Slides

for Part IA CST 2013/14

Discrete Mathematics For Computer Science

<cl.cam.ac.uk/teaching/1314/DiscMath>

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What are we up to ?

- Learn to read and write, and work with, mathematical arguments.
- Doing some basic discrete mathematics.
- Getting a taste of computer science applications.

What is it that we do?

In general:

Mathematical models and methods to analyse problems that arise in computer science.

In particular:

Make and study mathematical constructions by means of definitions and theorems. We aim at understanding their properties and limitations.

Some friendly advice

by K. Houston from the Preface of

How to Think Like a Mathematician

- It's up to you.
- Think for yourself.
- Observe.
- Seek to understand.
- Collaborate.
- Be active.
- Question everything.
- Prepare to be wrong.
- Develop your intuition.
- Reflect.

Mathematical argument

Objectives

- To develop techniques for analysing and understanding mathematical statements.
- To be able to present logical arguments that establish mathematical statements in the form of clear proofs.
- To prove Fermat's Little Theorem, a basic result in the theory of numbers that has many applications in computer science.

* Typo in printed notes

Proofs in practice

We are interested in examining the following statement:

The product of two odd integers is odd.

This seems innocuous* enough, but it is in fact full of baggage.
For instance, it presupposes that you know:

- what a statement is;
- what the integers $(\ldots, -1, 0, 1, \ldots)$ are, and that amongst them there is a class of odd ones $(\ldots, -3, -1, 1, 3, \ldots)$;
- what the product of two integers is, and that this is in turn an integer.

More precisely put, we may write:

If m and n are odd integers then so is $m \cdot n$.

which further presupposes that you know:

- what variables are;
- what

  if . . . then . . .

  statements are, and how one goes about proving them;
- that the symbol "$\cdot$" is commonly used to denote the product operation.

Even more precisely, we should write

For all integers m and n, if m and n are odd then so is $m \cdot n$.

which now additionally presupposes that you know:

- what

  for all . . .

  statements are, and how one goes about proving them.

Thus, in trying to understand and then prove the above statement, we are assuming quite a lot of *mathematical jargon* that one needs to learn and practice with to make it a useful, and in fact very powerful, tool.

*typo in printed notes

Some mathematical jargon

Statement

A sentence that is either true or * false — but not both.

Example 1

$$\text{‘}e^{i\pi} + 1 = 0\text{’}$$

Non-example

‘This statement is false’

Predicate

A statement whose truth depends on the value of one or more variables.

Example 2

1. ‘$e^{ix} = \cos x + i \sin x$’

2. *‘the function f is differentiable’*

Theorem

A very important true statement.

Proposition

A less important but nonetheless interesting true statement.

Lemma

A true statement used in proving other true statements.

Corollary

A true statement that is a simple deduction from a theorem or proposition.

Example 3

1. *Fermat's Last Theorem*

2. *The Pumping Lemma*

Conjecture

A statement believed to be true, but for which we have no proof.

Example 4

1. *Goldbach's Conjecture*
2. *The Riemann Hypothesis*
3. *Schanuel's Conjecture*

Proof

Logical explanation of why a statement is true; a method for establishing truth.

Logic

The study of methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.

Example 5

1. *Classical predicate logic*
2. *Hoare logic*
3. *Temporal logic*

Axiom

A basic assumption about a mathematical situation.

Axioms can be considered facts that do not need to be proved (just to get us going in a subject) or they can be used in definitions.

Example 6

1. *Euclidean Geometry*
2. *Riemannian Geometry*
3. *Hyperbolic Geometry*

Definition

An explanation of the mathematical meaning of a word (or phrase).

The word (or phrase) is generally defined in terms of properties.

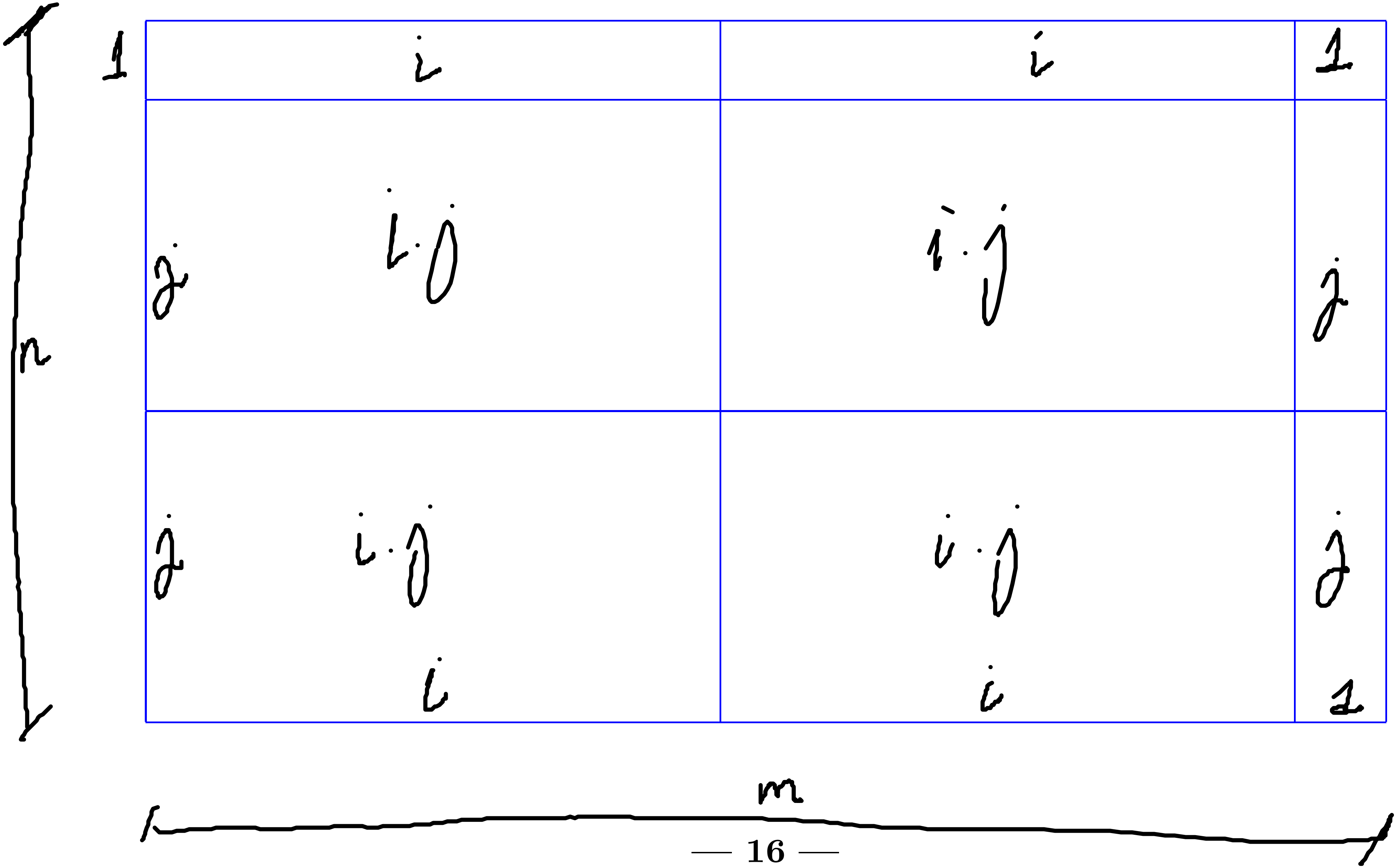
Warning: It is vitally important that you can recall definitions precisely. A common problem is not to be able to advance in some problem because the definition of a word is unknown.

Definition, theorem, intuition, proof in practice

Definition 7 *An integer is said to be* odd *whenever it is of the form* $2 \cdot i + 1$ *for some (necessarily unique) integer* i.

Proposition 8 *For all integers* m *and* n*, if* m *and* n *are odd then so is* $m \cdot n$.

Intuition:



~~Proof~~

$$m \cdot n = (2i+1)(2j+1)$$

$$= 4ij + 2i + 2j + 1$$

$$= 2(2ij + i + j) + 1$$

QED.

PROOF OF Proposition 8: Let m and n be odd integers, with m of the form $2i+1$ and n of the form $2j+1$, for i and j integers. Consider $m \cdot n = (2i+1) \cdot (2j+1)$

$$= \dots$$

$$= 2(2i \cdot j + i + j) + 1$$

which is of the form $2k+1$ (for $k = 2ij + i + j$), hence it is odd. □