

The C1x and C++11 concurrency model

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Sequential consistency

ISO C1x/C++11 concurrency

Sequential consistency

Pthreads

ISO C1x/C++11 concurrency

Sequential consistency

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Java

ISO C1x/C++11 concurrency

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Expose hardware model (e.g. ClightTSO)

ISO C1x/C++11 concurrency

Sequential consistency

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Expose hardware model (e.g. ClightTSO)

C++11/C1x: SC for data race free programs, almost...

C++11: the next C++

1300 page prose specification defined by the ISO.

The design is a detailed compromise:

- hardware/compiler implementability
- useful abstractions
- broad spectrum of programmers

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We fixed serious problems in both C++11 and C1x, both now finalised.

The C1x/C++11 memory model

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- top level
- sequential execution
- simple concurrency
- expert concurrency
- very expert concurrency

How may a program execute?

The memory model is factored out from a symbolic operational semantics.

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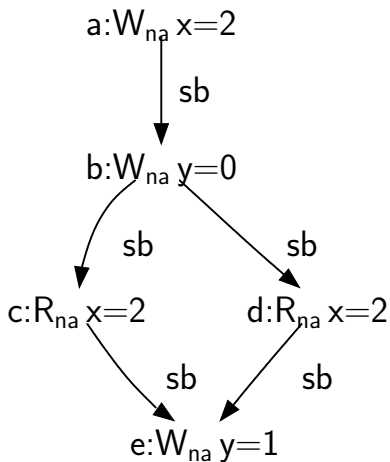
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The memory model is factored out from a symbolic operational semantics.

1. $P \mapsto E_1, \dots, E_n$
2. $E_i \mapsto X_{i1}, \dots, X_{im}$
3. is there an X_{ij} with a race? (actually, several kinds...)

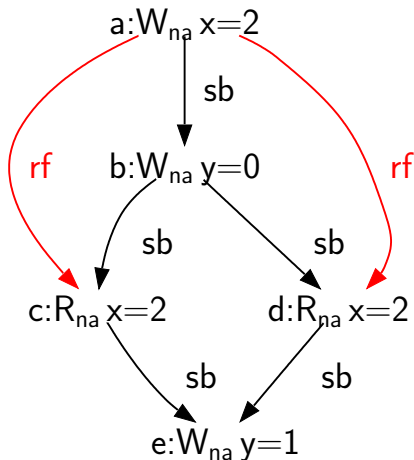
A single threaded program

```
int main() {  
    int x = 2;  
    int y = 0;  
    y = (x==x);  
    return 0; }
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The relations of a pre-execution

Each symbolic execution, E_i , contains:

sb – *sequenced before*

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Each full execution, X_{ij} , also has:

rf – *reads from*

sc – *SC order*

mo – *modification order*

A data race

```
int y, x = 2;
```

```
x = 3;          | y = (x==3);
```

a:W_{na} x=2

asw,rf

asw

b:W_{na} x=3

c:R_{na} x=2

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Simple concurrency: Decker's example and SC

```
atomic_int x = 0;
atomic_int y = 0;

x.store(1, seq_cst); | y.store(1, seq_cst);
y.load(seq_cst);     | x.load(seq_cst);
```

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atomic_int x = 0;
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c:W_{sc} y=1

sb



d:R_{sc} x=0

e:W_{sc} x=1

sb



f:R_{sc} y=0

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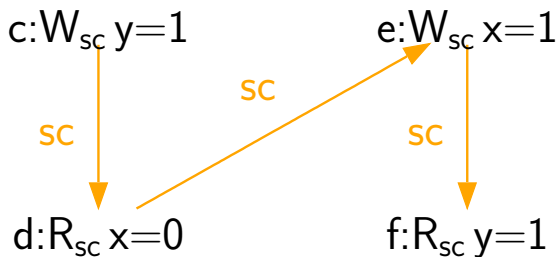
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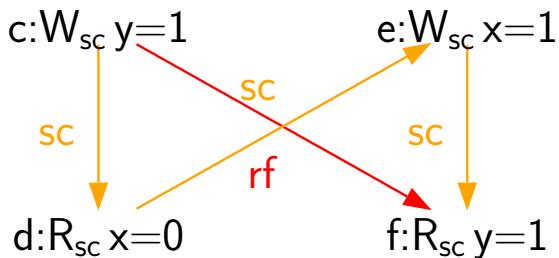
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```
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```



SC atomics

Read the last write in SC order.

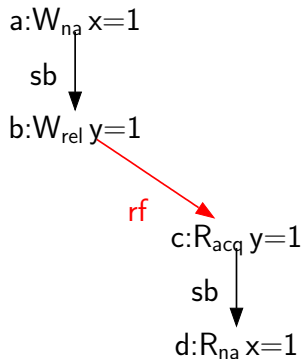


Using only seq_cst reads and writes gives SC.

(Initialization is not seq_cst though...)

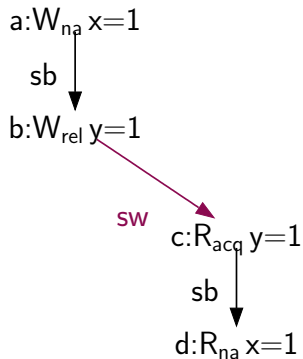
Expert concurrency: The release-acquire idiom

```
// sender | // receiver  
x = ...  | while (0 == y.load(acquire));  
y.store(1, release); | r = x;
```



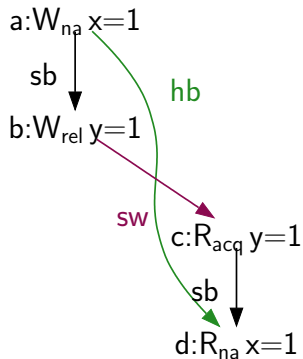
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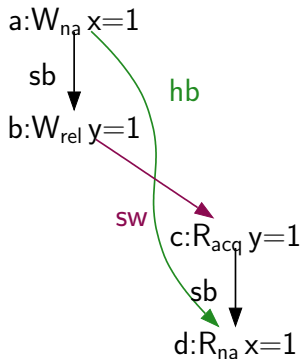
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$$\begin{array}{l} \xrightarrow{\text{simple-happens-before}} = \\ \left(\xrightarrow{\text{sequenced-before}} \cup \xrightarrow{\text{synchronizes-with}} \right)^+ \end{array}$$

Locks and unlocks

Unlocks and locks synchronise too:

```
int x, r;
mutex m;

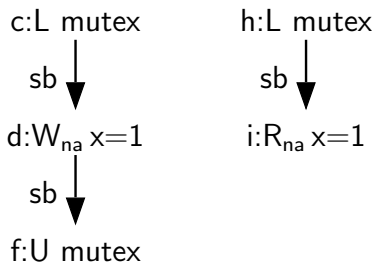
m.lock();           | m.lock();
x = ...            | r = x;
m.unlock();        |
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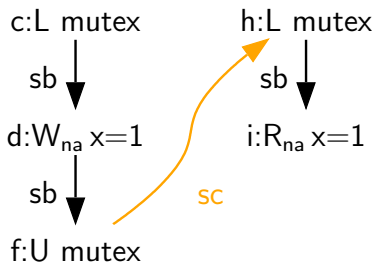


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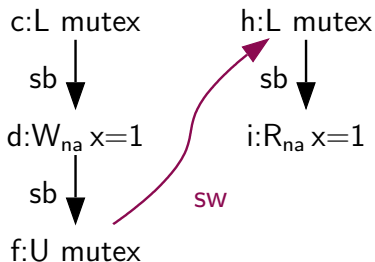
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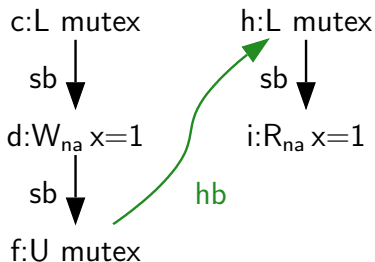


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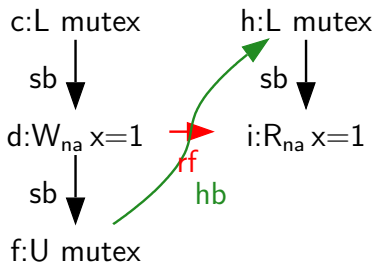
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```



Happens before is key to the model

Non-atomic loads read the most recent write in happens before. (This is unique in DRF programs)

The story is more complex for atomics, as we shall see.

Data races are defined as an absence of happens before.

A data race

```
int y, x = 2;
```

```
x = 3;          | y = (x==3);
```

a:W_{na} x=2

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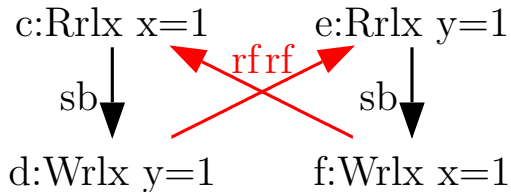
Data race definition

```
let data_races actions hb =  
  { (a, b) —  $\forall a \in actions \ b \in actions \mid$   
    (a = b)  
    same_location a b  
    (is_write a is_write b)  
    (same_thread a b)  
    (is_atomic_action a is_atomic_action b)  
    ((a, b)  $\in$  hb (b, a)  $\in$  hb) }
```

A program with a data race has undefined behaviour.

Relaxed writes: load buffering

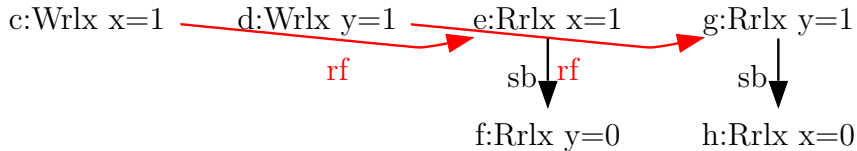
```
x.load(relaxed);      | y.load(relaxed);  
y.store(1, relaxed); | x.store(1, relaxed);
```



No synchronisation cost, but weakly ordered.

Relaxed writes: independent reads, independent writes

```
atomic_int x = 0;
atomic_int y = 0;
x.store(1, relaxed); | y.store(2, relaxed); | x.load(relaxed); | y.load(relaxed);
                    | y.load(relaxed); | x.load(relaxed);
```



Expert concurrency: fences avoid excess synchronisation

```
// sender          | // receiver  
x = ...           | while (0 == y.load(acquire));  
y.store(1, release); | r = x;
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```
// sender          | // receiver
x = ...           | while (0 == y.load(relaxed));
y.store(1, release); | fence(acquire);
                   | r = x;
```

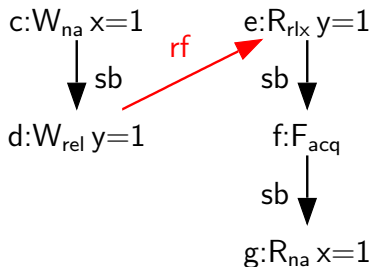
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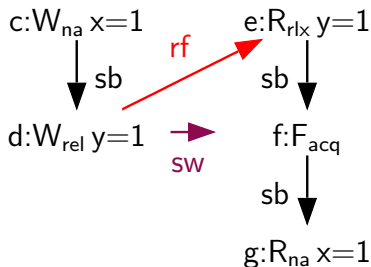
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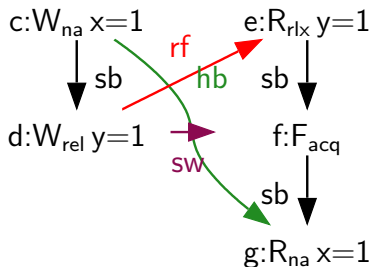
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Expert concurrency: modification order

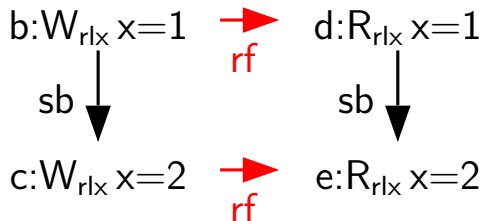
Modification order is a per-location total order over atomic writes of any memory order.

| | | |
|-----------------------------------|--|-------------------------------|
| <code>x.store(1, relaxed);</code> | | <code>x.load(relaxed);</code> |
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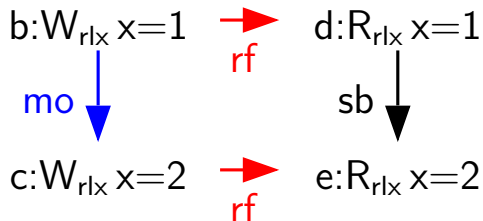
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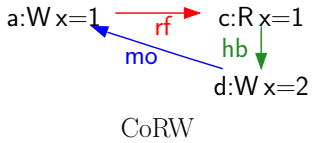
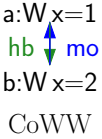
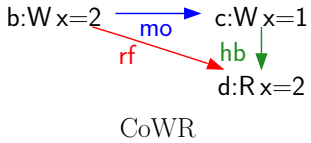
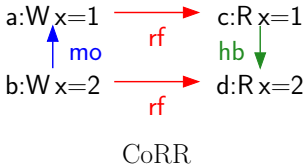
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Coherence and atomic reads

All forbidden!



Atomics cannot read from later writes in happens before.

Read-modify-writes

A successful `compare_exchange` is a read-modify-write.

Read-modify-writes read the last write in mo:

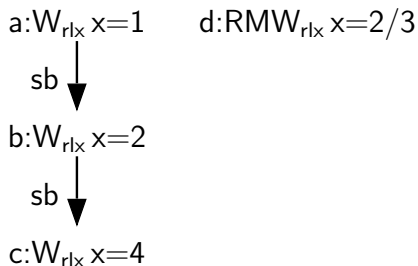
```
x.store(1, relaxed); | compare_exchange(&x, 2, 3, relaxed, relaxed);  
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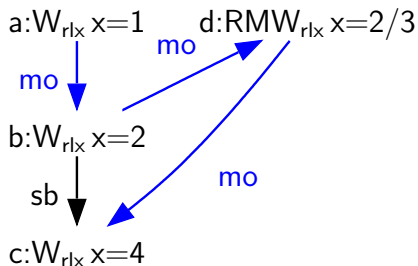


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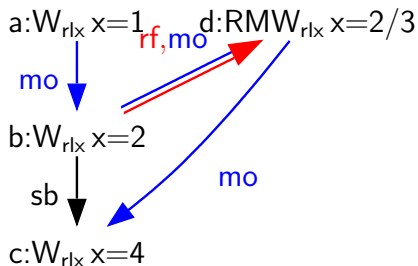


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Very expert concurrency: consume

Weaker than acquire

Stronger than relaxed

Non-transitive happens before! (only fully transitive through data dependence, dd)

The model as a whole

C1x and C++11 support many modes of programming:

- sequential

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Mathematizing C++ concurrency. M. Batty, S. Owens, S. Sarkar, P. Sewell, and T. Weber. In Proc. 38th ACM SIGACT-SIGPLAN Symposium on Principles of Programming Languages (POPL), 2011.

The full model

| | | | |
|-----|-----|-----|-----|
| 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 |
| 3 | 3 | 3 | 3 |
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Theorems

Are C1x and C++11 hopelessly complicated?

Programmers cannot be given this model!

With a formal definition, we can do proof, and even mechanise it.

What do we need to prove?

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What do we need to prove?

- implementability
- simplifications
- libraries

Implementability

Can we compile to x86?

Implementability

Can we compile to x86?

| Operation | x86 Implementation |
|--------------------|--------------------|
| load(non-seq_cst) | mov |
| load(seq_cst) | lock xadd(0) |
| store(non-seq_cst) | mov |
| store(seq_cst) | lock xchg |
| fence(non-seq_cst) | no-op |

x86-TSO is stronger and simpler.

Top level comparison

Recall the C/C++ semantics for program P :

1. $P \mapsto E_1, \dots, E_n,$

Top level comparison

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1. $P \mapsto E_1, \dots, E_n$, each an E_{opsem}

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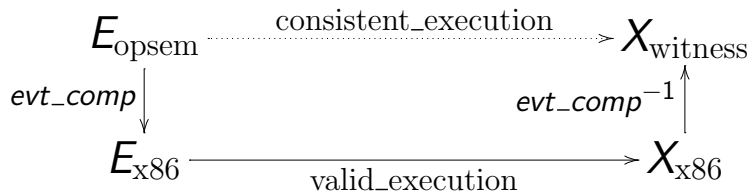
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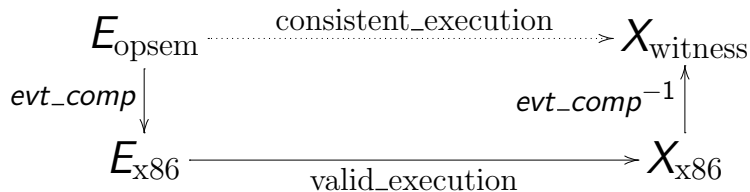
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There is not a DRF semantics.

Theorem



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We have a mechanised proof that C1x/C++11 behaviour is preserved.

Implementability

Can we compile to IBM Power?

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| C++0x Operation | POWER Implementation |
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| Non-atomic Load | ld |
| Load Relaxed | ld |
| Load Consume | ld (and preserve dependency) |
| Load Acquire | ld; cmp; bc; isync |
| Load Seq Cst | sync; ld; cmp; bc; isync |
| Non-atomic Store | st |
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Clarifying and compiling C/C++ concurrency: from C++0x to POWER. M. Batty, K. Memarian, S. Owens, S. Sarkar, and P. Sewell. In Proc. 39th ACM SIGPLAN-SIGACT Symposium on Principles of Programming Languages (POPL), 2012.

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```
atomic_int x = 0;
atomic_int y = 0;
if (1 == x.load(seq_cst)) | if (1 == y.load(seq_cst))
    atomic_init(&y, 1);      |    atomic_init(&x, 1);
```

atomic_init is a non-atomic write, and in C1x/C++11 they race...

Provide simplified models for higher level constructs.

Formal description of mutual exclusion in terms of happens-before.

We need libraries that provide a simpler model to programmers.

CPPMEM

helps explore and understand the model

Code in, all executions out

Confidence and speed

Communication

How may a program execute in CPPMEM?

1. $P \mapsto E_1, \dots, E_n$ — tracking constraints
2. $E_j \mapsto X_{i_1}, \dots, X_{i_m}$ — automatically uses formal model
3. is there an X_{ij} with a race?

Refinements to the standards

The current state of the standard

Fixed:

- Happens-before
- Coherence
- seq_cst atomics were more broken

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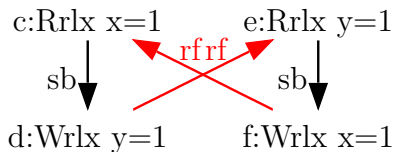
- Happens-before
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- seq_cst atomics were more broken

Not fixed:

- Self satisfying conditionals
- seq_cst atomics are still not SC

Self-satisfying conditionals

```
r1 = x.load(mo_relaxed);   |   r2 = y.load(mo_relaxed);  
if (r1 == 42)              |   if (r2 == 42)  
    y.store(r1, mo_relaxed); |   x.store(42, mo_relaxed);
```



...but it's not all bad!

Syntactic divide supported by simpler memory models.

Increasingly reasonable, consistent specification.

Remaining problems far less serious than Java.

Implementable above key architectures.

Conclusion

It's OK to like the C++0x memory model design

Our formal model lets us make fun things (go use it!)

- Optimized compilation?
- Static analysis?
- Dynamic analysis?
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- Program logics?

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Nitpicking C++ concurrency. J. C. Blanchette, T. Weber, M. Batty, S. Owens, and S. Sarkar. In Proc. 13th International ACM SIGPLAN Symposium on Principles and Practice of Declarative Programming (PPDP), 2011.