Exercises for Artificial Intelligence II

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1 Making decisions

1. Prove the result mentioned on slide 161:

$$\mathtt{VPI}_E(E', E'') = \mathtt{VPI}_E(E') + \mathtt{VPI}_{E,E'}(E'').$$

2. Evil Robot is teaching himself surgery. He believes that there are two treatments, t_1 and t_2 suitable for his first patient, each having three possible outcomes: `cure`, `death` and `amputation`. These have utilities of 100, -1000 and -250 respectively. Evil robot thinks that t_1 has probabilities 0.8, 0.1 and 0.1 respectively for the three outcomes and treatment t_2 has probabilities 0.75, 0.05 and 0.2. Compute the expected utility of each treatment.

   Evil Robot has been studying hard, and has learned that an unpleasant test T is available that might help him choose the better treatment. The test has a cost to the patient of -50, while the cost of not performing it is -2. (Evil robot will nonetheless conduct some slightly unpleasant tests.) He estimates that the probability of the test being positive is 0.7. He also thinks that, armed with a positive test he can give t_1 outcome probabilities of 0.9, 0.01 and 0.09 respectively, and t_2 outcome probabilities of 0.85, 0.02 and 0.13. If test T is negative then the outcome probabilities are unchanged. (He does the other tests just for fun.)

   In the interest of the patient, should Evil Robot use test T?

3. Exam question: 2007, paper 8, question 9.

4. Exam question: 2011, paper 8, question 8.

5. Exam question: 2013, paper 8, question 2.

2 HMMs

1. Derive the equation

$$b_{t+1:T} = \mathbf{S}\mathbf{E}_{t+1} b_{t+2:T}$$

   for the backward message in a hidden Markov model (lecture slide 208).

2. Explain why the backward message update should be initialized with the vector $(1, \ldots, 1)$.

3. Establish how the prior $\Pr(S_0)$ should be included in the derivation of the Viterbi algorithm. (This is mentioned on slide 192, but no detail is given.)

4. A hidden Markov model has transition matrix $S_{ij} = \Pr(S_{t+1} = s_j | S_t = s_i)$ where

$$\mathbf{S} = \begin{pmatrix} 0.2 & 0.4 & 0.4 \\ 0.1 & 0.6 & 0.3 \\ 0.8 & 0.1 & 0.1 \end{pmatrix}.$$

In any state we observe one of the symbols $\triangle$, $\bigtriangledown$, $\bigcirc$, $\square$ with the following probabilities:

	$\triangle$	$\bigtriangledown$	$\bigcirc$	$\square$
s_1	0.7	0.1	0.1	0.1
s_2	0.3	0.2	0.4	0.1
s_3	0.4	0.2	0.2	0.2

Prior probabilites for the states are $\Pr(s_1) = 0.3$, $\Pr(s_2) = 0.3$ and $\Pr(s_3) = 0.4$. We observe the sequence of symbols

$$\bigcirc \bigcirc \square \triangle \triangle \square \bigtriangledown \square.$$

Use the Viterbi algorithm to infer the most probable sequence of states generating this sequence.

5. Exam question: 2005, paper 9, question 8.
6. Exam question: 2008, paper 9, question 5.
7. Exam question: 2010, paper 7, question 4.
8. Exam question: 2013, paper 7, question 2.