L11 : Algebraic Path Problems with Applications to Internet Routing
Lecture 14

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Sobrinho's encoding of the Gao/Rexford rules

Additive component uses min with

- 0 is the type of a *downstream* route,
- 1 is the type of a *peer* route, and
- 2 is the type of an *upstream* route.
- ∞ is the type of no route.

Multiplicative component

	0	1	2	∞
0	0	∞	∞	∞
1	1	∞	∞	∞
2	2	2	2	∞
∞	∞	∞	∞	∞

Note that this is not associative! In addition, this models just the "local preference" component of BGP. Not this must be combined with a lexicographic product. Can we improve on this?

Important properties for algebraic structures of the form $(S, \oplus, F, \overline{0}, \overline{1})$

property	definition
D	$\forall a, b \in S,\ f \in F\ :\ f(a \oplus b) = f(a) \oplus f(b)$
INFL	$\forall a \in S,\ f \in F\ :\ a \leq f(a)$
S.INFL	$\forall a \in S,\ F \in F\ :\ a \neq \overline{0} \implies a < f(a)$
K	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b) \implies a = b$
$K_{\overline{0}}$	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b) \implies (a = b \vee f(a) = \overline{0})$
C	$\forall a, b \in S,\ f \in F\ :\ f(a) = f(b)$
$C_{\overline{0}}$	$\forall a, b \in S,\ f \in F\ :\ f(a) \neq f(b) \implies (f(a) = \overline{0} \vee f(b) = \overline{0})$

Stratified Shortest-Paths Metrics

Metrics

$$(s,\ d) \ \text{or} \ \infty$$

- $s \neq \infty$ is a *stratum level* in $\{0, 1,\ 2, \ldots,\ m-1\}$,
- d is a "shortest-paths" distance,
- Routing metrics are compared lexicographically

$$(s_1,\ d_1) < (s_2,\ d_2) \iff (s_1 < s_2) \vee (s_1 = s_2 \wedge d_1 < d_2)$$

Stratified Shortest-Paths Policies

Policy has form (f, d)

$$\begin{array}{rcl}(f, d)(s, d') & = & \langle f(s), d + d' \rangle \\ (f, d)(\infty) & = & \infty\end{array}$$

where

$$\langle s, t \rangle = \begin{cases} \infty & (\text{if } s = \infty) \\ (s, t) & (\text{otherwise}) \end{cases}$$

Constraint on Policies

$$(f,\ d)$$

- Either f is inflationary and $0 < d$,
- or f is strictly inflationary and $0 \leq d$.

Why?

$$(\text{S.INFL}(S) \vee (\text{INFL}(S) \wedge \text{S.INFL}(T))) \implies \text{S.INFL}(S \,\vec{\times}_{\overline{0}}\, T).$$

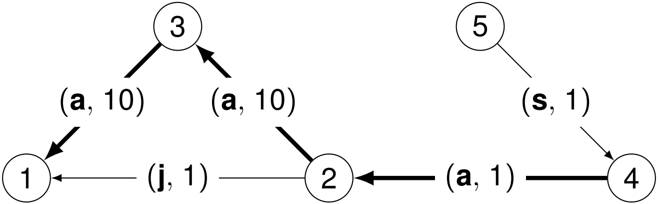
All Inflationary Policy Functions for Three Strata

	0	1	2	D	K_∞	C_∞		0	1	2	D	K_∞	C_∞
a	0	1	2	$\star$	$\star$		**m**	2	1	2			
b	0	1	∞	$\star$	$\star$		**n**	2	1	∞		$\star$	
c	0	2	2	$\star$			**o**	2	2	2	$\star$		$\star$
d	0	2	∞	$\star$	$\star$		**p**	2	2	∞	$\star$		$\star$
e	0	∞	2		$\star$		**q**	2	∞	2			$\star$
f	0	∞	∞	$\star$	$\star$	$\star$	**r**	2	∞	∞	$\star$	$\star$	$\star$
g	1	1	2	$\star$			**s**	∞	1	2		$\star$	
h	1	1	∞	$\star$		$\star$	**t**	∞	1	∞		$\star$	$\star$
i	1	2	2	$\star$			**u**	∞	2	2			$\star$
j	1	2	∞	$\star$	$\star$		**v**	∞	2	∞		$\star$	$\star$
k	1	∞	2		$\star$		**w**	∞	∞	2		$\star$	$\star$
l	1	∞	∞	$\star$	$\star$	$\star$	**x**	∞	∞	∞	$\star$	$\star$	$\star$

Almost shortest paths

	0	1	2	D	K_∞	interpretation
a	0	1	2	$\star$	$\star$	$+0$
j	1	2	∞	$\star$	$\star$	$+1$
r	2	∞	∞	$\star$	$\star$	$+2$
x	∞	∞	∞	$\star$	$\star$	$+3$
b	0	1	∞	$\star$	$\star$	filter 2
e	0	∞	2		$\star$	filter 1
f	0	∞	∞	$\star$	$\star$	filter 1, 2
s	∞	1	2		$\star$	filter 0
t	∞	1	∞		$\star$	filter 0, 2
w	∞	∞	2		$\star$	filter 0, 1

Shortest paths with filters, over $\mathbf{INF}_3$



Note that the path $5, 4, 2, 1$ with weight $(1,\ 3)$ would be the globally best path from node 5 to node 1. But in this case, poor node 5 is left with no path! The locally optimal solution has $\mathbf{R}(5,\ 1) = \infty$.

Both D and $K_{\overline{0}}$

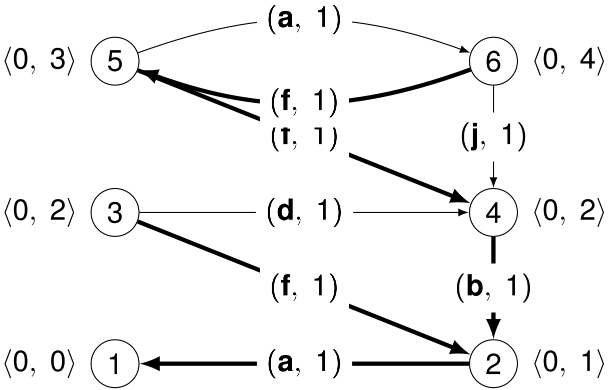
This makes combined algebra distributive!

	0	1	2
a	0	1	2
b	0	1	∞
d	0	2	∞
f	0	∞	∞
j	1	2	∞
l	1	∞	∞
r	2	∞	∞
x	∞	∞	∞

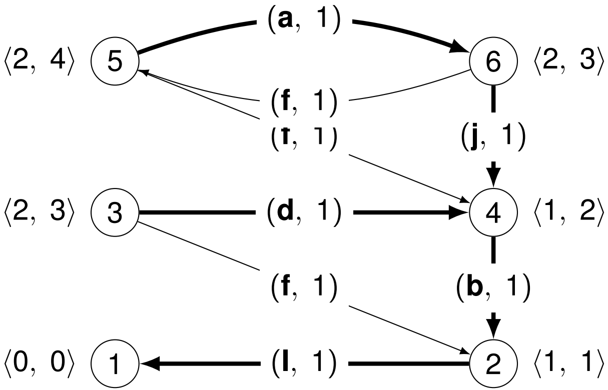
Why?

$$(\mathrm{D}(S) \wedge \mathrm{D}(T) \wedge \mathrm{K}_{\overline{0}}(S)) \implies \mathrm{D}(S \vec{\times}_{\overline{0}} T)$$

Example 1



Example 2



BGP : standard view

- 0 is the type of a *downstream* route,
- 1 is the type of a *peer* route, and
- 2 is the type of an *upstream* route.

	0	1	2
f	0	∞	∞
l	1	∞	∞
o	2	2	2

“Autonomous” policies

	0	1	2	D	K_∞
f	0	∞	∞	$\star$	$\star$
h	1	1	∞	$\star$	
l	1	∞	∞	$\star$	$\star$
o	2	2	2	$\star$	
p	2	2	∞	$\star$	
q	2	∞	2		
r	2	∞	∞	$\star$	$\star$
t	∞	1	∞		$\star$
u	∞	2	2		
v	∞	2	∞		$\star$
w	∞	∞	2		$\star$
x	∞	∞	∞	$\star$	$\star$