Term type rules

$$\frac{\Gamma(x) = \mathbb{P}}{\Gamma \vdash x : \mathbb{P}} \qquad \frac{\Gamma, x : \mathbb{P} \vdash t : \mathbb{P}}{\Gamma \vdash rec\ x\, t : \mathbb{P}} \qquad \frac{\Gamma \vdash t_j : \mathbb{P} \quad \text{all } j \in I}{\Gamma \vdash \Sigma_{i \in I} t_i : \mathbb{P}}$$

$$\frac{\Gamma \vdash t : \mathbb{P}}{\Gamma \vdash .t : .\mathbb{P}} \qquad \frac{\Gamma \vdash u : .\mathbb{P} \quad \Gamma, x : \mathbb{P} \vdash t : \mathbb{Q}}{\Gamma \vdash [u > .x \Rightarrow t] : \mathbb{Q}}$$

$$\frac{\Gamma, x : \mathbb{P} \vdash t : \mathbb{Q}}{\Gamma \vdash \lambda x\, t : \mathbb{P} \to \mathbb{Q}} \qquad \frac{\Gamma \vdash t : \mathbb{P} \to \mathbb{Q} \quad \Gamma \vdash u : \mathbb{P}}{\Gamma \vdash t\, u : \mathbb{Q}}$$

$$\frac{\Gamma \vdash t : \mathbb{P}_b \qquad b \in A}{\Gamma \vdash b\, t : \Sigma_{a \in A} a \mathbb{P}_a} \qquad \frac{\Gamma \vdash t : \Sigma_{a \in A} a \mathbb{P}_a \quad b \in A}{\Gamma \vdash \pi_b(t) : \mathbb{P}_b}$$

$$\frac{\Gamma \vdash t : \mathbb{P}_j[\mu \vec{P}\, \vec{\mathbb{P}} / \vec{P}]}{\Gamma \vdash t : \mu_j \vec{P}\, \vec{\mathbb{P}}} \qquad \frac{\Gamma \vdash t : \mu_j \vec{P}\, \vec{\mathbb{P}}}{\Gamma \vdash t : \mathbb{P}_j[\mu \vec{P}\, \vec{\mathbb{P}} / \vec{P}]}$$

Action type rules

$$\bullet\mathbb{P} : \bullet : \mathbb{P} \qquad \frac{u : \mathbb{P} \qquad \mathbb{Q} : q : \mathbb{Q}'}{\mathbb{P} \to \mathbb{Q} : (u \mapsto q) : \mathbb{Q}'} \qquad \frac{\mathbb{P}_a : p : \mathbb{P}'}{\Sigma_{a \in A} a\mathbb{P}_a : a\,p : \mathbb{P}'} \qquad \frac{\mathbb{P}_j[\mu\vec{P}\,\vec{\mathbb{P}}/\vec{P}] : p : \mathbb{P}'}{\mu_j\vec{P}\,\vec{\mathbb{P}} : p : \mathbb{P}'}$$

Transition rules

$$\frac{\mathbb{P} : t[rec\, x\, t/x] \xrightarrow{p} t'}{\mathbb{P} : rec\, x\, t \xrightarrow{p} t'} \qquad \frac{\mathbb{P} : t_j \xrightarrow{p} t'}{\mathbb{P} : \Sigma_{i \in I} t_i \xrightarrow{p} t'}\ j \in I$$

$$\frac{}{.\mathbb{P} : .t \xrightarrow{\bullet} t} \qquad \frac{.\mathbb{P} : u \xrightarrow{\bullet} u' \quad \mathbb{Q} : t[u'/x] \xrightarrow{q} t'}{\mathbb{Q} : [u > .x \Rightarrow t] \xrightarrow{q} t'}$$

$$\frac{\mathbb{Q} : t[u/x] \xrightarrow{p} t'}{\mathbb{P} \to \mathbb{Q} : \lambda x\, t \xrightarrow{u \mapsto p} t'} \qquad \frac{\mathbb{P} \to \mathbb{Q} : t \xrightarrow{u \mapsto p} t'}{\mathbb{Q} : t\, u \xrightarrow{p} t'}$$

$$\frac{\mathbb{P}_a : t \xrightarrow{p} t'}{\Sigma_{a \in A} a\mathbb{P}_a : a\, t \xrightarrow{a\, p} t'} \qquad \frac{\Sigma_{a \in A} a\mathbb{P}_a : t \xrightarrow{a\, p} t'}{\mathbb{P}_a : \pi_a(t) \xrightarrow{p} t'}$$

$$\frac{\mathbb{P}_j[\mu \vec{P}\, \vec{\mathbb{P}}/\vec{P}] : t \xrightarrow{p} t'}{\mu_j \vec{P}\, \vec{\mathbb{P}} : t \xrightarrow{p} t'}$$

Example untyped derivation

Remember: $\mathbf{nil} = \sum_{i \in \emptyset} P_i$

$$\mathbf{.}b\mathbf{.nil} \stackrel{\cdot}{\rightarrow} b\mathbf{.nil}$$

Example untyped derivation

Remember: $\mathbf{nil} = \sum_{i \in \emptyset} P_i$

$$\frac{\mathbf{.}b\mathbf{.nil} \xrightarrow{\cdot} b\mathbf{.nil}}{a\mathbf{.}b\mathbf{.nil} \xrightarrow{a\cdot} b\mathbf{.nil}}$$

Example untyped derivation

Remember: $\mathbf{nil} = \sum_{i \in \emptyset} P_i$

$$\dfrac{\dfrac{}{\mathbf{.}b\mathbf{.nil} \xrightarrow{\cdot} b\mathbf{.nil}}}{\dfrac{a\mathbf{.}b\mathbf{.nil} \xrightarrow{a\cdot} b\mathbf{.nil}}{a\mathbf{.}b\mathbf{.nil} + c\mathbf{.nil} \xrightarrow{a\cdot} b\mathbf{.nil}}}$$

Example untyped derivation

Remember: $\mathbf{nil} = \sum_{i \in \emptyset} P_i$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{.}b\mathbf{.nil} \stackrel{\cdot}{\rightarrow} b\mathbf{.nil}}}{a\mathbf{.}b\mathbf{.nil} \stackrel{a\cdot}{\rightarrow} b\mathbf{.nil}}}{a\mathbf{.}b\mathbf{.nil} + c\mathbf{.nil} \stackrel{a\cdot}{\rightarrow} b\mathbf{.nil}}}{\pi_a(a\mathbf{.}b\mathbf{.nil} + c\mathbf{.nil}) \stackrel{\cdot}{\rightarrow} b\mathbf{.nil}}$$

Example untyped derivation

Remember: $\mathbf{nil} = \sum_{i \in \emptyset} P_i$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{.b.\mathbf{nil} \stackrel{\cdot}{\rightarrow} b.\mathbf{nil}}}{a.b.\mathbf{nil} \xrightarrow{a.} b.\mathbf{nil}}}{a.b.\mathbf{nil} + c.\mathbf{nil} \xrightarrow{a.} b.\mathbf{nil}} \qquad \dfrac{\dfrac{\dfrac{}{.\mathbf{nil} \stackrel{\cdot}{\rightarrow} \mathbf{nil}}}{b.\mathbf{nil} \xrightarrow{b.} \mathbf{nil}}}{a'\,b.\mathbf{nil} \xrightarrow{a'\,b.} \mathbf{nil}}}{[\pi_a(a.b.\mathbf{nil} + c.\mathbf{nil}) > .x \Rightarrow a'x] \xrightarrow{a'\,b.} \mathbf{nil}}$$

Recursion: Type assignment

Let $\mathbb{P} = \mu P\ a.P + bP$ in the following

$$\dfrac{\dfrac{\dfrac{x : \mathbb{P} \vdash \mathbf{nil} : \mathbb{P}}{x : \mathbb{P} \vdash .\mathbf{nil} : .\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : \mathbb{P}}$$

$$\dfrac{\dfrac{x : \mathbb{P} \vdash x : \mathbb{P}}{x : \mathbb{P} \vdash bx : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash bx : \mathbb{P}}$$

Recursion: Type assignment

Let $\mathbb{P} = \mu P\ a.P + bP$ in the following

$$\dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{P} \vdash \mathbf{nil} : \mathbb{P}}{x : \mathbb{P} \vdash .\mathbf{nil} : .\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : \mathbb{P}} \qquad \dfrac{\dfrac{x : \mathbb{P} \vdash x : \mathbb{P}}{x : \mathbb{P} \vdash bx : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash bx : \mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} + bx : \mathbb{P}}$$

Recursion: Type assignment

Let $\mathbb{P} = \mu P\ a.P + bP$ in the following

$$\dfrac{\dfrac{\dfrac{\dfrac{x : \mathbb{P} \vdash \mathbf{nil} : \mathbb{P}}{x : \mathbb{P} \vdash .\mathbf{nil} : .\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash a.\mathbf{nil} : \mathbb{P}} \qquad \dfrac{\dfrac{x : \mathbb{P} \vdash x : \mathbb{P}}{x : \mathbb{P} \vdash bx : a.\mathbb{P} + b\mathbb{P}}}{x : \mathbb{P} \vdash bx : \mathbb{P}}}{\dfrac{x : \mathbb{P} \vdash a.\mathbf{nil} + bx : \mathbb{P}}{\vdash \mathrm{rec}\ x\ a.\mathbf{nil} + bx : \mathbb{P}}}$$

Abstraction and application

$$\dfrac{\dfrac{\dfrac{}{\mathbf{.nil} \xrightarrow{\cdot} \mathbf{nil}}}{a\mathbf{.nil} \xrightarrow{a\cdot} \mathbf{nil}}}{a\mathbf{.nil} + \mathbf{nil} \xrightarrow{a\cdot} \mathbf{nil}}}{c(a\mathbf{.nil} + \mathbf{nil}) \xrightarrow{ca\cdot} \mathbf{nil}}$$

Abstraction and application

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{.nil} \xrightarrow{\cdot} \mathbf{nil}}}{a\mathbf{.nil} \xrightarrow{a\cdot} \mathbf{nil}}}{a\mathbf{.nil} + \mathbf{nil} \xrightarrow{a\cdot} \mathbf{nil}}}{c(a\mathbf{.nil} + \mathbf{nil}) \xrightarrow{ca\cdot} \mathbf{nil}}}{\lambda y\ c(a\mathbf{.nil} + y) \xrightarrow{\mathbf{nil} \mapsto ca\cdot} \mathbf{nil}}$$

Abstraction and application

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{.nil} \xrightarrow{\cdot} \mathbf{nil}}}{a\mathbf{.nil} \xrightarrow{a\cdot} \mathbf{nil}}}{a\mathbf{.nil} + \mathbf{nil} \xrightarrow{a\cdot} \mathbf{nil}}}{c(a\mathbf{.nil} + \mathbf{nil}) \xrightarrow{ca\cdot} \mathbf{nil}}}{\lambda y\ c(a\mathbf{.nil} + y) \xrightarrow{\mathbf{nil} \mapsto ca\cdot} \mathbf{nil}}}{\lambda x \lambda y\ c(x + y) \xrightarrow{a\mathbf{.nil} \mapsto (\mathbf{nil} \mapsto ca\cdot)} \mathbf{nil}}$$

Abstraction and application

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathbf{.nil} \stackrel{\cdot}{\to} \mathbf{nil}}}{a\mathbf{.nil} \xrightarrow{a\mathbf{.}} \mathbf{nil}}}{a\mathbf{.nil} + \mathbf{nil} \xrightarrow{a\mathbf{.}} \mathbf{nil}}}{c(a\mathbf{.nil} + \mathbf{nil}) \xrightarrow{ca\mathbf{.}} \mathbf{nil}}}{\lambda y\ c(a\mathbf{.nil} + y) \xrightarrow{\mathbf{nil} \mapsto ca\mathbf{.}} \mathbf{nil}}}{\lambda x \lambda y\ c(x + y) \xrightarrow{a\mathbf{.nil} \mapsto (\mathbf{nil} \mapsto ca\mathbf{.})} \mathbf{nil}}}{(\lambda x \lambda y\ c(x + y))(a\mathbf{.nil}) \xrightarrow{\mathbf{nil} \mapsto ca\mathbf{.}} \mathbf{nil}}$$

Abstraction and application (with types)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{.\mathbb{0} : \mathbf{.nil} \xrightarrow{\cdot} \mathbf{nil}}}{a.\mathbb{0} : a\mathbf{.nil} \xrightarrow{a.} \mathbf{nil}}}{a.\mathbb{0} + b.\mathbb{0} : a\mathbf{.nil} + \mathbf{nil} \xrightarrow{a.} \mathbf{nil}}}{c(a.\mathbb{0} + b.\mathbb{0}) : c(a\mathbf{.nil} + \mathbf{nil}) \xrightarrow{ca.} \mathbf{nil}}}{b.\mathbb{0} \to c(a.\mathbb{0} + b.\mathbb{0}) : \lambda y\ c(a\mathbf{.nil} + y) \xrightarrow{\mathbf{nil} \mapsto ca.} \mathbf{nil}}}{a.\mathbb{0} \to b.\mathbb{0} \to c(a.\mathbb{0} + b.\mathbb{0}) : \lambda x \lambda y\ c(x + y) \xrightarrow{a\mathbf{.nil} \mapsto (\mathbf{nil} \mapsto ca.)} \mathbf{nil}}}{b.\mathbb{0} \to c(a.\mathbb{0} + b.\mathbb{0}) : (\lambda x \lambda y\ c(x + y))(a\mathbf{.nil}) \xrightarrow{\mathbf{nil} \mapsto ca.} \mathbf{nil}}$$