# Topics in Concurrency

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# Concurrency and distribution

- Computation is becoming increasingly distributed, concurrent and interactive
  - boundaries of computation becoming increasingly unclear,
  - behaviour of systems increasingly difficult to reproduce
- $\rightsquigarrow$ problems such as how to structure and understand distributed computation, how to ensure correctness (e.g. security) of processes in an uncontrolled environment
- Concurrency theory is a broad and active field for research, but
- Present ideas of process and logics for distributed computation are too crude to address all problems . . .

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  - behaviour of systems increasingly difficult to reproduce
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- Concurrency theory is a broad and active field for research, but
- Present ideas of process and logics for distributed computation are too crude to address all problems . . . However there are attempts:

**topics in concurrency**

- Theories of processes, logics & model checking, security, mobility

# Topics in Concurrency

- Simple parallelism and non-determinism
- Communicating processes
  - Milner's CCS (Calculus of Communicating Systems)
  - Bisimulation
- Specification logics for processes
  - modal $\mu$-calculus
  - CTL
  - model checking [Concurrency workbench]
- Petri nets
  - events, causal dependence, independence
- Mobile processes
  - Higher-order processes: process passing, location
- Security protocols
  - SPL (Security Protocol Language)
  - Petri net semantics
  - Proofs of secrecy and authentication

Chapter 1 in the lecture notes revises relevant topics from Discrete Mathematics (well-founded induction and Tarski's fixed point theorem).

# While programs

$$c ::= \texttt{skip} \mid X := a \mid \texttt{if}\ b\ \texttt{then}\ c_1\ \texttt{else}\ c_2 \mid c_0; c_1 \mid \texttt{while}\ b\ \texttt{do}\ c$$

- States $\sigma \in \Sigma$ are functions from locations to values
- Configurations: $\langle c, \sigma \rangle$ and $\sigma$
- Rules describe a single step of execution:

$$\frac{\langle c_0, \sigma \rangle \to \langle c_0', \sigma' \rangle}{\langle c_0; c_1, \sigma \rangle \to \langle c_0'; c_1, \sigma' \rangle} \qquad \frac{\langle c_0, \sigma \rangle \to \sigma'}{\langle c_0; c_1, \sigma \rangle \to \langle c_1, \sigma' \rangle}$$

$$\frac{\langle b, \sigma \rangle \to \text{true} \qquad \langle c, \sigma \rangle \to \langle c', \sigma' \rangle}{\langle \texttt{while}\ b\ \texttt{do}\ c, \sigma \rangle \to \langle c'; \texttt{while}\ b\ \texttt{do}\ c, \sigma' \rangle}$$

$$\vdots$$

# Parallel commands

Syntax extended with parallel composition:

$$c ::= \ldots \mid c_0 \parallel c_1$$

Rules:

$$\frac{\langle c_0, \sigma \rangle \rightarrow \langle c_0', \sigma' \rangle}{\langle c_0 \parallel c_1, \sigma \rangle \rightarrow \langle c_0' \parallel c_1, \sigma' \rangle}$$

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- Parallelism $\rightsquigarrow$ Non-determinism
- Behaviour of $\parallel$-commands not a partial function from states to states; when are two $\parallel$-commands equivalent? [Congruence?]
- Parallelism by non-deterministic interleaving
- "communication by shared variables"

*Study of parallelism (or concurrency)*
*includes*
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What about the converse?

*Can we explain parallelism (or concurrency)*
*in terms of non-determinism?*

# The language of Guarded Commands (Dijkstra)

- Boolean expressions: $b$
- Arithmetic expressions: $a$
- Commands:

$$c ::= \texttt{skip} \mid \texttt{abort} \mid X := a \mid c_0; c_1 \mid \texttt{if } gc \texttt{ fi} \mid \texttt{do } gc \texttt{ od}$$

- Guarded commands:

$$\begin{array}{lcll} gc & ::= & b \to c & \text{guard} \\ & \mid & gc_0 \,[\!]\, gc_1 & \text{alternative} \end{array}$$

# Operational semantics

- Assume given rules for evaluating Booleans and assignments.
- **Guarded commands:**

$$\frac{\langle b, \sigma \rangle \rightarrow \mathit{true}}{\langle b \rightarrow c, \sigma \rangle \rightarrow \langle c, \sigma \rangle}$$

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$$\frac{\langle gc_0, \sigma \rangle \to \langle c, \sigma' \rangle}{\langle gc_0 \,[]\, gc_1, \sigma \rangle \to \langle c, \sigma' \rangle} \qquad \frac{\langle gc_1, \sigma \rangle \to \langle c, \sigma' \rangle}{\langle gc_0 \,[]\, gc_1, \sigma \rangle \to \langle c, \sigma' \rangle}$$

introduces non-determinism

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$$\frac{\langle b, \sigma\rangle \to \mathit{false}}{\langle b \to c, \sigma\rangle \to \mathrm{fail}}$$

fail is a new configuration

$$\frac{\langle gc_0, \sigma\rangle \to \mathrm{fail} \qquad \langle gc_1, \sigma\rangle \to \mathrm{fail}}{\langle gc_0 \mathbin{[\!]} gc_1, \sigma\rangle \to \mathrm{fail}}$$

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$$\frac{\langle gc_0, \sigma\rangle \to \mathrm{fail} \qquad \langle gc_1, \sigma\rangle \to \mathrm{fail}}{\langle gc_0 \,[\!]\, gc_1, \sigma\rangle \to \mathrm{fail}}$$

- **Commands:**

  $\texttt{abort}$ has no rules

- Conditional:

$$\frac{\langle gc, \sigma \rangle \rightarrow \langle c, \sigma' \rangle}{\langle \texttt{if } gc \texttt{ fi}, \sigma \rangle \rightarrow \langle c, \sigma' \rangle}$$

  no rule in case $\langle gc, \sigma \rangle \rightarrow \text{fail}$; then conditional behaves like $\texttt{abort}$

- Loop:

$$\frac{\langle gc, \sigma \rangle \rightarrow \text{fail}}{\langle \texttt{do } gc \texttt{ od}, \sigma \rangle \rightarrow \sigma}$$

$$\frac{\langle gc, \sigma \rangle \rightarrow \langle c, \sigma' \rangle}{\langle \texttt{do } gc \texttt{ od}, \sigma \rangle \rightarrow \langle c; \texttt{do } gc \texttt{ od}, \sigma' \rangle}$$

  in case $\langle gc, \sigma \rangle \rightarrow \text{fail}$, the loop behaves like $\texttt{skip}$:

$$\langle \texttt{skip}, \sigma \rangle \rightarrow \sigma$$

The process

$$\texttt{do}\ b_1 \rightarrow c_1\ [\!]\ \ldots\ [\!]\ b_n \rightarrow c_n\ \texttt{od}$$

is a form of (non-deterministically interleaved) parallel composition

$$\boxed{b_1 \rightarrow c_1} \parallel \ldots \parallel \boxed{b_n \rightarrow c_n}$$

in which each $c_i$ occurs atomically (i.e. uninterruptedly) provided $b_i$ holds each time it starts

$\leadsto$

| | |
|---|---|
| UNITY | (Misra and Chandy) |
| Hardware languages | (Staunstrup) |

# Examples

- Computing maximum:

```
if
   X ≥ Y → MAX = X
   []
   Y ≥ X → MAX = Y
fi
```

- Euclid's algorithm:

```
do
   X > Y → X := X − Y
   []
   Y > X → Y := Y − X
od
```

# Examples

- Computing maximum:

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if
   X ≥ Y → MAX = X
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do
   X > Y → X := X − Y
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   Y > X → Y := Y − X
od
```

Have

$$\{X = m \wedge Y = n \wedge m > 0 \wedge n > 0\}$$
$$Euclid$$
$$\{X = Y = gcd(m, n)\}$$

*. . . guarded commands support a neat Hoare-style logic*

- Invariant:

$$gcd(m,n) = gcd(X,Y)$$

  On exiting loop, $X = Y$.
- Key properties:

$$\begin{array}{rcll} gcd(m,n) & = & gcd(m-n,n) & \text{if } m > n \\ gcd(m,n) & = & gcd(m,n-m) & \text{if } n > m \\ gcd(m,m) & = & m & \end{array}$$

- Recalling:

$$gcd(m,n) \mid m,n$$

  and

$$\ell \mid m,n \implies \ell \mid gcd(m,n)$$

# Synchronized communication (Hoare, Milner)

handshake.jpg

Communication by "handshake",
with possible exchange of value,
localised to process-process (CSP)
or to a channel (CCS, OCCAM)

[Abstracts away from the protocol underlying coordination/"handshake" in the implementation]

# Extending GCL with synchronization

- Allow processes to send and receive values on channels

| | |
|---|---|
| $\alpha!a$ | evaluate expression *a* and send value on channel $\alpha$ |
| $\alpha?X$ | receive value on channel $\alpha$ and store it in *X* |

- All interaction between parallel processes is by sending / receiving values on channels
- Communication is synchronized and unicast
- Allow send and receive in commands *c* and in guards *gc*:

$$\texttt{do}\ \underbrace{Y < 100 \wedge \alpha?X}_{gc} \rightarrow \underbrace{\alpha!(X * X) \parallel Y := Y + 1}_{c}\ \texttt{od} \text{ is allowed}$$

- Language close to OCCAM and CSP

# Extending GCL with synchronization

Transitions now carry labels.

$$\frac{}{\langle \alpha?X, \sigma\rangle \xrightarrow{\alpha?n} \sigma[n/X]} \qquad \frac{\langle a, \sigma\rangle \to n}{\langle \alpha!a, \sigma\rangle \xrightarrow{\alpha!n} \sigma}$$

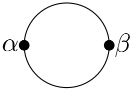
$$\frac{\langle c_0, \sigma\rangle \xrightarrow{\lambda} \langle c_0', \sigma'\rangle}{\langle c_0 \parallel c_1, \sigma\rangle \xrightarrow{\lambda} \langle c_0' \parallel c_1, \sigma'\rangle}$$ ($\lambda$ might be empty label) $+$ symmetric

$$\frac{\langle c_0, \sigma\rangle \xrightarrow{\alpha?n} \langle c_0', \sigma'\rangle \qquad \langle c_1, \sigma\rangle \xrightarrow{\alpha!n} \langle c_1', \sigma\rangle}{\langle c_0 \parallel c_1, \sigma\rangle \to \langle c_0' \parallel c_1', \sigma'\rangle}$$ $+$symmetric

$$\frac{\langle c, \sigma\rangle \xrightarrow{\lambda} \langle c', \sigma'\rangle}{\langle c \setminus \alpha, \sigma\rangle \xrightarrow{\lambda} \langle c' \setminus \alpha, \sigma'\rangle} \ \lambda \not\equiv \alpha?n \text{ or } \alpha!n$$

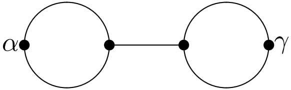
# Examples

- forwarder:




do $a?X \to \beta!X$od

- buffer capacity 2:

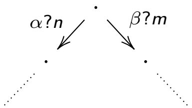



$$
\begin{array}{ll}
( & \texttt{do } \alpha?X \to \beta!X \texttt{ od} \\
\| & \texttt{do } \beta?X \to \gamma!X \texttt{ od } ) \setminus \beta
\end{array}
$$

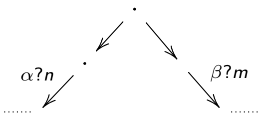
# Branching: internal vs external choice

- Extend the language, allowing Booleans to be attached to input/output actions
- Compare:

$\texttt{if}\ (\mathit{true} \wedge \alpha?X \to c_0)\ [\!]\ (\mathit{true} \wedge \beta?X \to c_1)\ \texttt{fi}$




$\texttt{if}\ (\mathit{true} \to (\alpha?X; c_0))\ [\!]\ (\mathit{true} \to (\beta?X; c_1))\ \texttt{fi}$




- Not equivalent processes w.r.t. their deadlock capabilities.