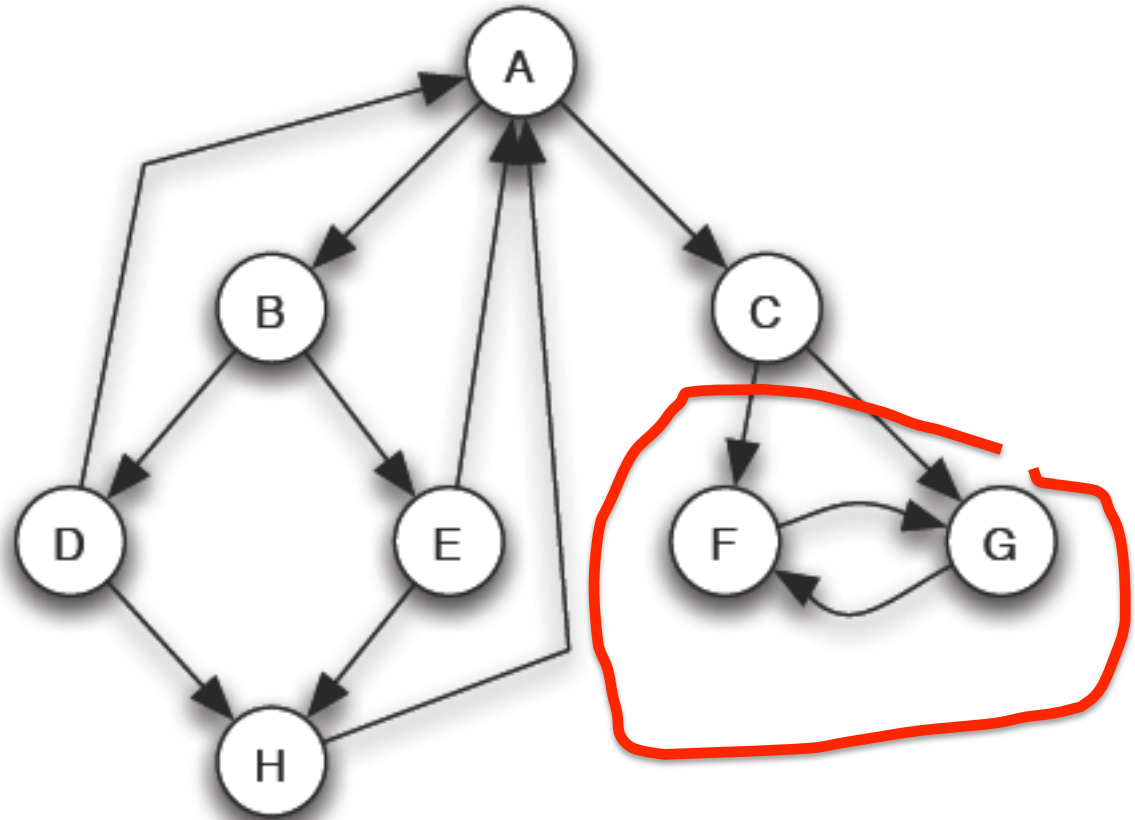


Problems with the basic PageRank

Dead ends



- F,G converge to $\frac{1}{2}$ and all the other nodes to 0



Solution: The REAL PageRank



- **[Scaled] Update Rule:**
 - Apply basic update rule. Then, scale down all values by scaling factor s [chosen between 0 and 1].
 - **[Total network PageRank value changes from 1 to s]**
 - Divide $1-s$ residual units of PageRank equally over all nodes: $(1-s)/n$ each.
- It can be proven that values converge again.
- Scaling factor usually chosen between 0.8 and 0.9

Search Ranking is very important to business



- A change in results in the search pages might mean loss of business
 - I.e., not appearing on first page.
- Ranking algorithms are kept very secret and changed continuously.

Examples of Google Bombs







Tue, Jan 27 2009 15:05 CET | by Rene Beekman | 1422 V

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Web Groups

[President of the United States - George W. Bush](#)  ★  · Government
Article from Encarta Encyclopedia provides an overview of Bush's life.
www.whitehouse.gov/president/ - 21k - [Cached](#) - [Similar pages](#)

[Historians vs. George W. Bush](#)  ★★★★★  · Politics
Of 415 historians who expressed a view of President Bush's administration to this point
success or **failure**, 338 classified it as a **failure** and 77 as a ...
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Congestive heart **failure** (CHF), congestive cardiac **failure** (CCF) or just heart **failure**, i
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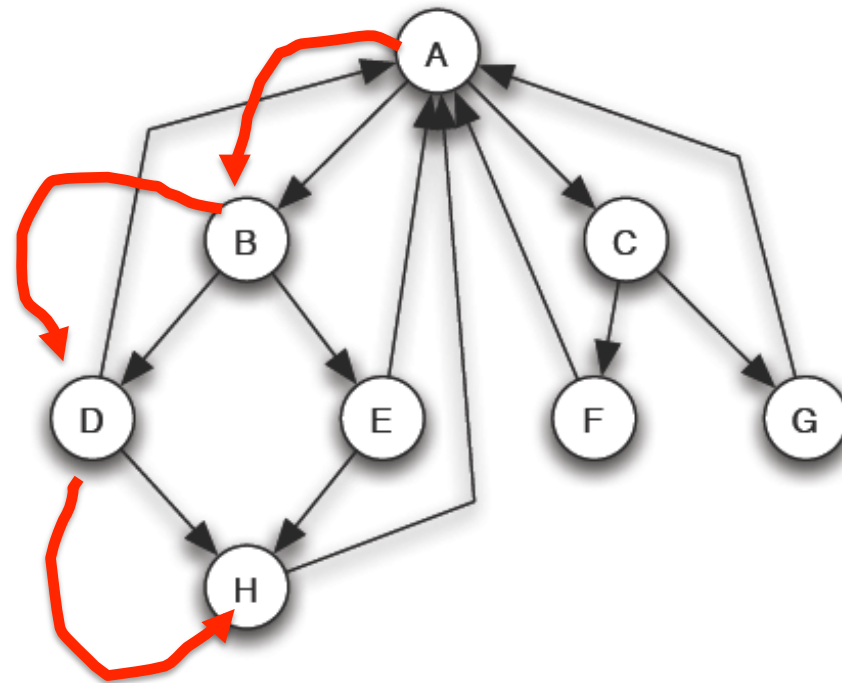
[ал или как да им спретнем един Googlebomb](#)
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пено правителство.
net/vile/failure/ - 91k - [Cached](#) - [Similar pages](#)

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Random Walks



- Starting from a node, follow one outgoing link with an equal probability



PageRank as Random Walk



- The probability of being at a page X after k steps of a random walk is precisely the PageRank of X after k applications of the Basic PageRank Update Rule
- Scaled Update Rule equivalent: follow a random outgoing link with probability s while with probability $1-s$ jump to a random node in the network.

References



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