Social and Technological Network Analysis

Lecture 4: Community Detection and Overlapping Communities

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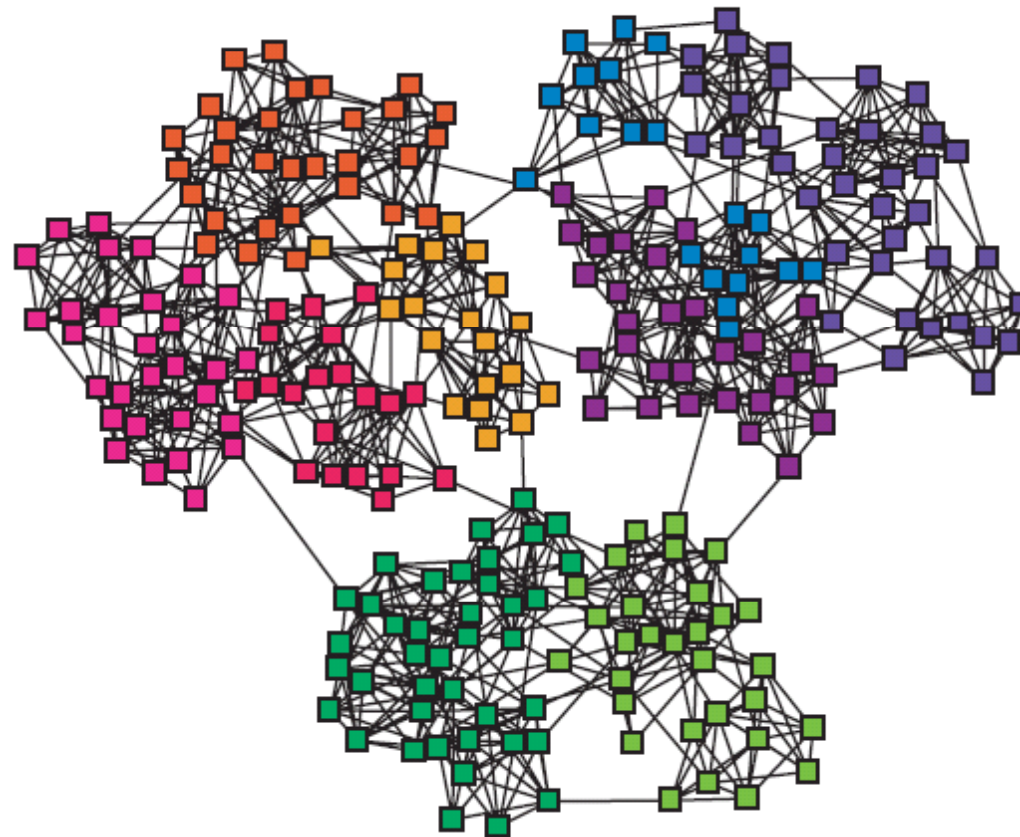# Communities

- Weak ties (Lecture 2) seemed to bridge groups of tightly coupled nodes (communities)
- How do we find these communities?

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In This Lecture

- We will describe a Community Detection method based on betweenness centrality.
- We will describe the concept of Modularity and Modularity Optimization.
- We will describe methods for overlapping community detection.

What is a Community?

Why do we want to find partitions/communities?

- Clustering web clients with similar interest or geographically near can improve performance
- Customers with similar interests could be clustered to help recommendation systems
- Clusters in large graphs can be used to create data structures for efficient storage of graph data to handle queries or path searches
- Study the relationship/mediation among nodes
  - Hierarchical organization study

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Example

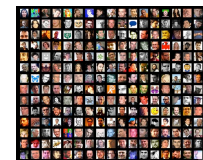Zachary's Karate club: 34 members of a club over 3 years. Edges: interaction outside the club
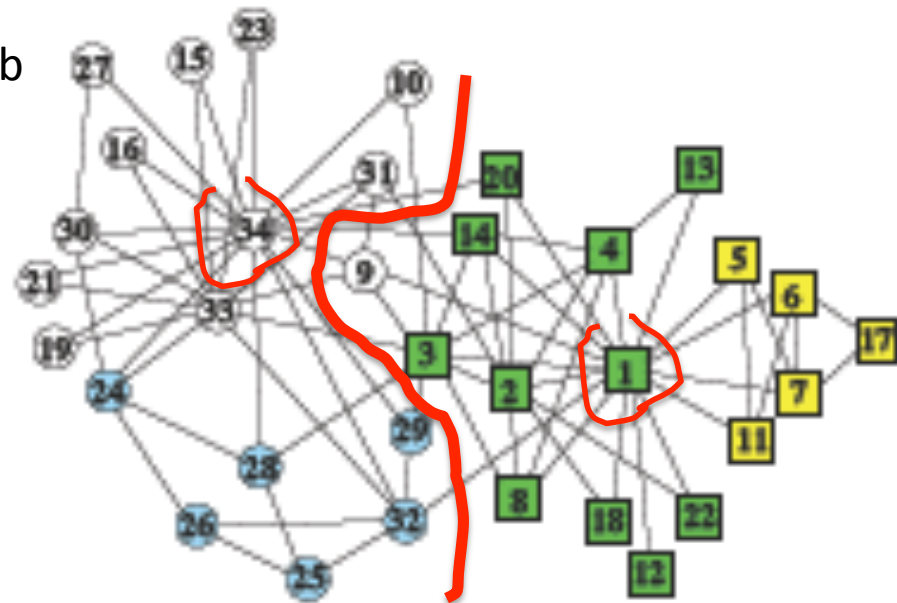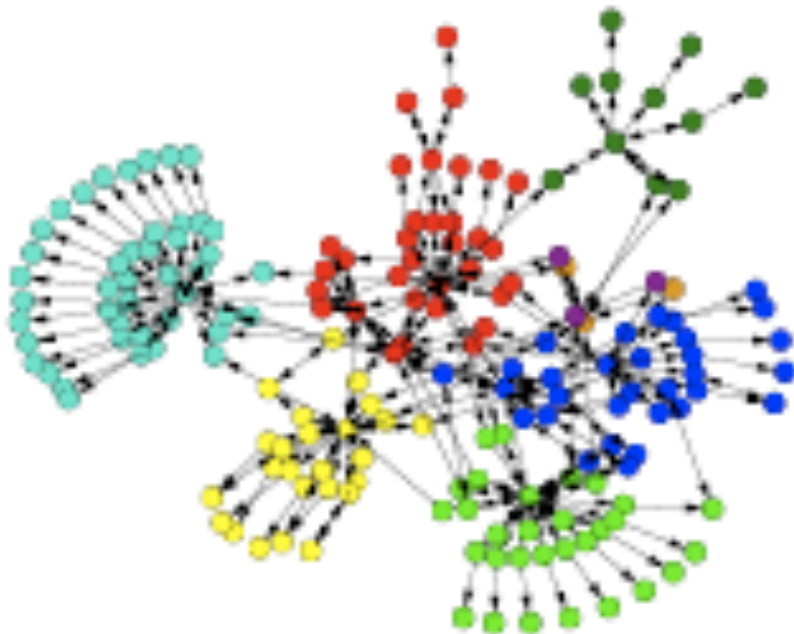
WWW: pages and hyperlinks
Identification of clusters can improve page ranking

Remove weak ties

- Local bridges connect weakly interacting parts of the network.
- What if we have many bridges: which do we remove first? Or there might be no bridges.

- Note: **Without those bridges paths between nodes would be longer.**
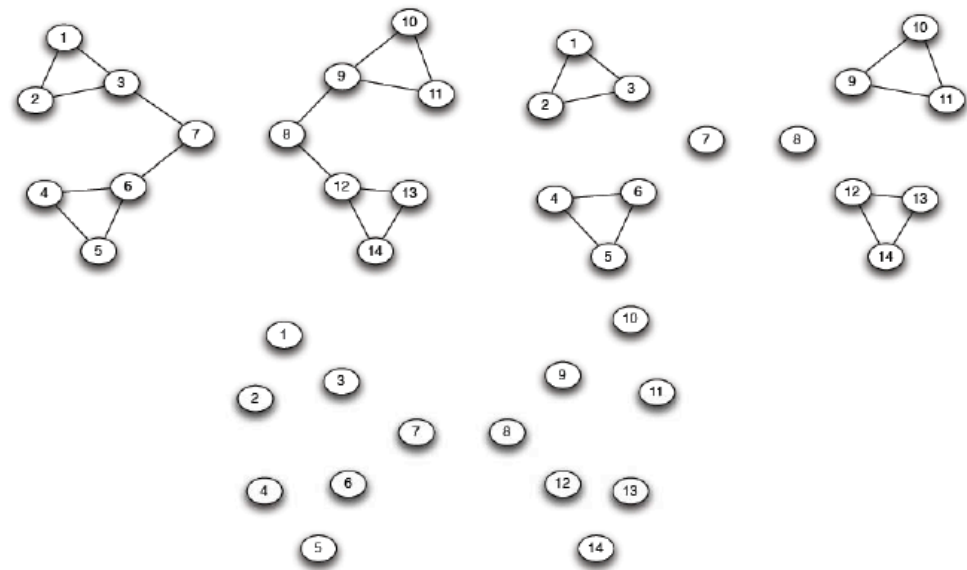
Edge Betweenness

- Edge Betweenness: the number of shortest paths between pairs of nodes that run along the edge.
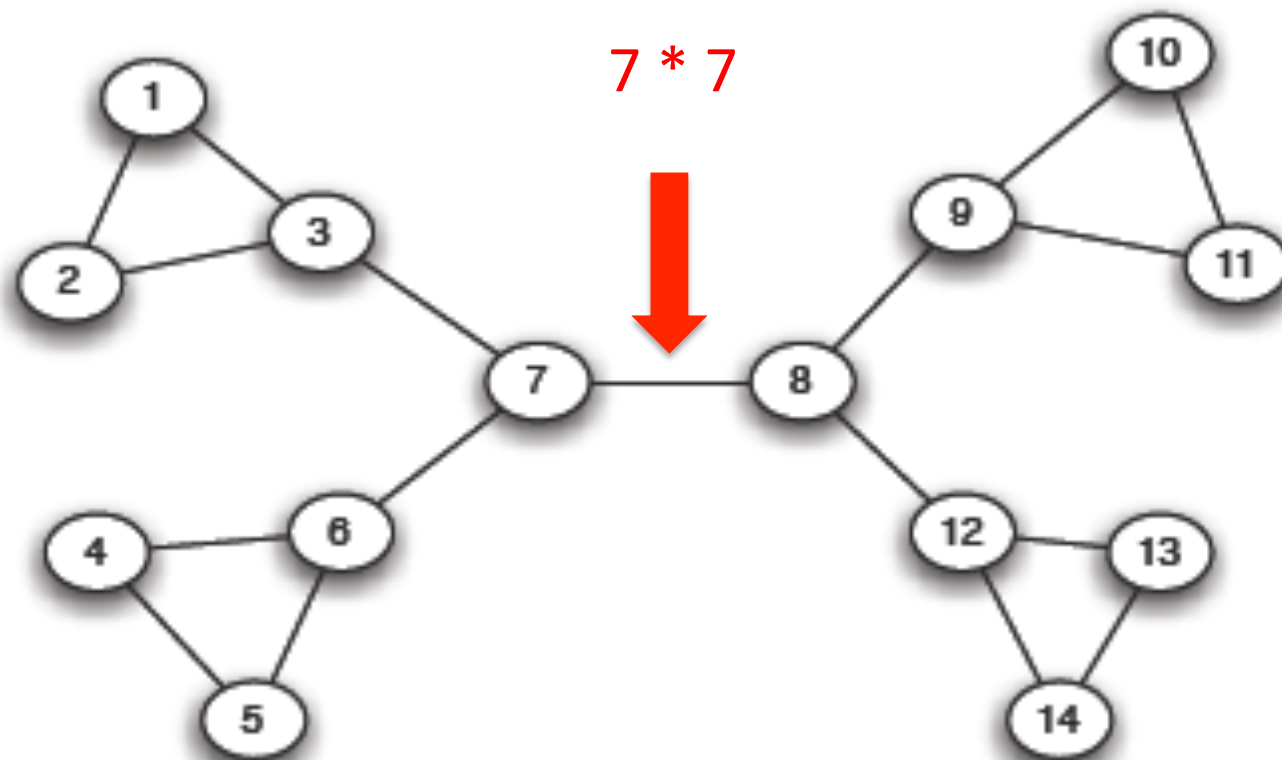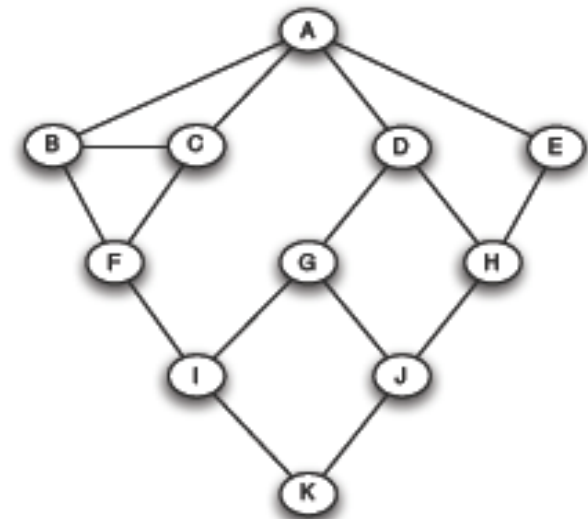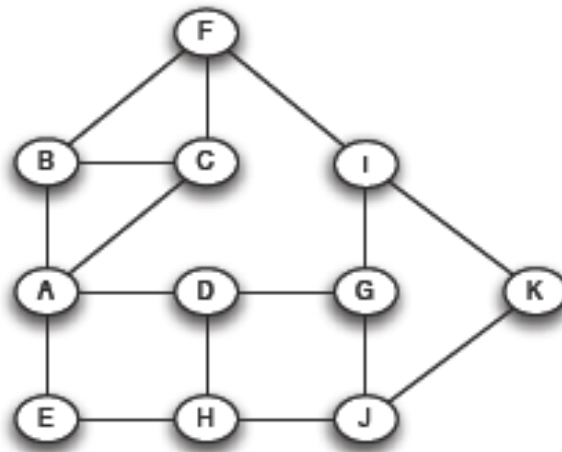
Algorithm of Girvan-Newmann (PNAS 2002)

- Calculate the betweenness of all edges
- Cut the edge with highest betweenness
- Recalculate edge betweenness
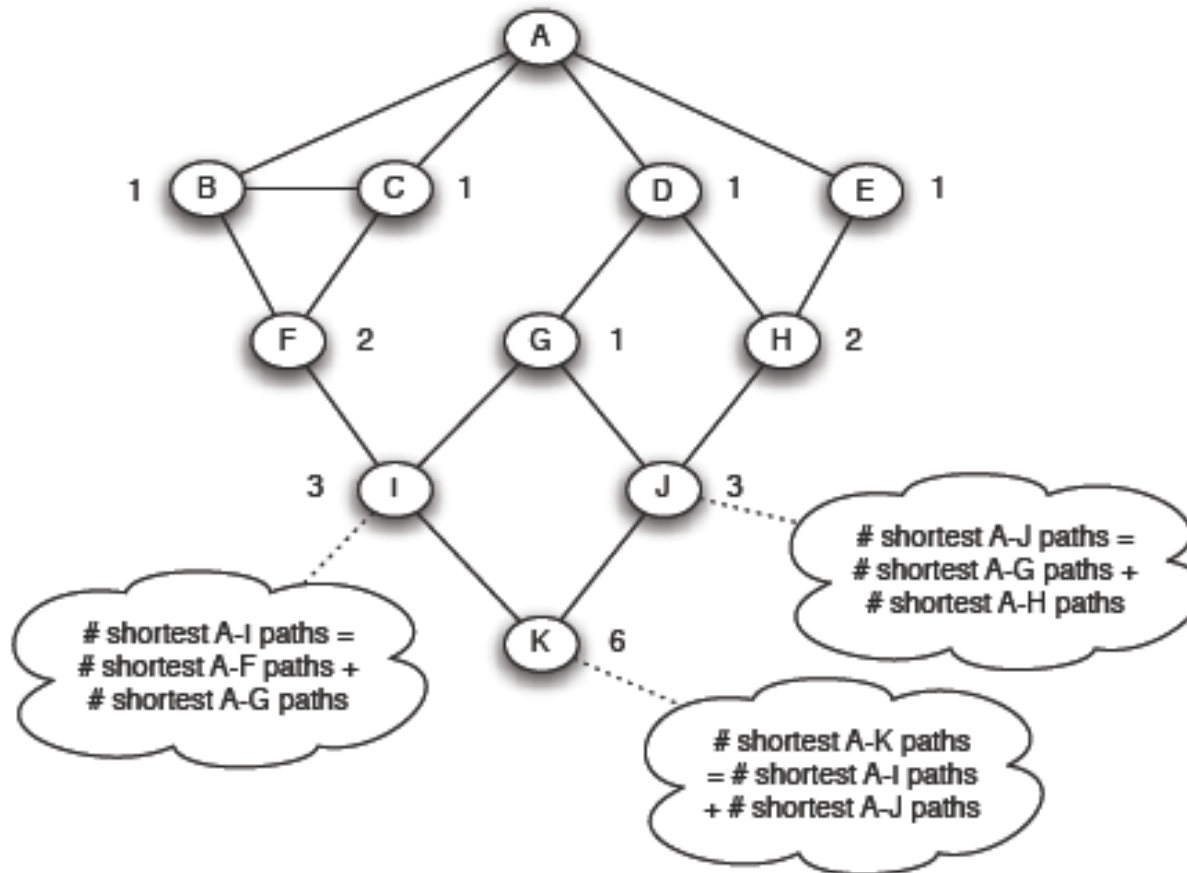
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How is the betweenness computed?

- Calculate the shortest paths from node A
  - BFS search from A.
  - Determine number of shortest paths from A to each node.

Calculating number of shortest paths

The graph shows nodes with the following labels and values:

- 1 B — C 1
- D 1 — E 1
- F 2 — G 1 — H 2
- 3 I — J 3
- K 6

shortest A-I paths =
shortest A-F paths +
shortest A-G paths

shortest A-J paths =
shortest A-G paths +
shortest A-H paths

shortest A-K paths
= # shortest A-I paths
+ # shortest A-J paths

Calculating flows
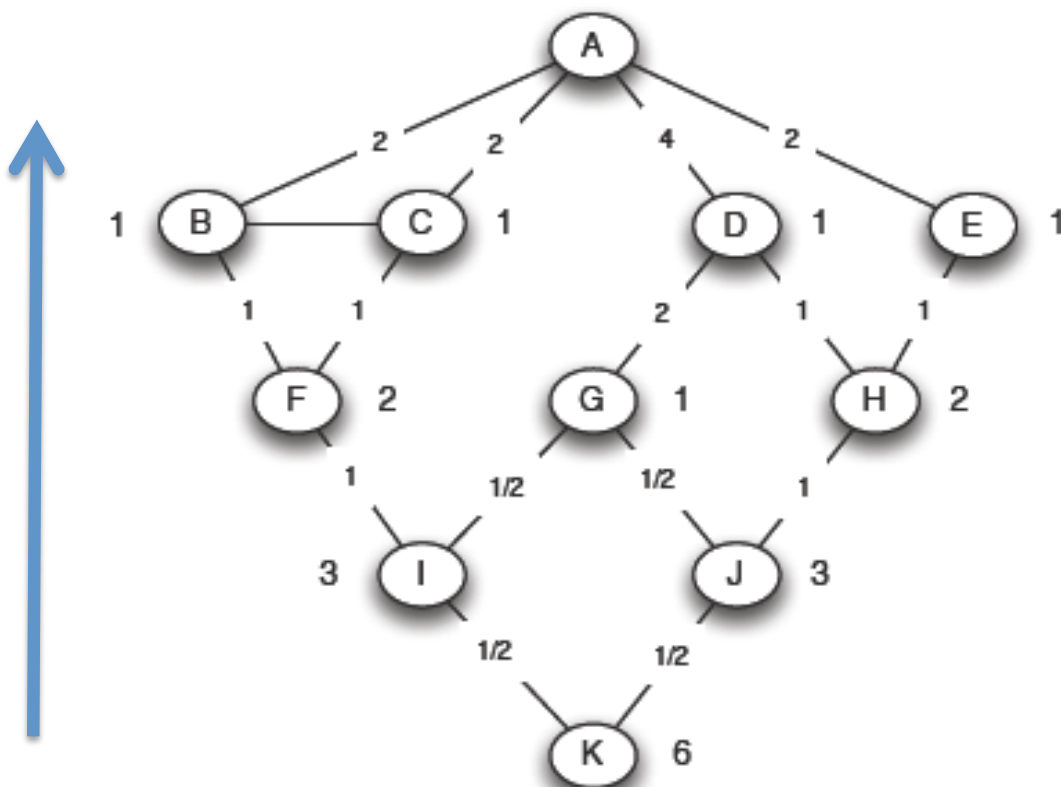
When we get to a node X in the breadth-first search structure, working up from the bottom, we add up all the flows arriving from edges directly below X, plus 1 for the flow destined for X itself. We then divide this up over the edges leading upward from X, in proportion to the number of shortest paths coming through each.

Calculating Edge Betweenness

- Build one of these graphs for each node in the graph
- Sum the values on the edges on each graph to obtain the edge betweenness

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Edge: deletion
When do we stop?

- How do we know when to stop?

- When X communities have been detected?
- When the level of cohesion inside a community has reached Y?

- There is no prescriptive way for every case
- There are also many other ways of detecting communities.

Modularity

- Perhaps a good measure of when to stop is when for each community the "cohesion" within the community is higher than outside...
- *Q= (edges inside the community)- (expected number of edges inside the community for a random graph with same node degree distribution as the given network)*

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Modularity on a randomized graph calculation

Ka
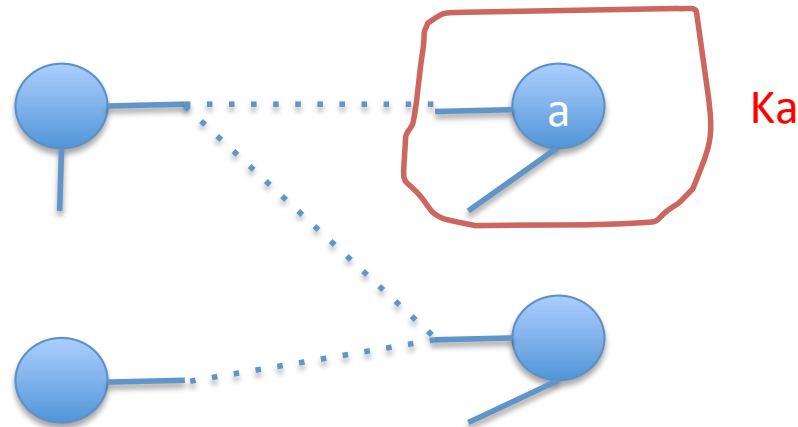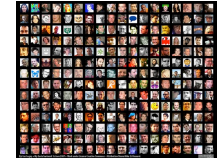
The expected number of edges in the randomized version of the graph where nodes are rewired:

$$\frac{k_a k_b}{2m}$$

m is the number of edges of the graph = ½ sum(ki)
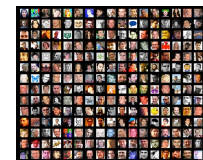
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Modularity (2)

- Number of edges **inside** a community:

$$\frac{1}{2}\sum_{a,b} A_{a,b}\,\delta(c_a, c_b)$$

- Where:
- $A_{a,b}$ is 1 if there is an edge a->b,
- $\delta(c_a , c_b)$ is the Kronecker Delta (1 if c_a is equal to c_b)

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Modularity (3)

$$Q1 = \frac{1}{2}\sum_{a,b}A_{a,b}\delta(c_a,c_b) - \frac{1}{2}\sum_{a,b}\frac{k_a k_b}{2m}\delta(c_a,c_b)$$

$$Q1 = \frac{1}{2}\sum_{a,b}(A_{a,b} - \frac{k_a k_b}{2m})\delta(c_a,c_b)$$

$$Q = \frac{1}{2m}\sum_{a,b}(A_{a,b} - \frac{k_a k_b}{2m})\delta(c_a,c_b)$$

Fraction of edges over all edges m

Modularity (4)

- Modularity ranges from -1 to 1.
  - It is positive if the number of edges inside the group are more than the expected number.
  - Variation from 0 indicate difference with random case.

- Modularity can be used at each round of the Girvan-Newmann algorithm to check if it is time to stop. However the complexity of this is $O(m^2 n)$.
- Why don't we try to just maximize modularity?

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Modularity Optimization

- Finding the configuration with maximum modularity in a graph is an NP complete problem.
- However there are good approximation algorithms.

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Fast Modularity

- Start with a network of n communities of 1 node
- Calculate ΔQ for all possible community pairs
- Merge the pair of the largest increase in Q
- Repeat (2)&(3) until one community remains
- Cross cut the dendrogram where Q is maximum.
- This runs in $O((m + n)n)$.

- A further optimization runs in $O(m\, d\, \log n)$ [d depth of dendrogram].
- Most networks are sparse so m~n and d~log n
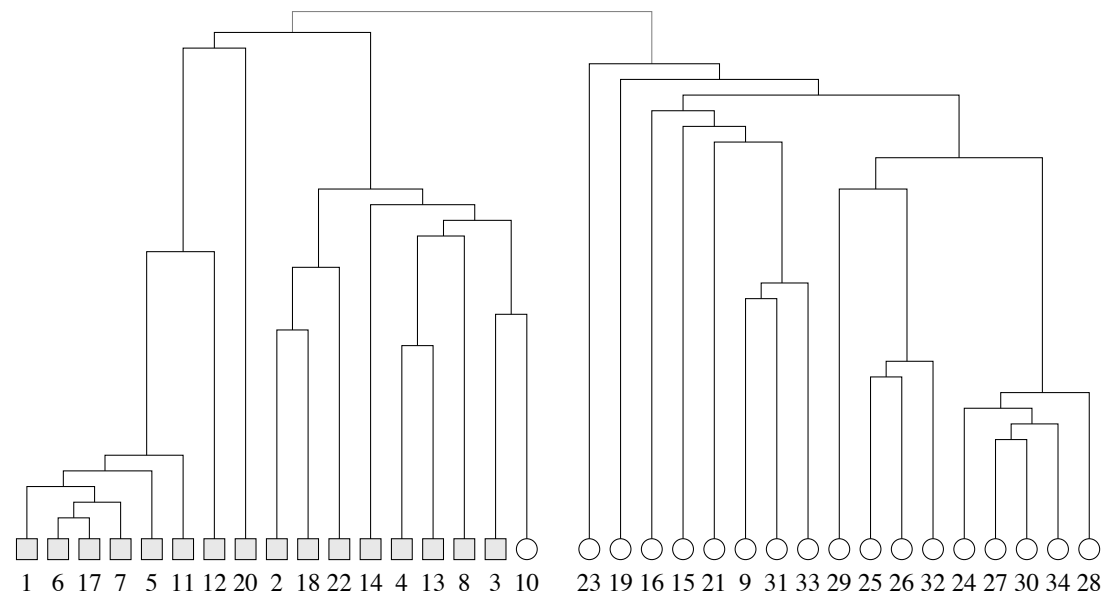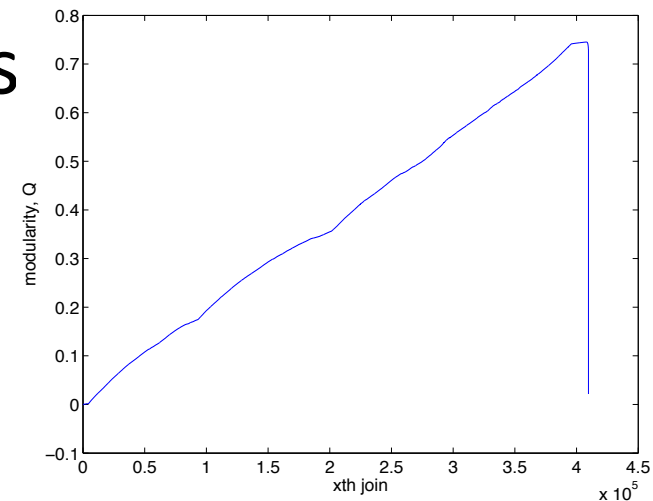
Example of Dendrogram

FIG. 2: Dendrogram of the communities found by our algorithm in the "karate club" network of Zachary [5, 17]. The shapes of the vertices represent the two groups into which the club split as the result of an internal dispute.

Application to Amazon Recommedations

- Network of products.
- A link between product a and product b if b was frequently purchased by buyers of a.
- 200000 nodes and 2M edges.
- Max when 1684 communities
- Mean size of 243 products

FIG. 1: The modularity Q over the course of the algorithm (the x axis shows the number of joins). Its maximum value is $Q = 0.745$, where the partition consists of 1684 communities.

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Amazon: Top Communities (87% of nodes)

| Rank	Size	Description
1	114538	General interest: politics; art/literature; general fiction; human nature; technical books; how things, people, computers, societies work, etc.
2	92276	The arts: videos, books, DVDs about the creative and performing arts
3	78661	Hobbies and interests I: self-help; self-education; popular science fiction, popular fantasy; leisure; etc.
4	54582	Hobbies and interests II: adventure books; video games/comics; some sports; some humor; some classic fiction; some western religious material; etc.
5	9872	classical music and related items
6	1904	children's videos, movies, music and books
7	1493	church/religious music; African-descent cultural books; homoerotic imagery
8	1101	pop horror; mystery/adventure fiction
9	1083	jazz; orchestral music; easy listening
10	947	engineering; practical fashion

TABLE I: The 10 largest communities in the Amazon.com network, which account for 87% of the vertices in the network.

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Amazon:
Community Size Distribution

- A power law distribution of community size

- (more on power laws in later lectures)

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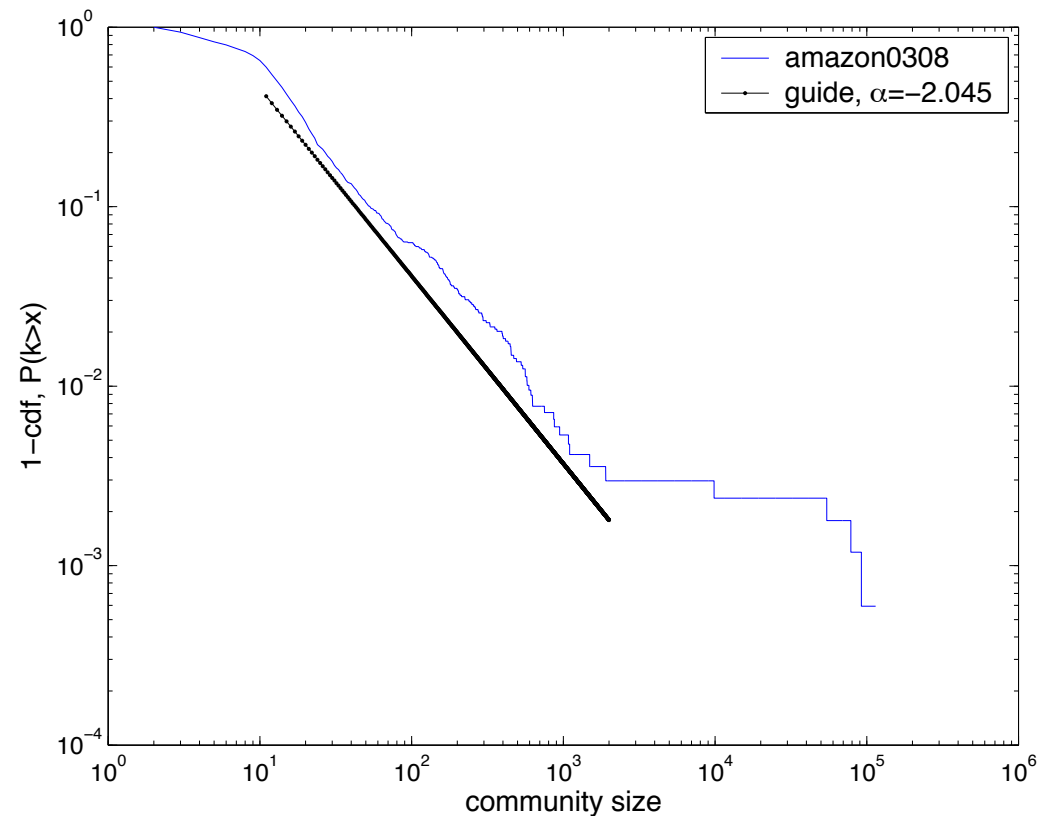
Limitations of Modularity

- Modularity is not a perfect measures
- It appears to depend on the number of links in the network (L).
- Problems for modules with a number of internal links of the order of $\sqrt{2L}$ or smaller.

- Intuition: modularity depends on links of a community to the "outside", ie the rest of the network.

S. Fortunato, S. Barthelemy. Resolution limit in community detection. Proc. Natl. Acad. Sci., 2007.

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Louvain Method

- The Louvain method is more efficient and more accurate.
- Step 1: for each node i consider neighbours (j) and evaluate gain in modularity of community if i moves to j's community. Do this for all nodes. Stop when no improvement can be achieved.
- Step 2: see each created community as a node and repeat step 1. Stop when maximum modularity is obtained.

Efficiency

- Extremely faster than other algorithms
- Complexity is linear on typical and sparse data.
  - Modularity is easy to compute and number of communities decreases drastically after a few steps.
  - Probability that 2 communities can be merged by just moving one node is very low (overcoming the problem of the resolution limit)

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Performance and Modularity results for various networks and approaches

	Karate	Arxiv	Internet	Web nd.edu	Phone	Web uk-2005	Web WebBase 2001
Nodes/links	34/77	9k/24k	70k/351k	325k/1M	2.6M/6.3M	39M/783M	118M/1B
CNM	.38/0s	.772/3.6s	.692/799s	.927/5034s	-/-	-/-	-/-
PL	.42/0s	.757/3.3s	.729/575s	.895/6666s	-/-	-/-	-/-
WT	.42/0s	.761/0.7s	.667/62s	.898/248s	.56/464s	-/-	-/-
Our algorithm	.42/0s	.813/0s	.781/1s	.935/3s	.769/134s	.979/738s	.984/152mn

Louvain over a telecom network in Belgium

The colours are different languages spoken by people. The intermediate node is one with a lot of language mixing.

Edges are calls. Each of these communities are more than 100 people.

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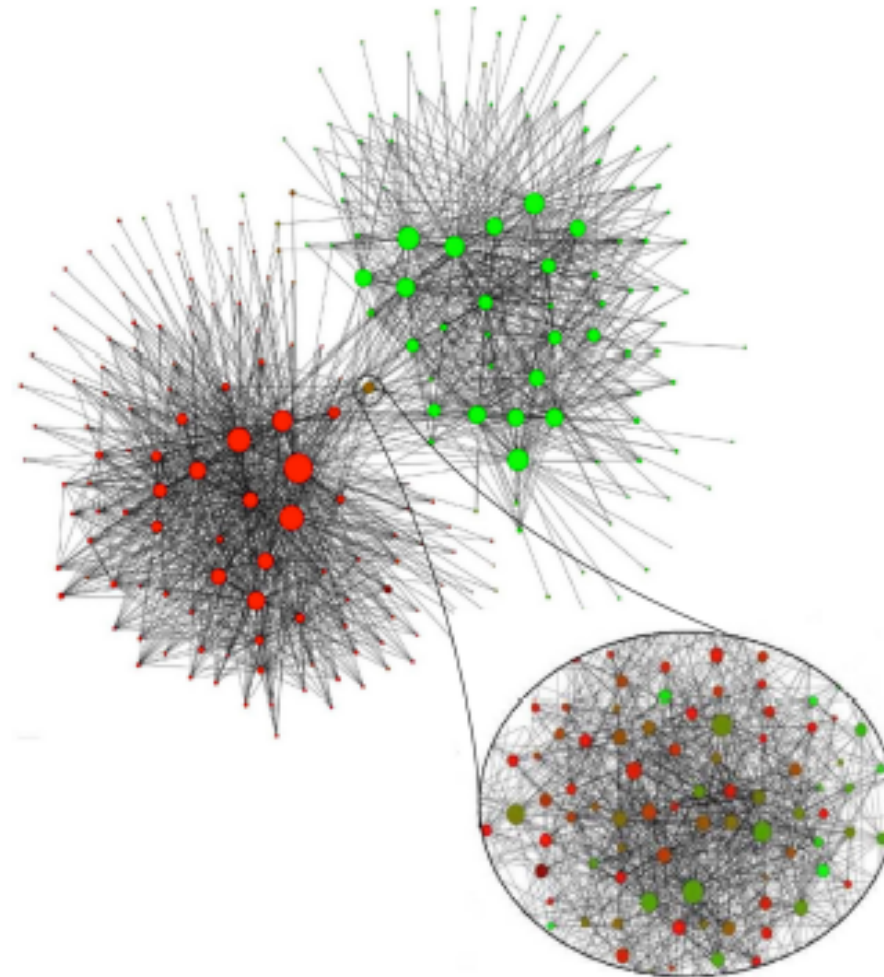
Overlapping Communities

- Community membership could overlap: a node could be part of more than 1 community.
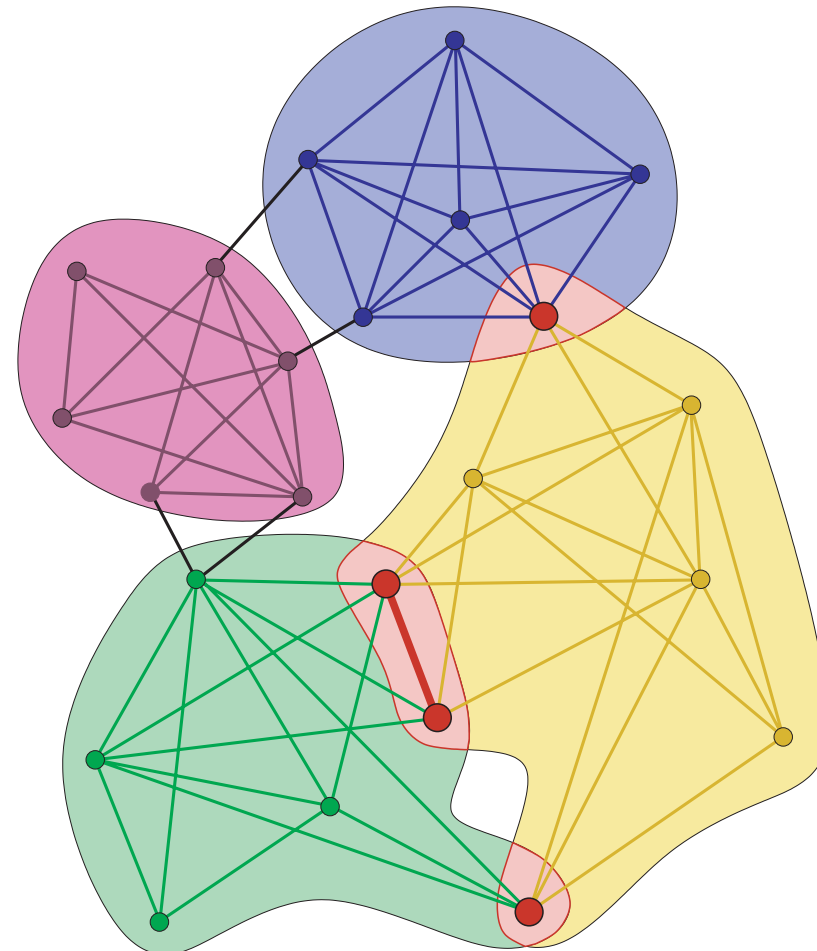
Nodes can belong to more than 1 social circle!
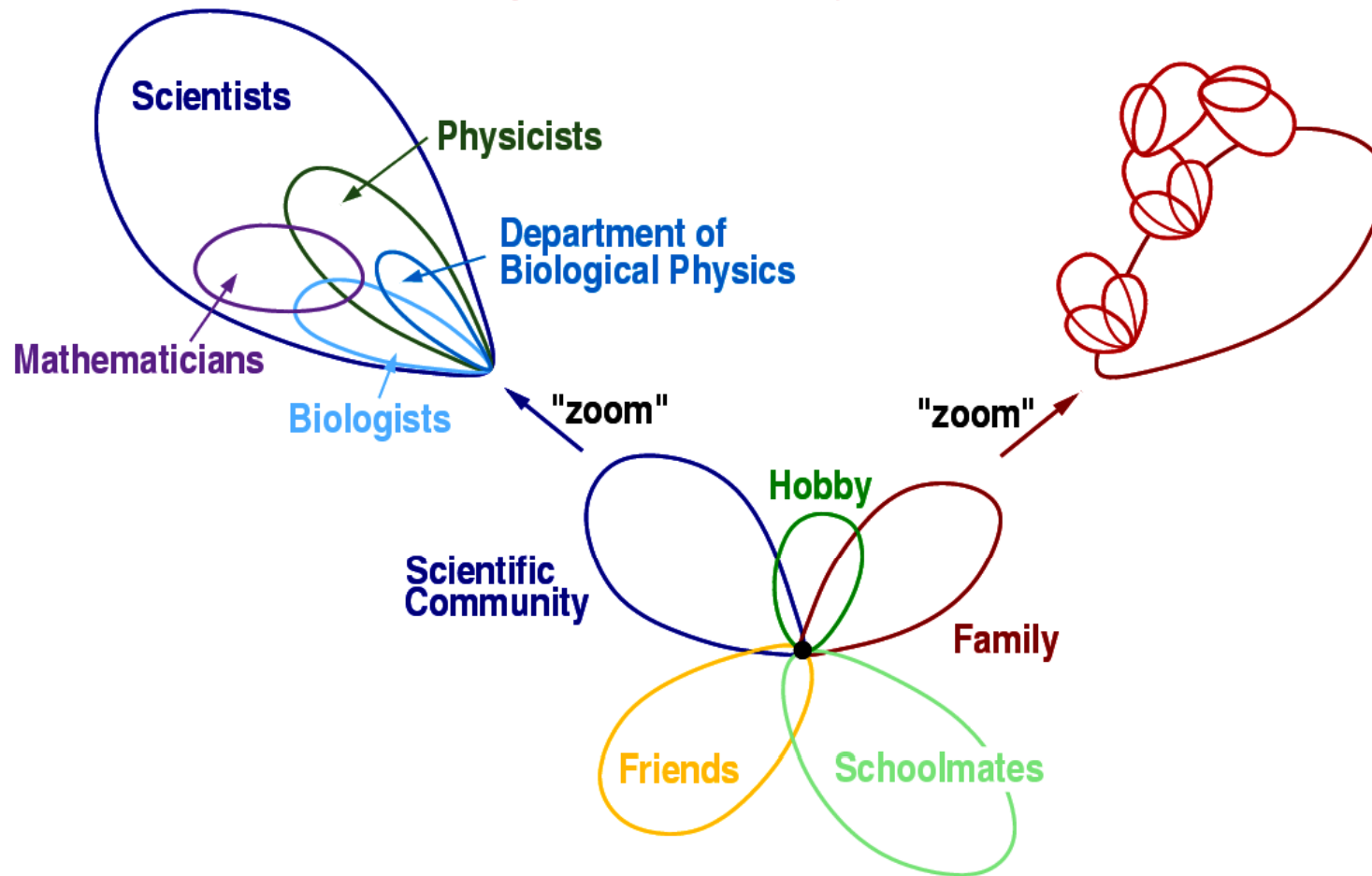
Clique Percolation Method: the idea (Palla 2005)

- Two nodes belong to the same community if they can be connected through adjacent *k*-cliques.
- A k-clique is a fully connected graph of k nodes.
- K-cliques are adjacent if they have k-1 overlapping nodes.
- K-clique community: nodes which can be reached through a sequence of adjacent k-cliques.

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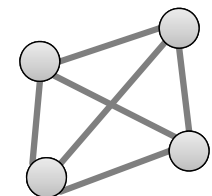
4-clique

Clique Percolation Method: The algorithm

- **Find the maximal cliques**
  - A maximal clique is a clique that cannot be extended by including one more adjacent vertex
  - This is complex but real networks are relatively sparse.
- **Build clique overlap matrix**
  - Each clique is a node
  - Connect two cliques if they overlap in at least *k-1* nodes
- **Communities:**
  - Connected components of the clique overlap matrix

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Example

Maximal cliques
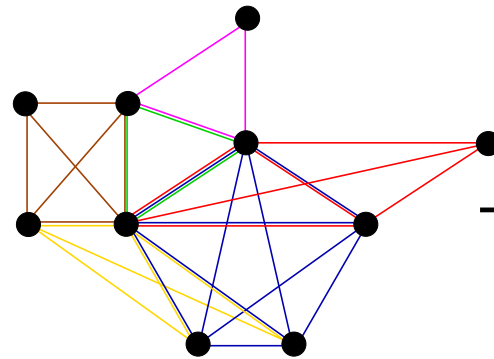
Overlap Matrix: elements are n. of overlapping nodes

k=4

Erase elements less than 4 on diagonal and less than 3 elsewhere

K-cliques

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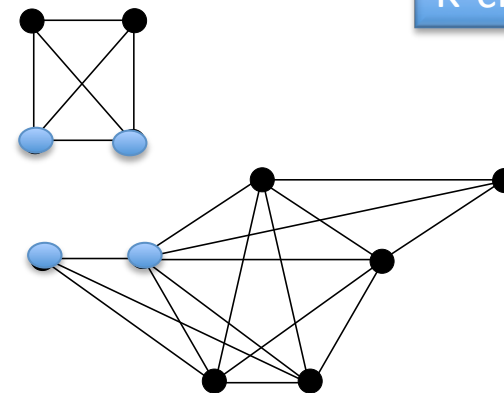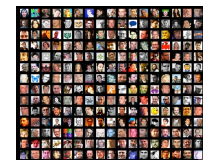
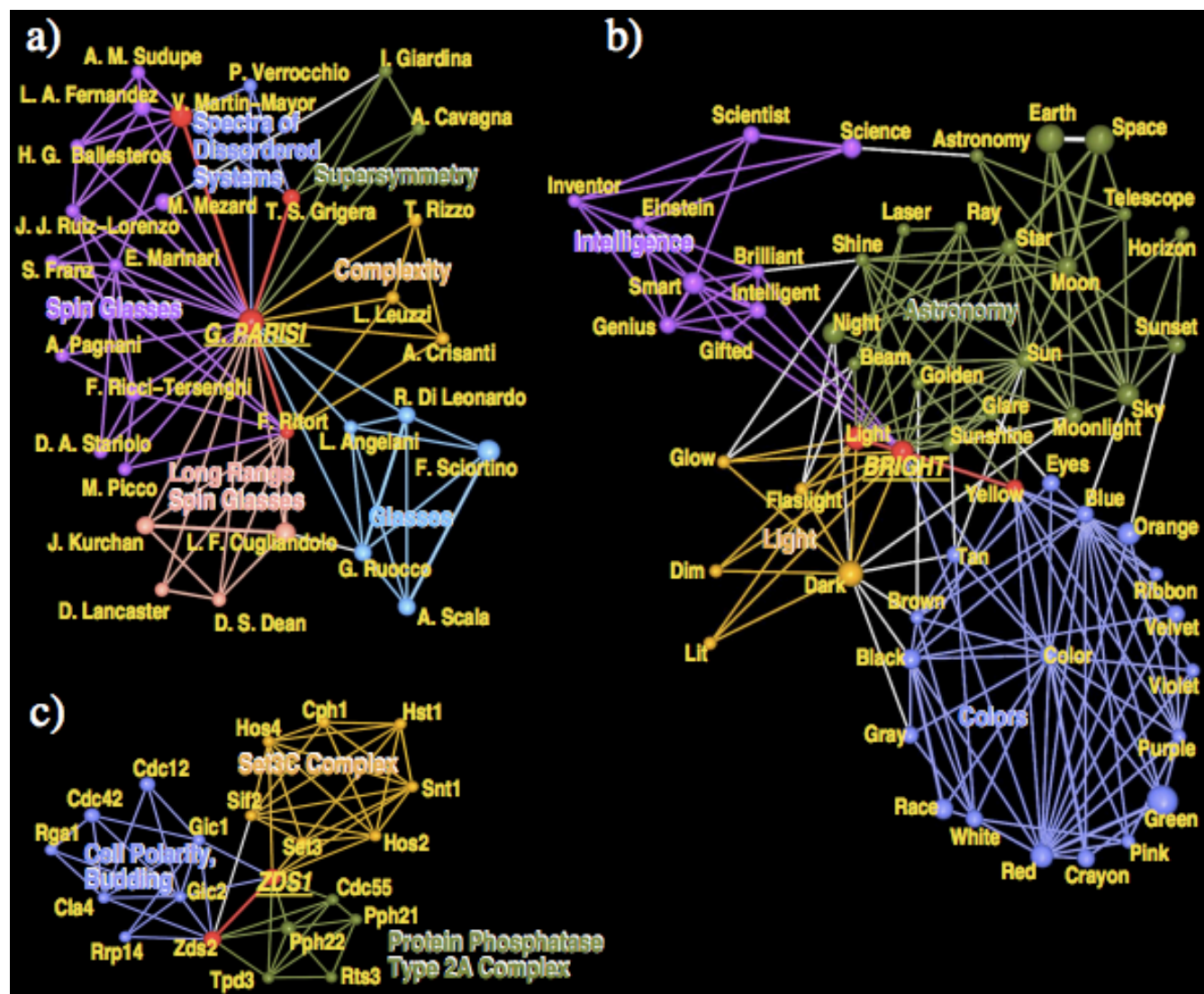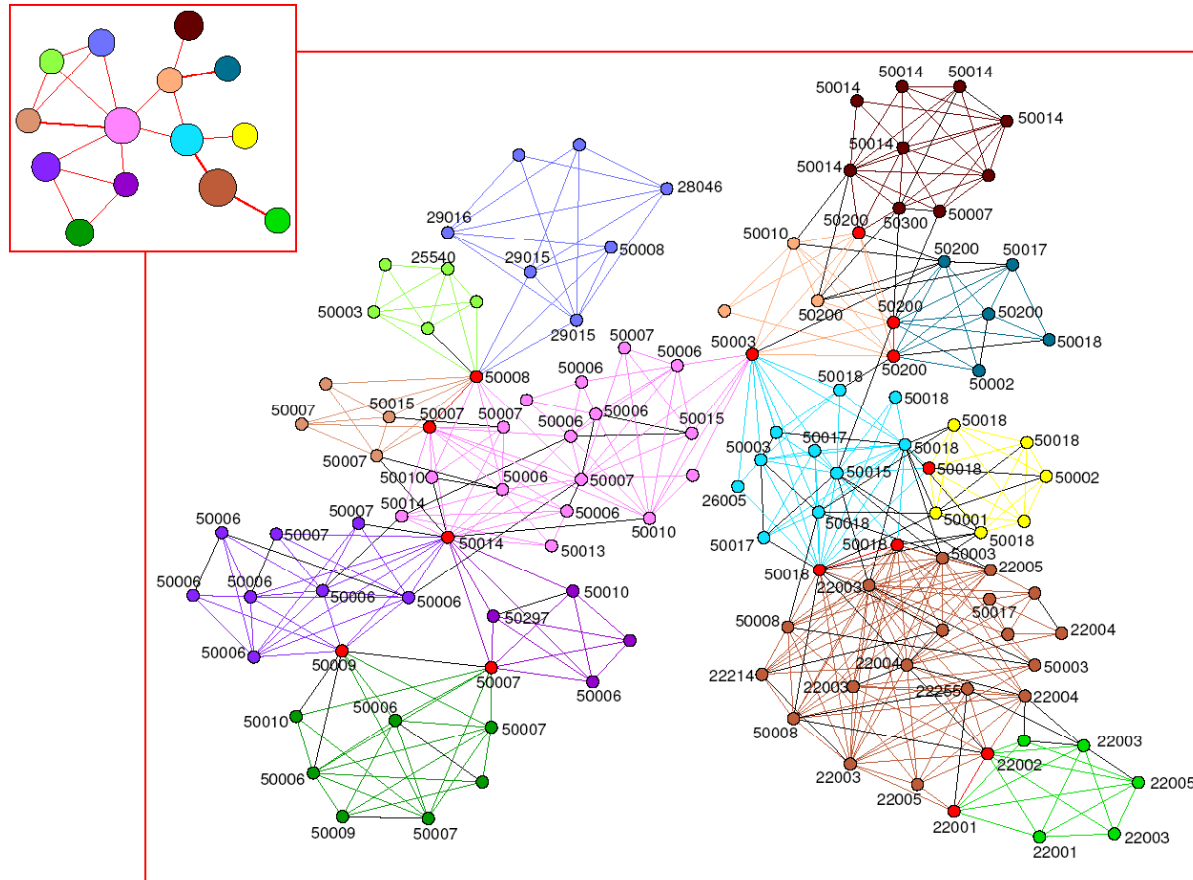# Application

Overlapping networks:
1) Parisi's coauthorship networks
2) Networks of "bright" in the word association network
3) Protein to protein interaction network

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Application:
Phone Call Network

From Leskovec
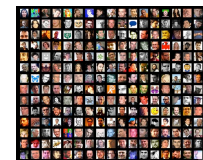
Community Detection and Weak Ties

- Twitter was analyzed trying to identify if the static network of followers gives information about the dynamics of retweeting and mentioning.
- Dataset: follower network (undirected), 2M users, and network of tweets, mention and retweets for 1 month.
- Some community detection methods are used to find clusters in the follower network.
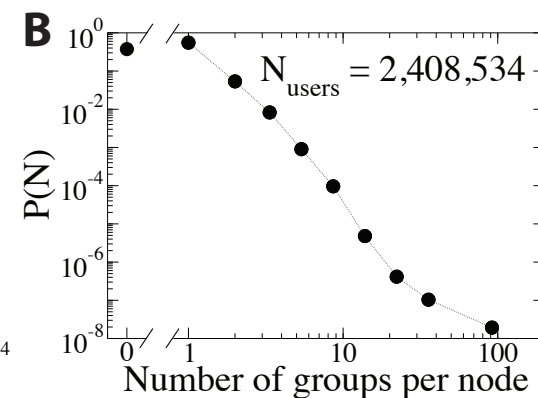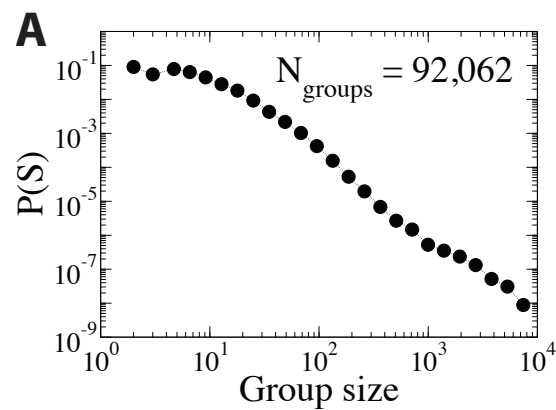
Sample

- Gray: followers
- Red: mentions
- Green: retweet
- 3 groups, one user between groups.

A

B

internal links   between groups   bridge links   no-group links
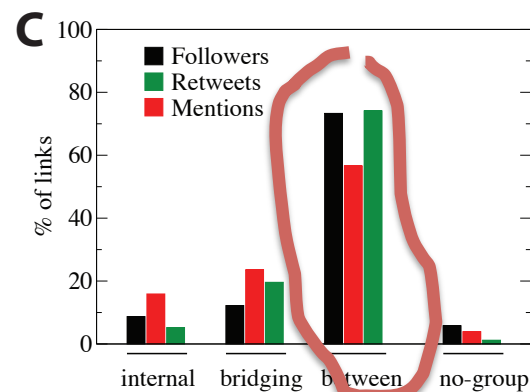
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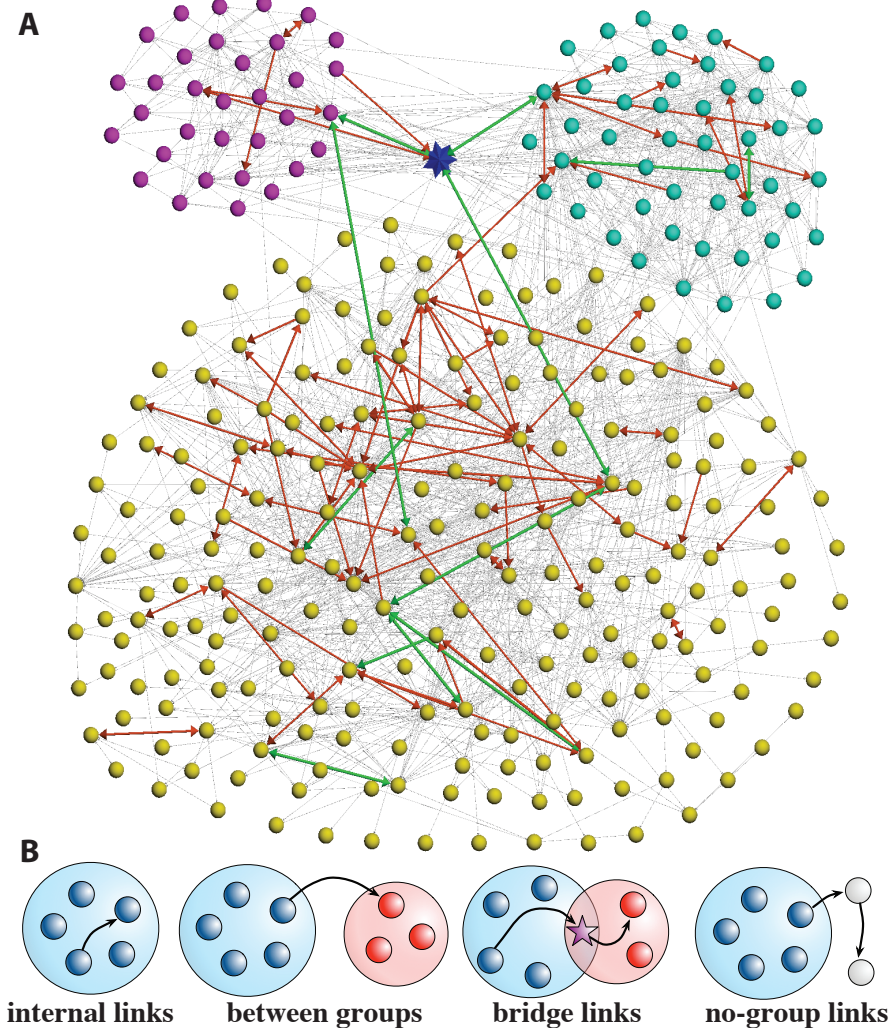
Some statistics

92,000 groups
Largest group: 10,000 users
37% users: no group

Mentions are double the followers in internal and bridging
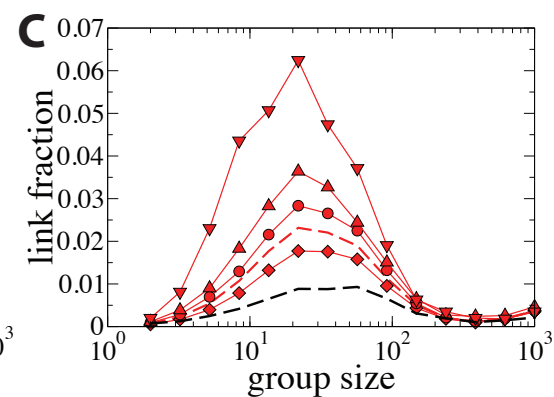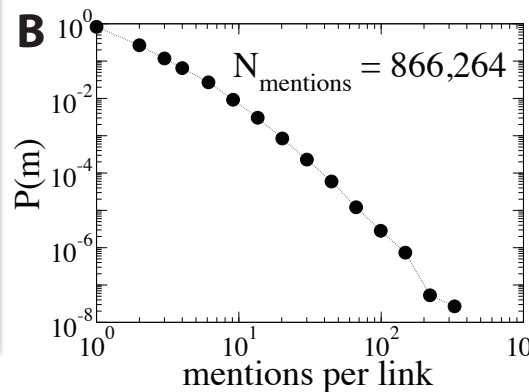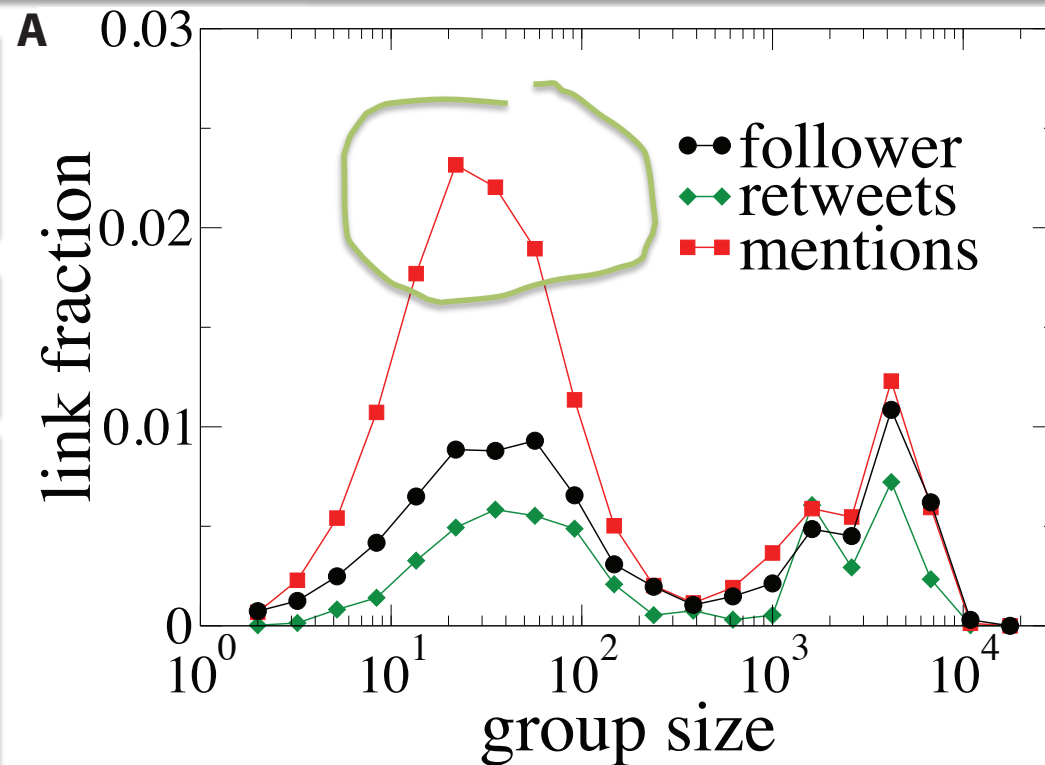
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Internal Links

Internal mentions are more than follower links with groups around 100.

The distribution of mentions over links is quite wide

C: The dashed curves are the total for the follower network (black) and for the links with mentions (red). Others (from bottom to top): fractions of links with: 1 non-reciprocated mentions (diamonds), 3 mentions (circles), 6 mentions (triangle up) and more than 6 reciprocated mentions (triangle down).
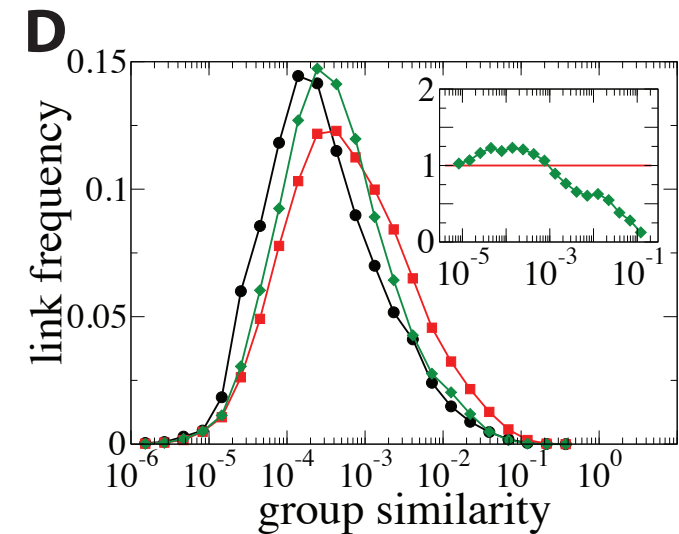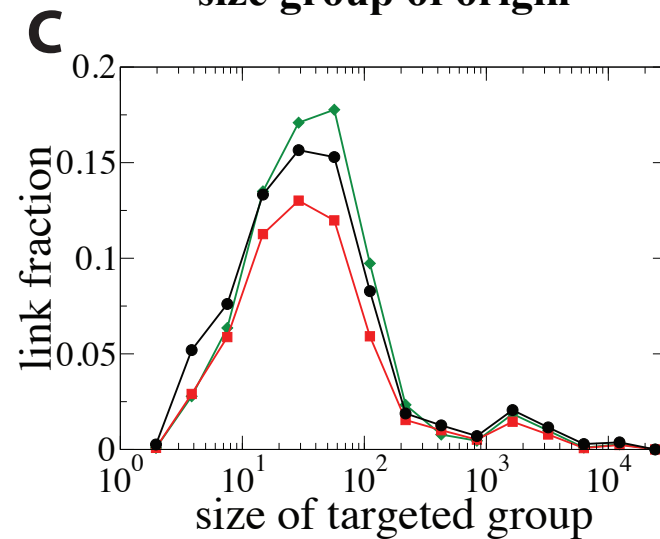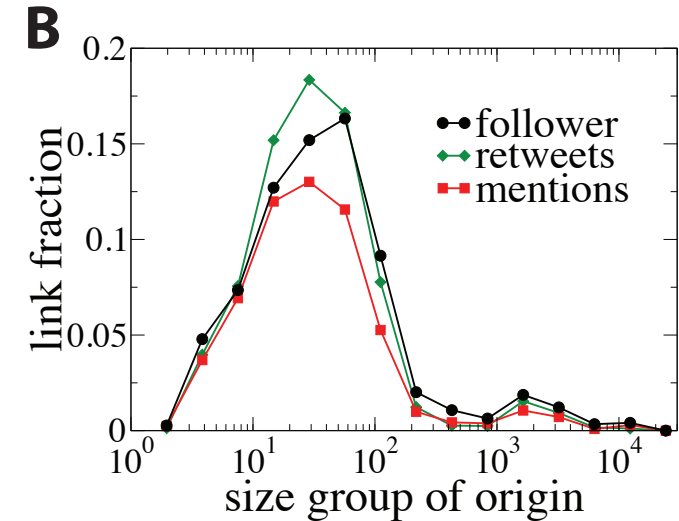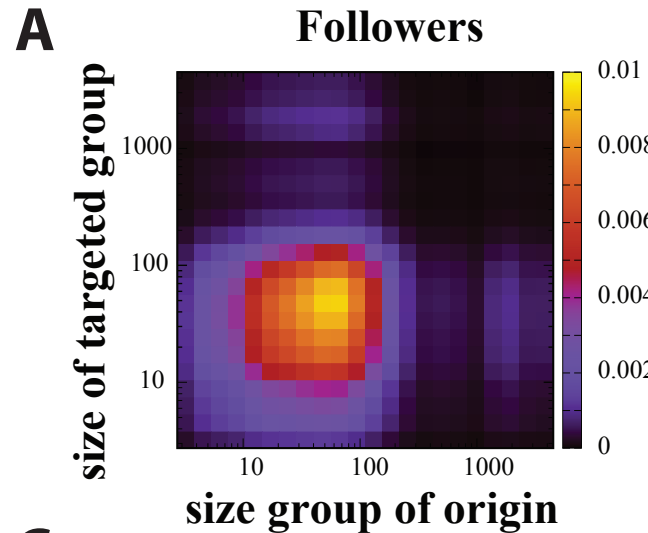

A link fraction vs group size — follower, retweets, mentions

B $P(m)$ vs mentions per link, $N_{mentions} = 866,264$

C link fraction vs group size

Links between groups

Occur between groups of <200 nodes

$$sim(A,B) = \frac{|\bigcap linksAandB|}{|\bigcup linksAandB|}$$

Retweets seem to occur more between groups than within! Weak ties!!!!! Retweets also seem to happen **between** less similar groups!

A Followers

size of targeted group

size group of origin

B

- follower
- retweets
- mentions

link fraction

size group of origin

C

link fraction

size of targeted group

D

link frequency

group similarity

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Bridge Links

A

B
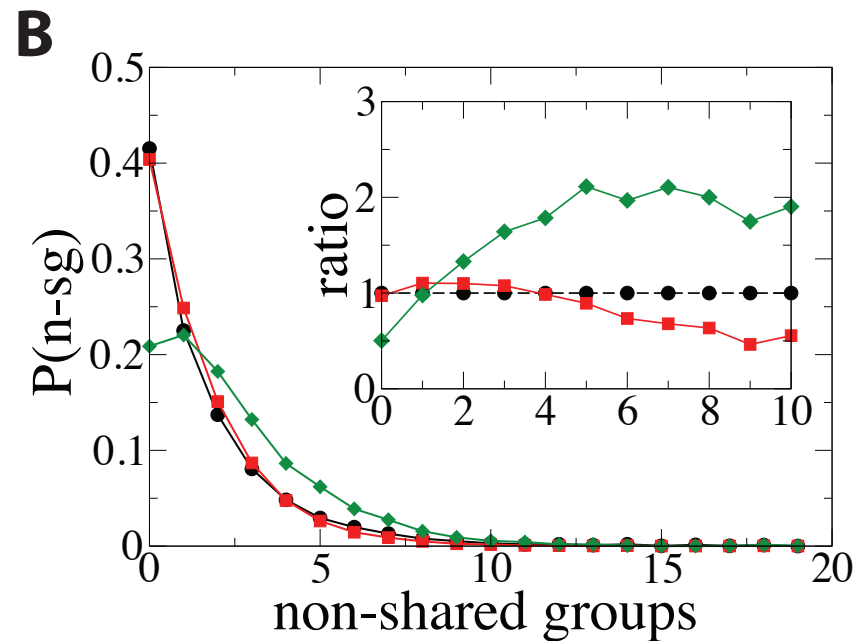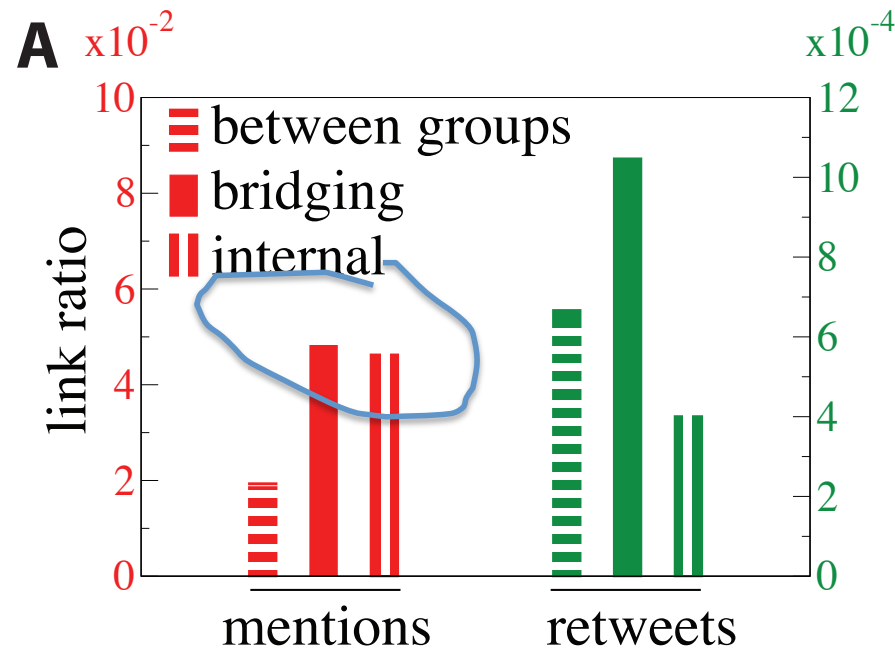
Retweets on a bridge increase with the number of groups assigned to the bridging nodes

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Discussion on findings

- There seems to be a correlation with the role of weak ties and the clustering done on the followers network

- Weak ties seem to be carrier of information (retweets) while internal group links seem to be more about mentions and communication

Summary

- We have discussed modularity based community detection as well as overlapping community detection.
- Many methods exist...

- We have shown cluster and weak ties analysis on an online social network dataset.

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References
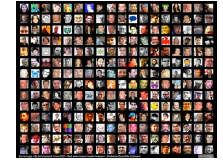
- M. Girvan and M. E. J. Newman. **Community structure in social and biological networks** Proc. Natl. Acad. Sci. USA, 99(12):7821–7826, June 2002.
- S. Fortunato. **Community detection in graphs**, Arxiv 2009.
- Michelle Girvan and Mark E. J. Newman. **Community structure in social and biological networks.** Proc. Natl. Acad. Sci. USA, 99(12):7821–7826, June 2002.
- M.E.J. Newman, M. Girvan. **Finding and evaluating community structure in networks.** Phys. Rev. E 69, 026113, 2004.
- A. Clauset, M.E.J. Newman, C. Moore. **Finding community structure in very large networks**. Phys. Rev. E 70, 066111, 2004.
- Vincent D Blondel, Jean-Loup Guillaume, Renaud Lambiotte, Etienne Lefebvre. **Fast unfolding of communities in large networks.** Journal of Statistical Mechanics: Theory and Experiment 2008 (10).
- G. Palla, I. Derényi, I. Farkas, and T. Vicsek. **Uncovering the overlapping community structure of complex networks in nature and society**. *Nature.* **435**, 814-818 (2005).
- A. Lancichinetti, F. Radicchi,3 J. Ramasco, S. Fortunato. **Finding statistically significant communities in networks.** PLOS One 2011; 6(4). (not discussed).
- P. Grabowicz, J. Ramasco, E. Moro, J. Pujol, V.. Eguiluz. **Social features of online networks: the strength of weak ties in online social media. arXiv:1107.4009. July 2011.**

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