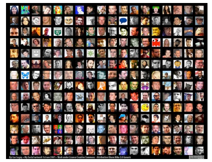

Social and Technological Network Analysis

Lecture 5: Structure of the Web, Search and Power Laws

Dr. Cecilia Mascolo

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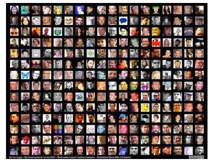

In This Lecture

- We describe power law networks and their properties and show examples of networks which are power law in nature, including the web.
- We present the preferential attachment model which allows the generation of power law networks.
- We study prediction of power laws
- We introduce search and PageRank

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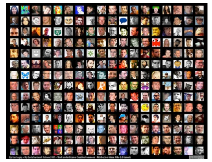
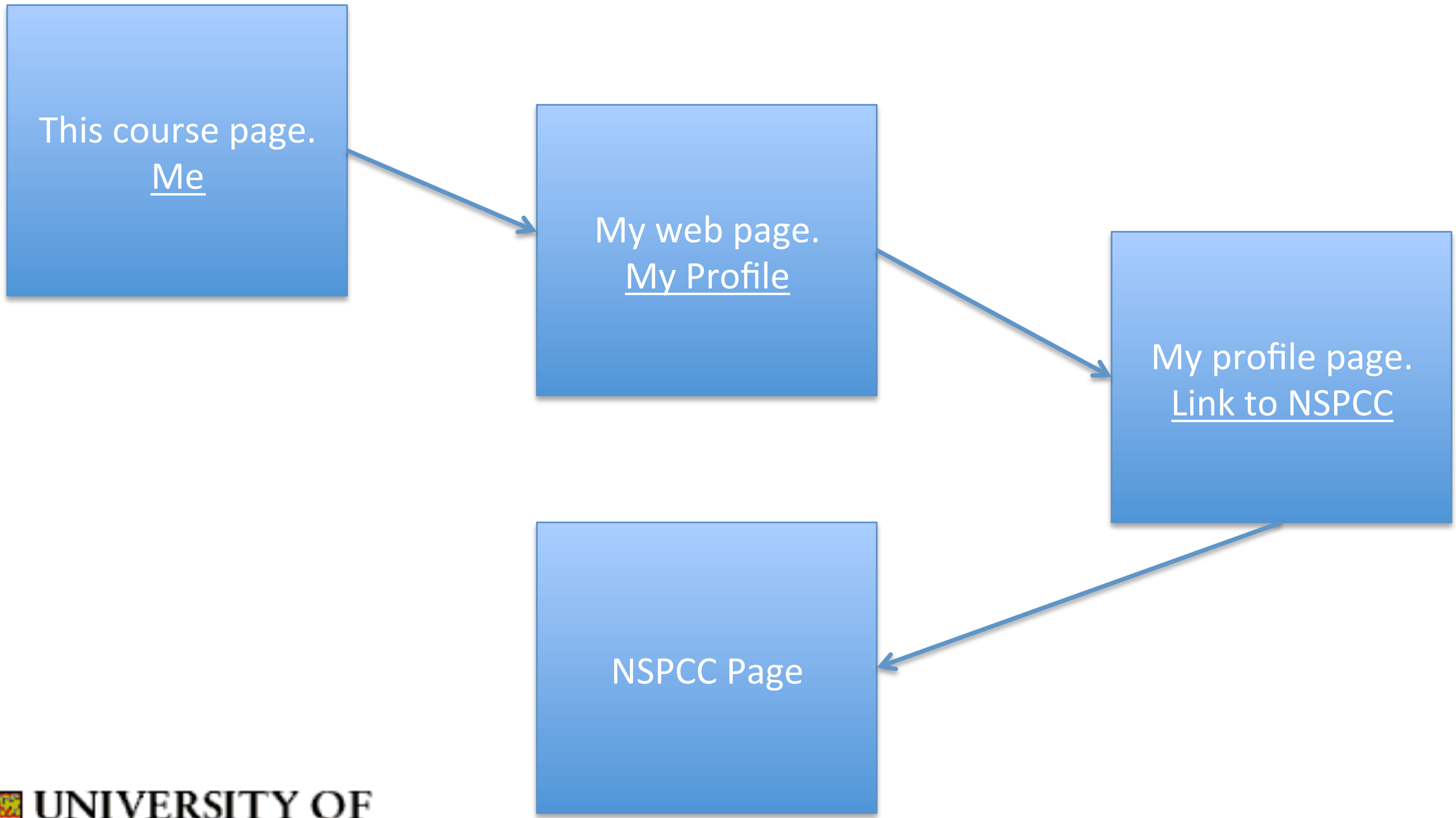


The Web is a Graph…



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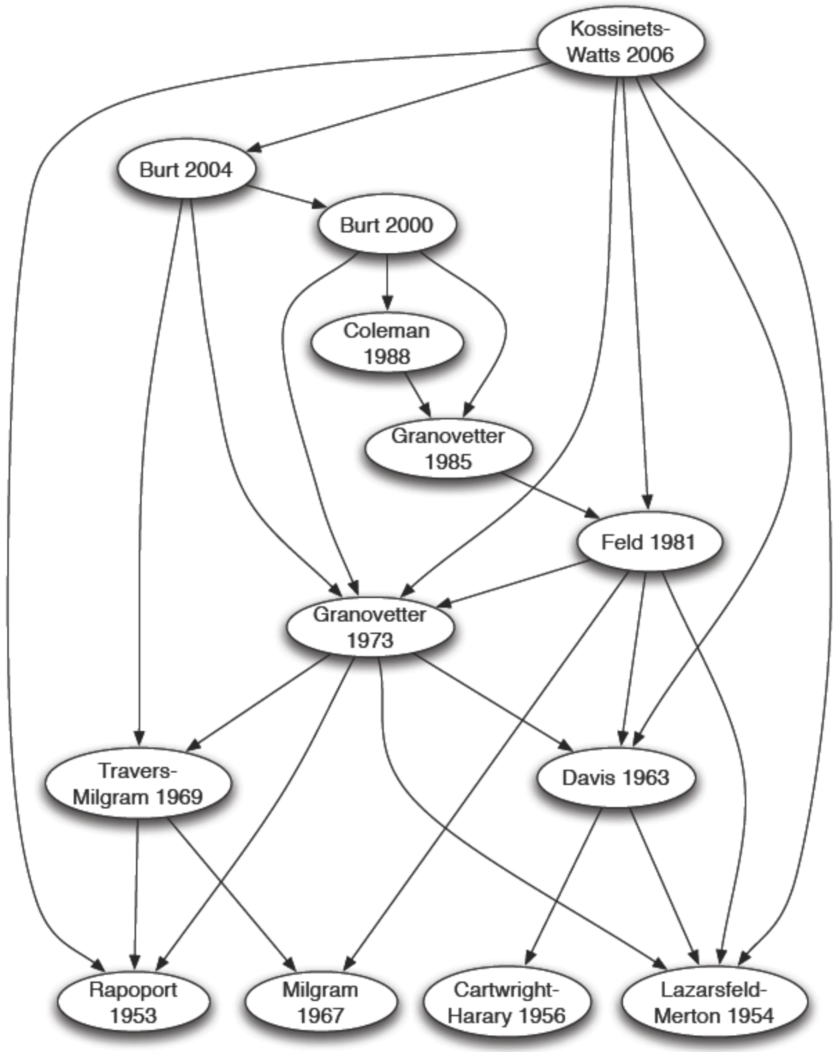
Precursor of hypertexts

- Citation networks of books and articles.
- Difference: links point only backwards in time



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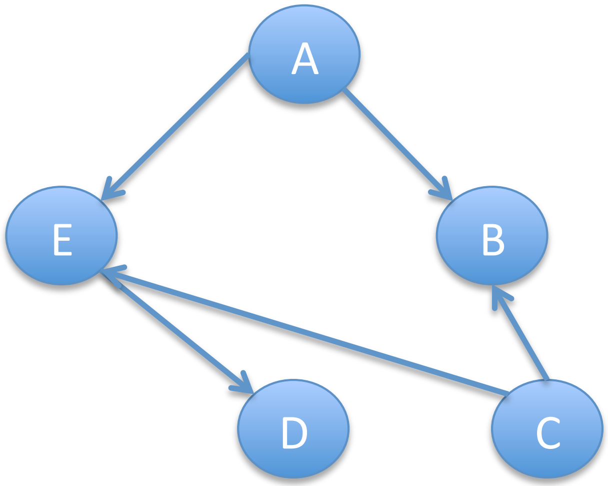
Web is a **Directed** Graph

- **Path**: A path from A to B exists if there is a sequence of nodes beginning with A and ending with B such that each consecutive pair of nodes is connected by an edge pointing in the forward direction.



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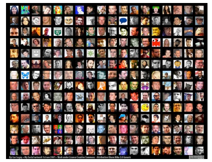
Strongly Connected Component



- A strongly connected component (SCC) in a directed graph is a subset of nodes such that:

  i) Every pair in the subset has a path to each other
  ii) The subset is not part of some larger subset with property i)
- Weakly connected component (WCC) is the connected component in **the undirected** graph derived from the directed graph.
  – Two nodes can be in the same WCC even if there no directed path between them.

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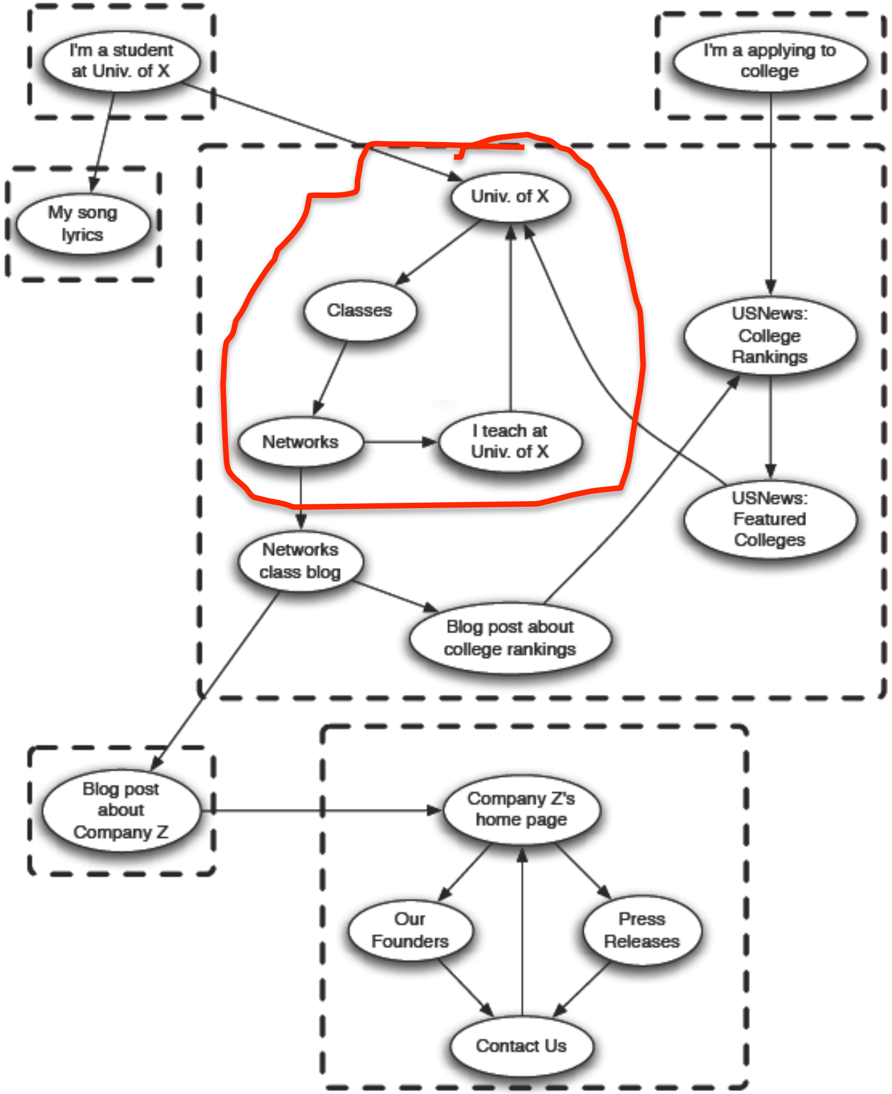
SCC example



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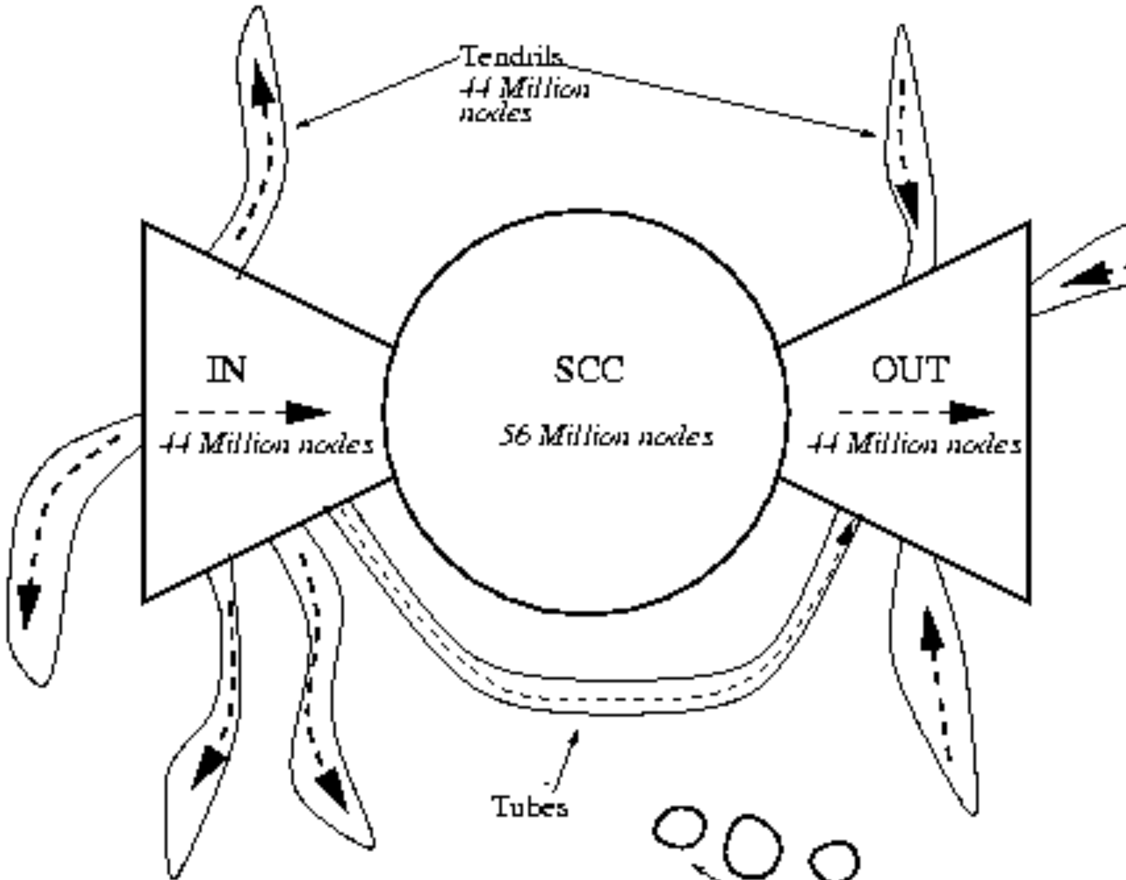
The Web

- Broder'00
- Data from Altavista (200 million pages)
- 186M nodes in the WCC (90% of links)



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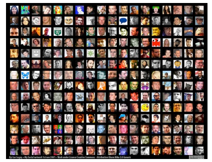

Popularity of Web Pages

- How do we expect the popularity of web pages to be distributed?
  - What fraction of web pages have k in-links?

  - If each page decides independently at random whether to link to any given other page then the n of in-links of a page is the sum of independent random quantities -> normal distribution
  - In this case, the number pages with k in-links decreases exponentially in k
  - Is this true for the Web?

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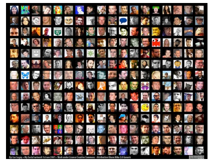
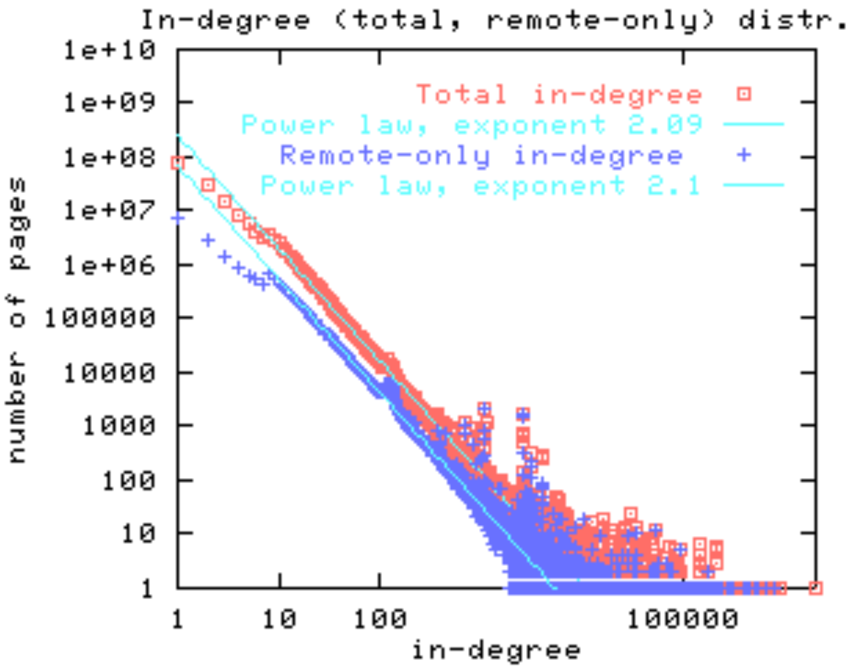
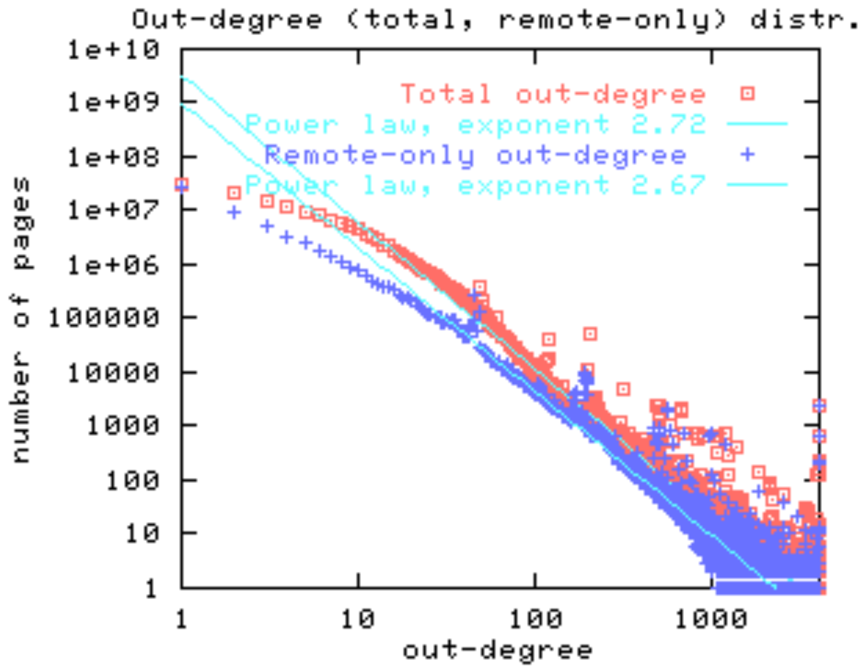
Degree distribution for the Web



- Finding: degree distr. proportional to ~$1/k^2$
- $1/k^2$ decreases much more slowly than a normal distribution





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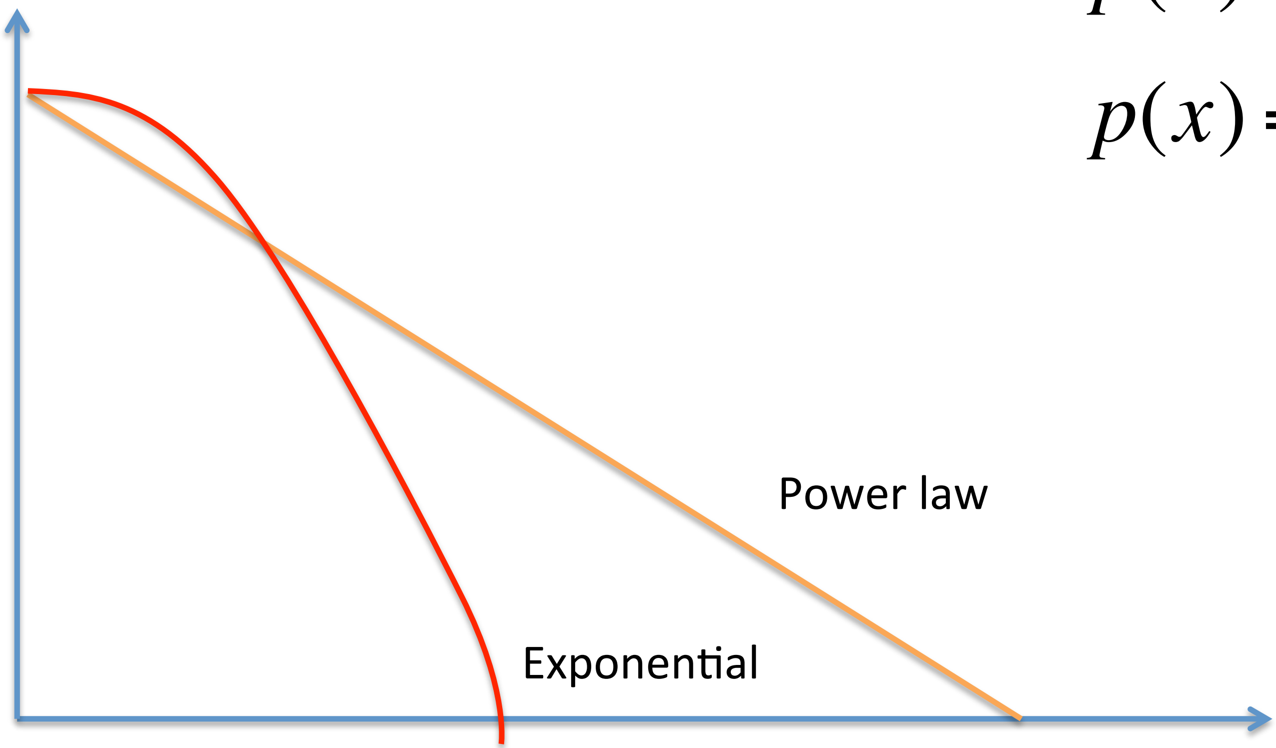
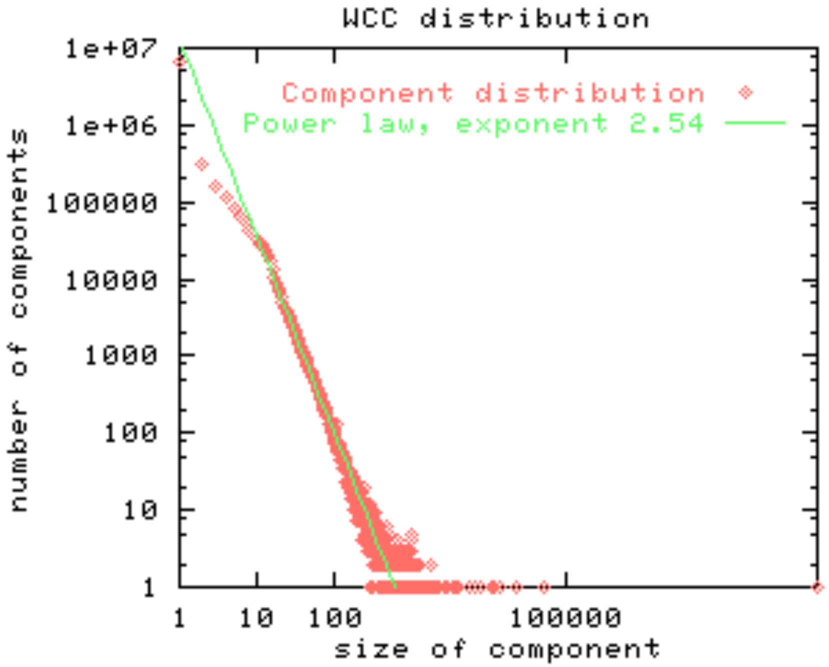
Power Law vs Exponential

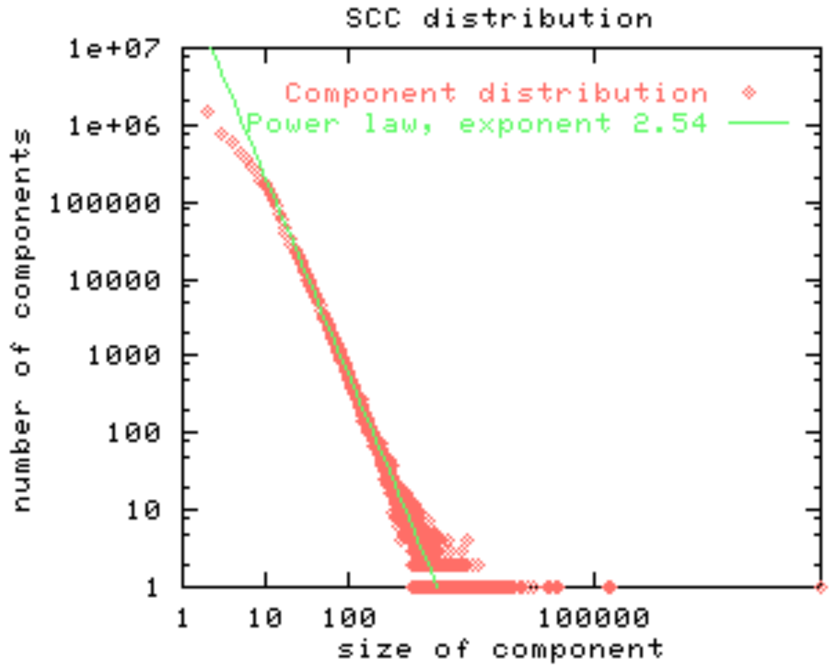
$$p(x) = x^{-\alpha}$$

$$p(x) = e^{-\lambda x}$$



Distribution of WCC and SCC





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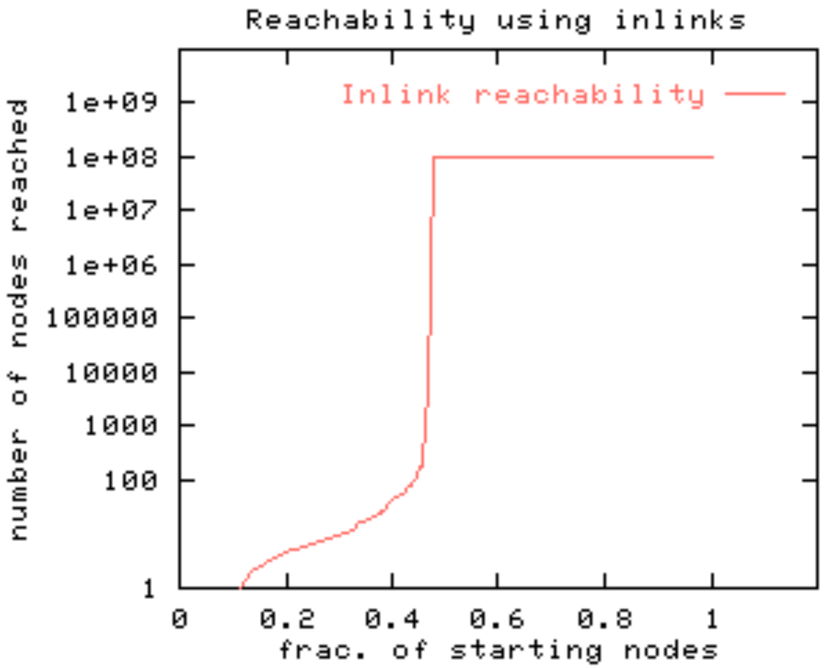
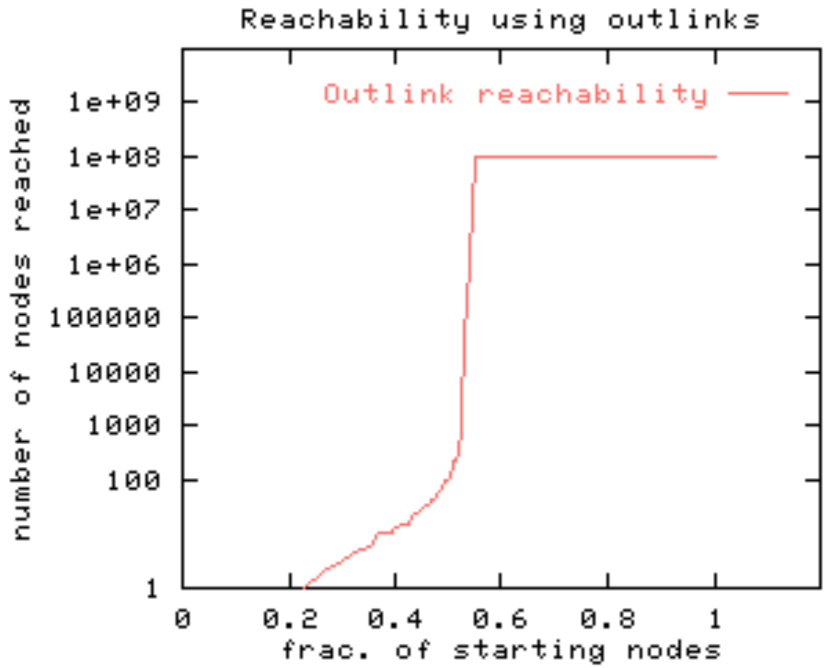
Reachability

- Followed links backwards and forward





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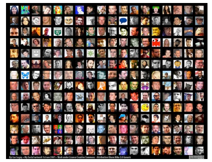
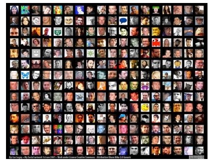
Diameter of the Web



- 75% of the time there is no directed path between two random nodes
- Average distance of existing paths: 16
- Average distance of undirected paths: 6.83

- Diameter in the SCC is at least 28

Power Laws aka Scale Free Networks



- We have seen that the degree distribution followed a straight line in log-log

$$\ln p_k = -\alpha \ln k + c$$

$$p_k = Ck^{-\alpha}$$

- α defines the slope of the curve
- α is typically between 2 and 3.

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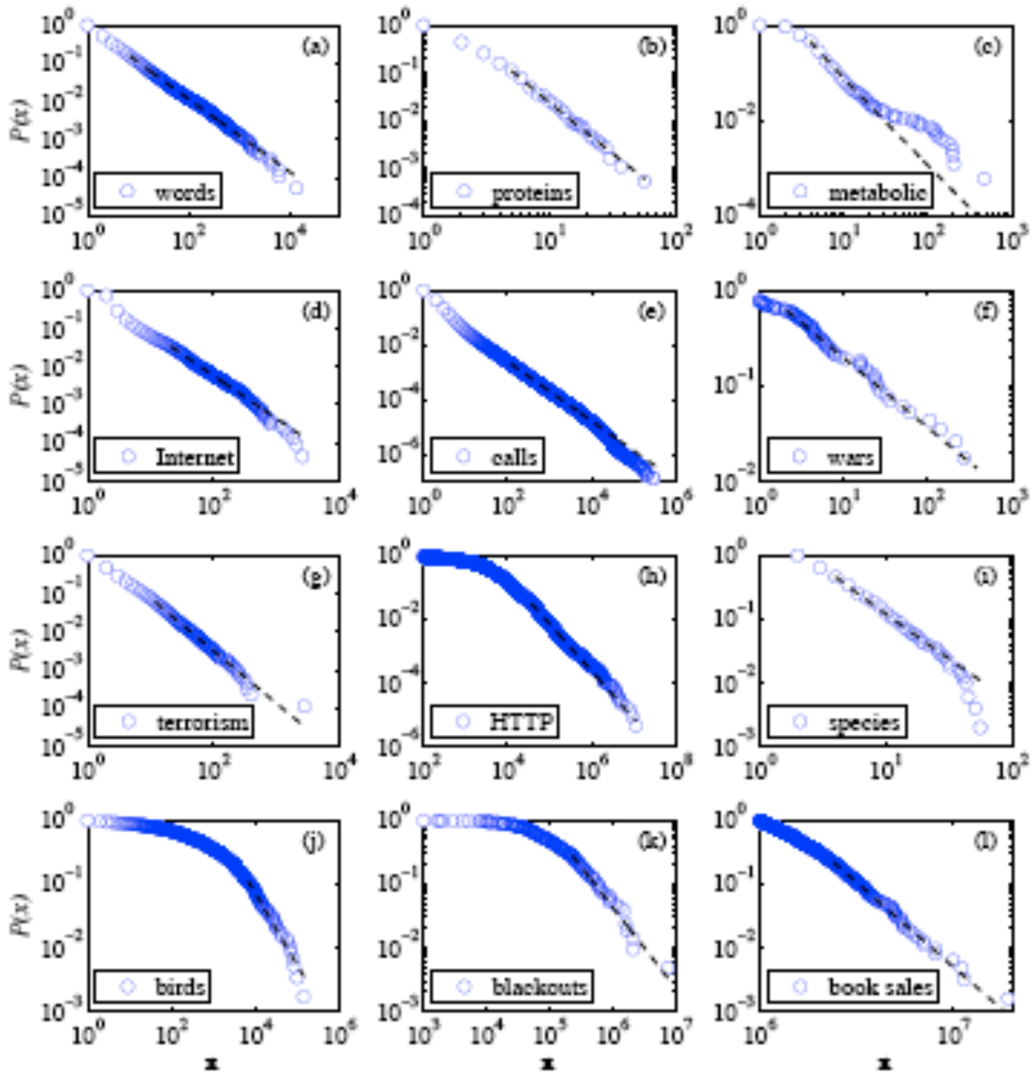
Power Laws in various domains



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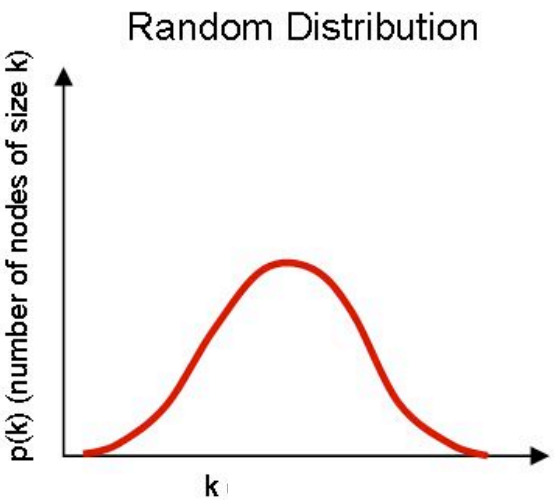
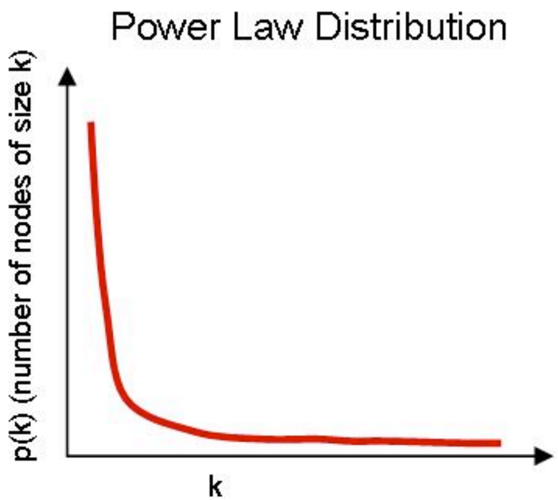
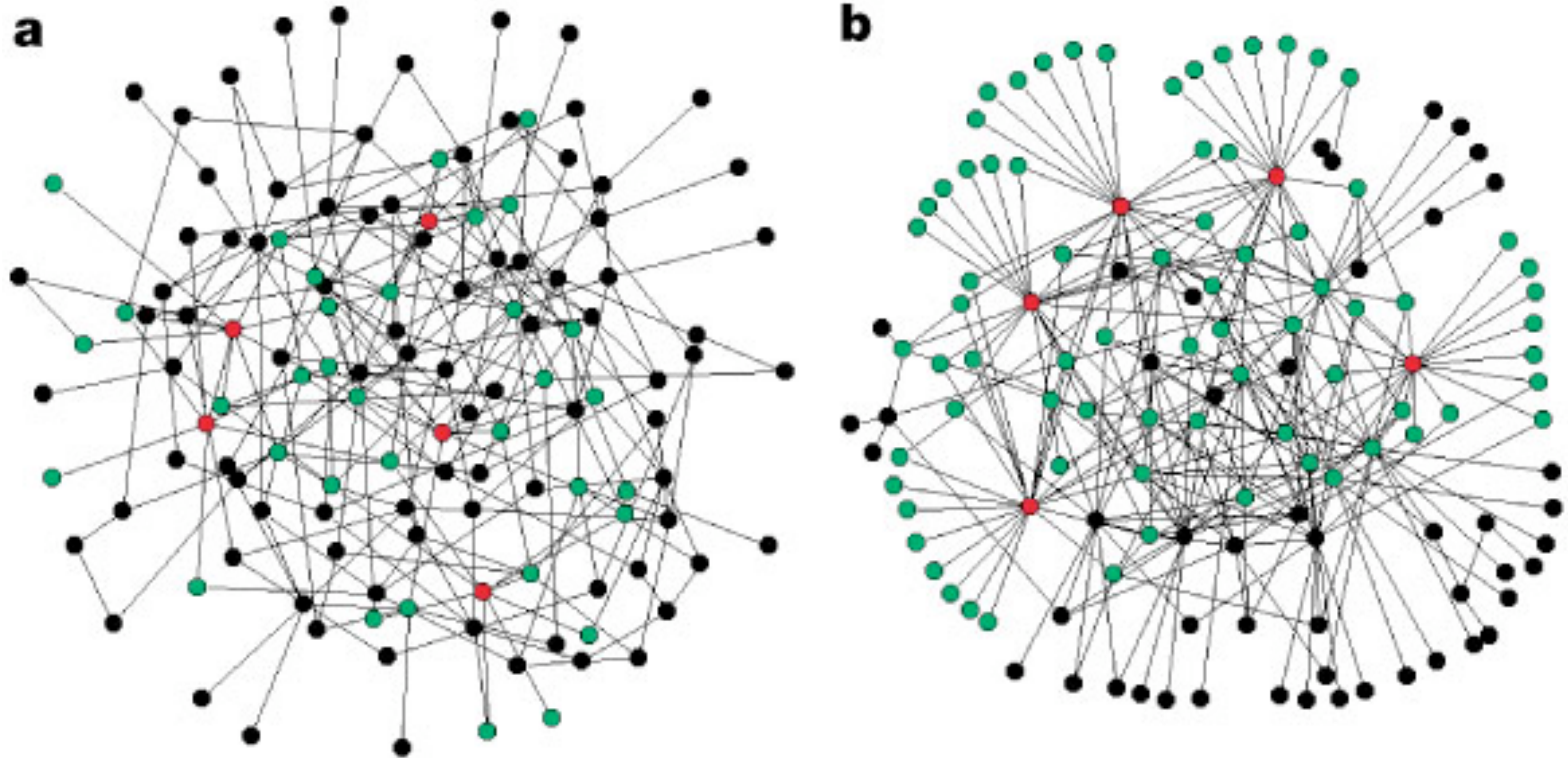
What does it mean?

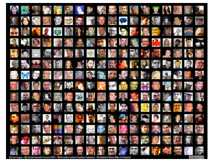




Random vs Power Law Networks



What's a good model for scale free networks



- Let's use the web network as example:
- Pages are created in order (1,2,3..)
- Page j created and it links to an earlier page in the following way:
  - With prob. p, j chooses page i at random and links it;
  - With prob. 1-p, j chooses page i and links to the page i points to.
  - Repeat.
- The middle step is essentially a copy of the node i behaviour…

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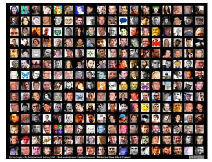
Preferential attachment



- Pages are created in order (1,2,3..)
- Page j created and it links to an earlier page in the following way:
  - With prob. p, j chooses page i at random and links it;
  - **With prob. 1-p, j chooses a page z with prob. proportional to z's current number of in-links and links to z (ie proportional to degree).**
  - Repeat.

Rich-get-richer model

If we run this for many pages the fraction of pages with k in-links will be distributed approximately according to a power law $1/k^c$
c depends on p

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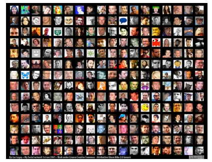

Intuition

- With probability 1-p page j chooses a page l with probability proportional to l’s number of inlinks and creates a link to l.
- This mechanism predicts that the growth happens so that
  - A page’s popularity growth at a rate proportional to its current value.
  - The rich get richer effect amplifies the larger values

Preferential Attachment

- What have we shown?
- There is a "copying" behaviour happening in these networks where node seem to emulate other nodes.
- This is shown true for selection of books, songs, web pages, movies etc.

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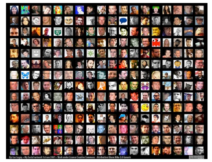
How predictable is the rich-get-richer process?



- Is the popularity of items in the power law predictable?
- Would a popular book still be popular if we go back in time and start the process again?
- Experiments show it would not…

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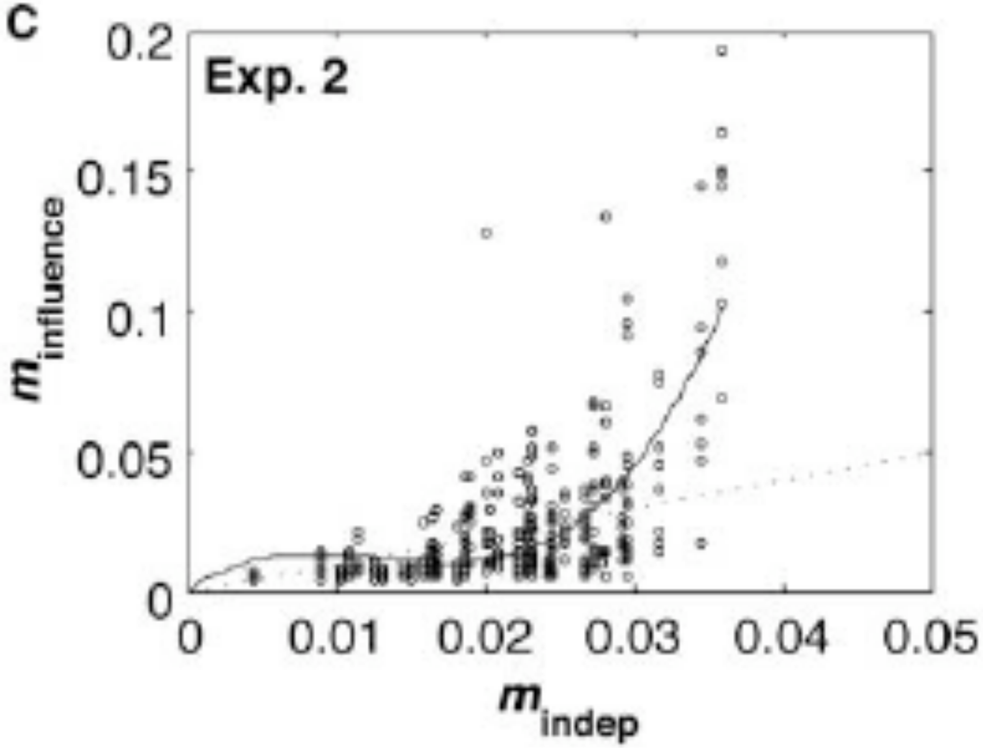
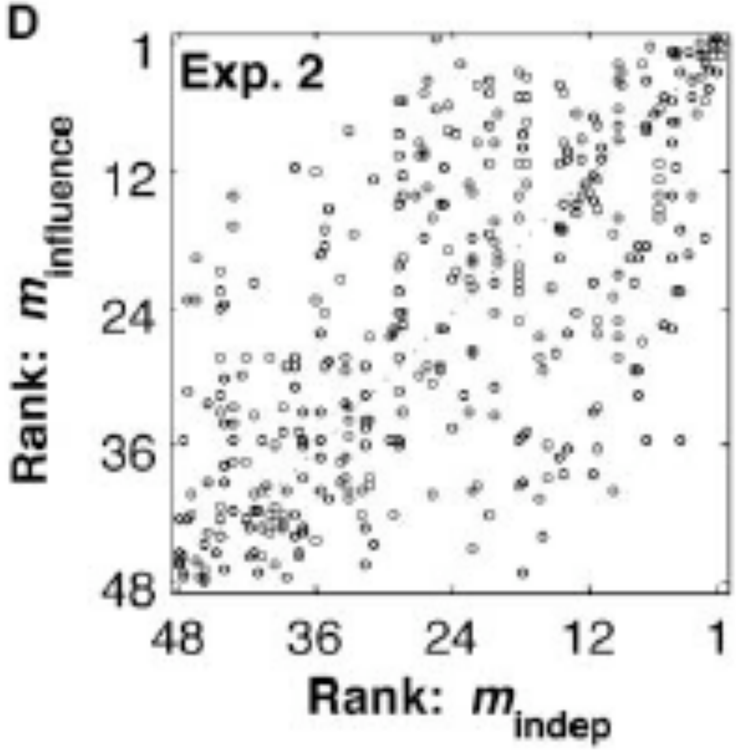
Unpredictability [Salganik et al 06]

- 48 songs, 14,000 participants, 8 servers





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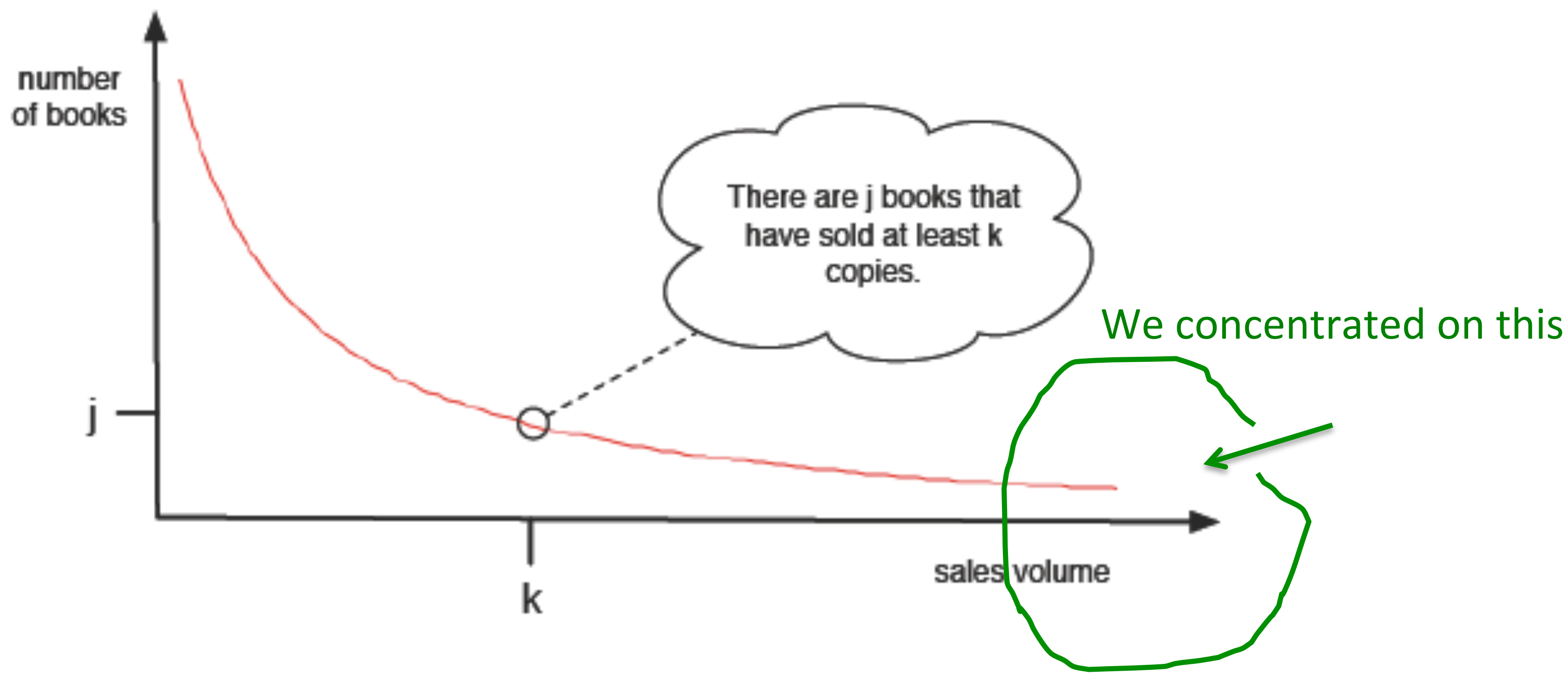
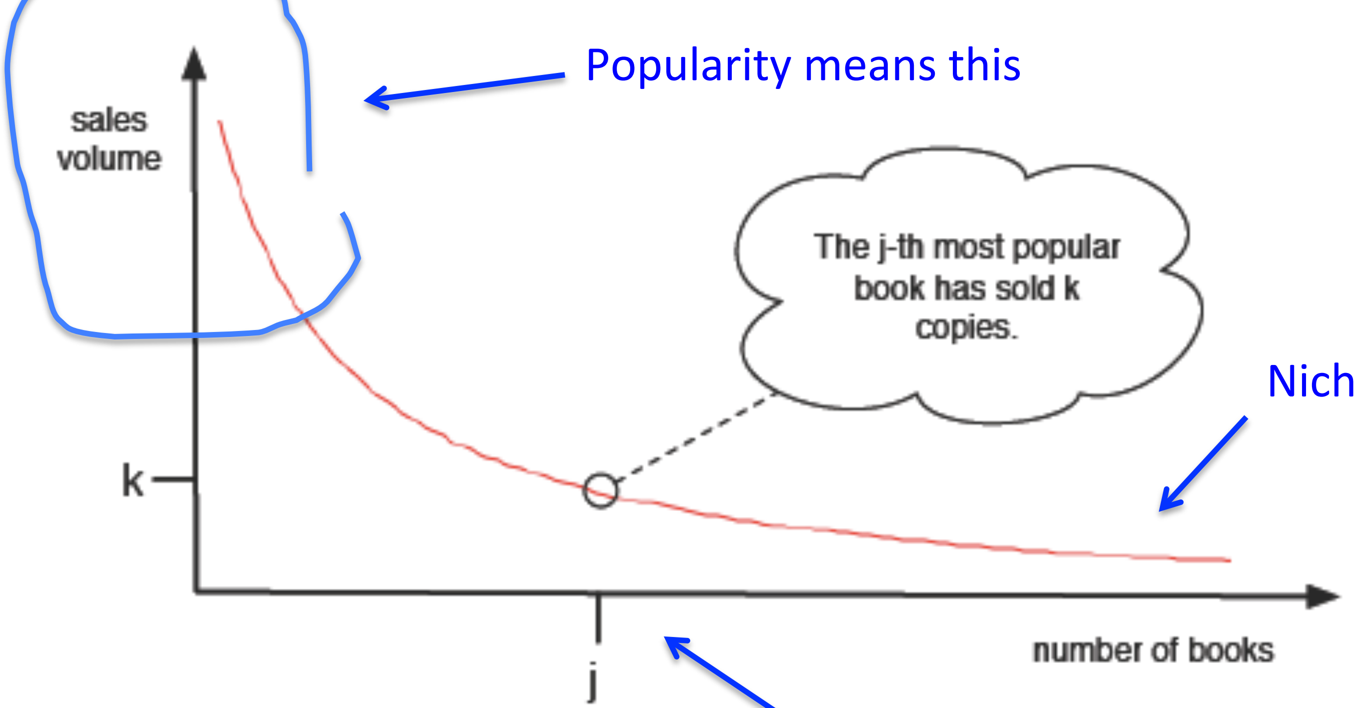
View of the curve

- The way we have seen the curve so far…



Let’s transform the function

- If the initial function is a power law, this one is too (we do not prove this)



Popularity means this

Niche tastes

Sale ranking

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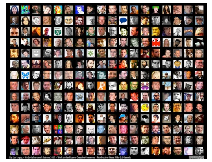
Search



- Information retrieval problem: synonyms (jump/leap), polysemy (Leopard), etc
- Now with the web: diversity in authoring introduces issues of common criteria for ranking documents
- The web offers abundance of information: whom do we trust as source?

- Still one issue: static content versus real time
  - World trade center query on 11/9/01
  - Twitter helps solving these issues these days

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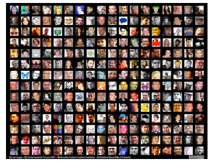
Automate the Search



- When searching "Computer Laboratory" on Google the first link is for the department's page.

- How does Google know this is the best answer?
- We could collect a large sample of pages relevant to "computer laboratory" and collect their votes through their links.
- The pages receiving more in-links are ranked first.
- But if we use **the network structure** more deeply we can improve results.

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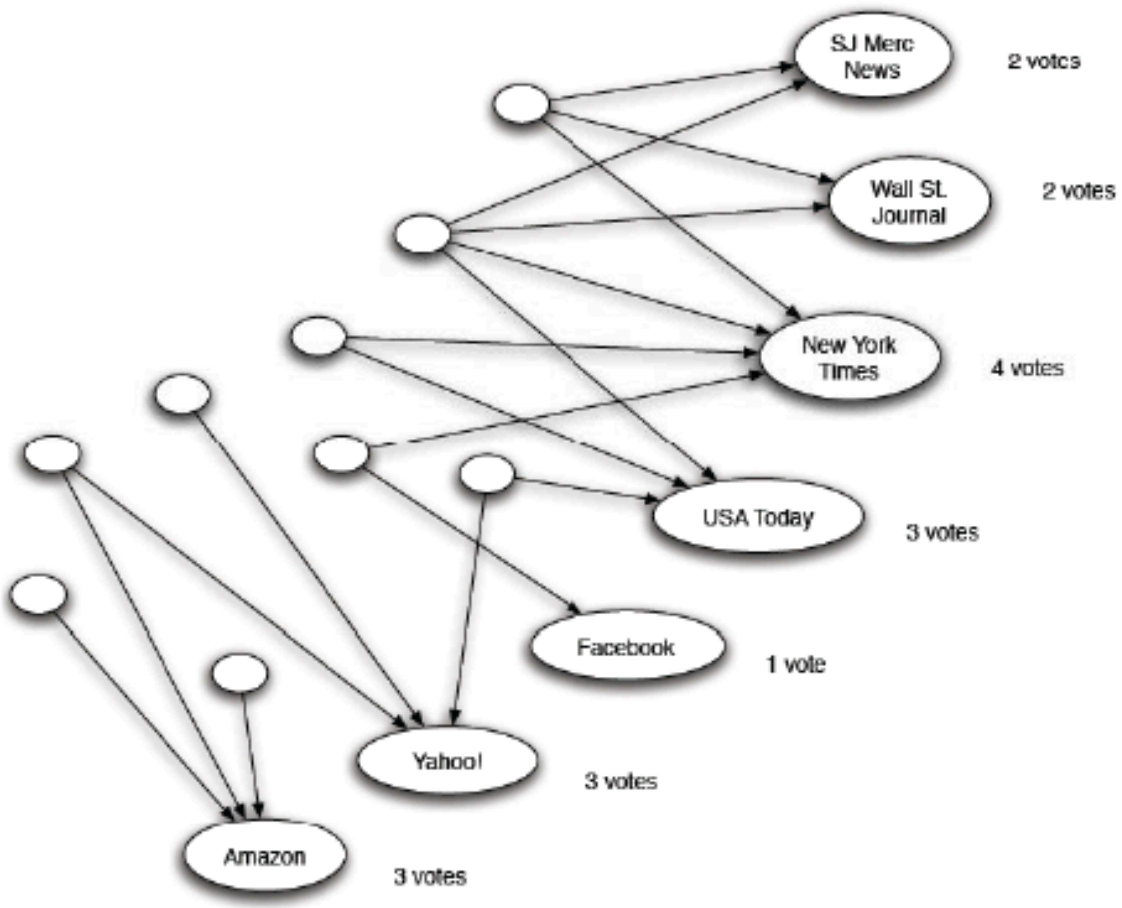
Example: Query "newspaper" Authorities

- Links are seen as votes.
- **Authorities** are established: the highly endorsed pages



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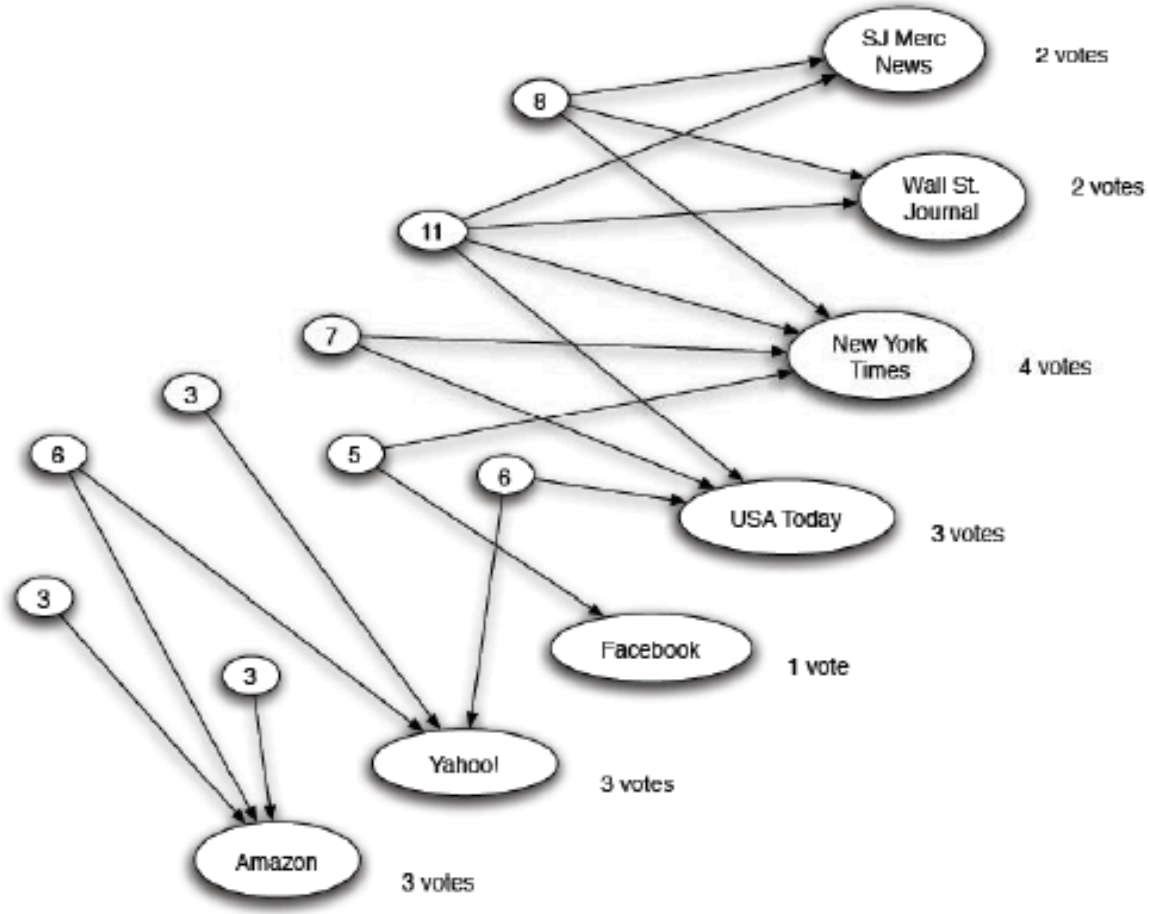
A Refinement: Hubs

- Numbers are reported back on the source page and aggregate.
- Hubs are high value lists



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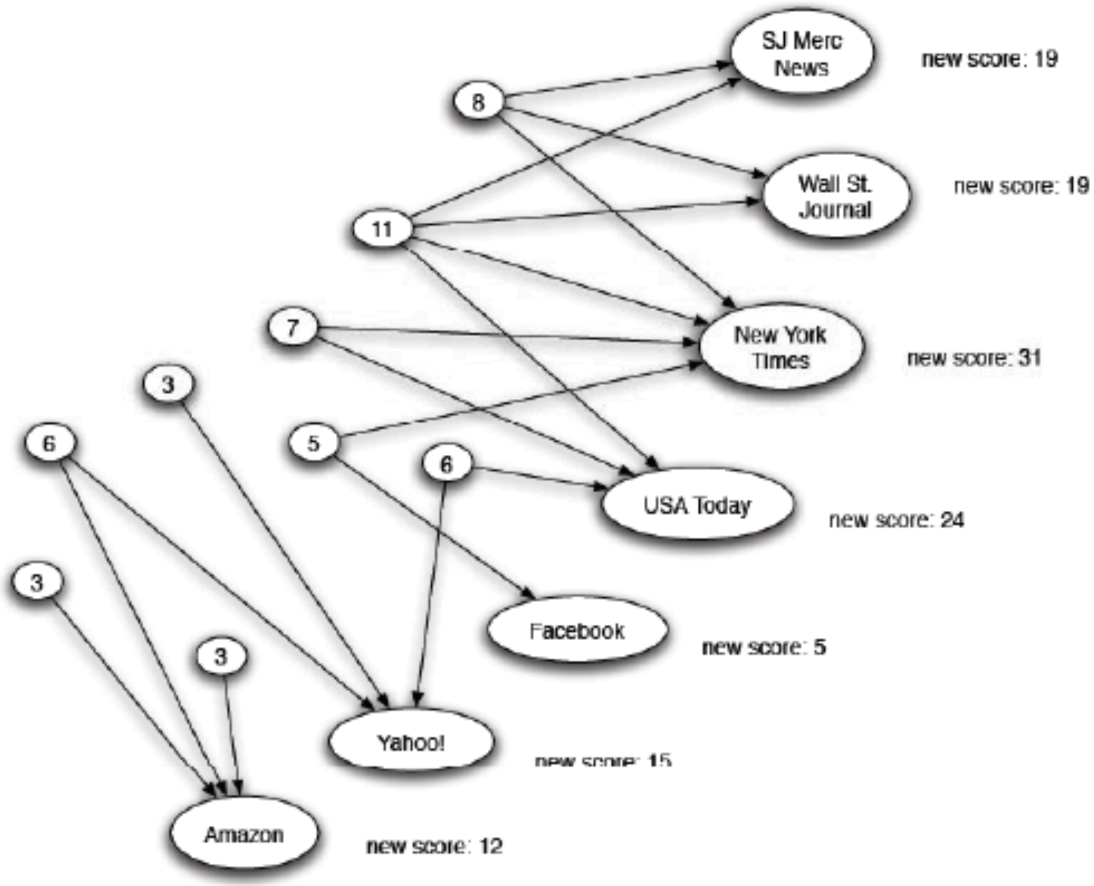
Principle of Repeated Improvement

- And we are now reweighting the authorities

- When do we stop?



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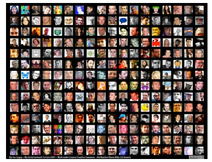
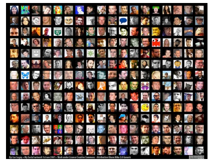
Repeating and Normalizing



- The process can be repeated
- Normalization:
  - Each authority score is divided by the sum of all authority scores
  - Each hub score is divided by the sum of all hub scores

More Formally: does the process converge?



- Each page has an authority a_i and a hub h_i score
- Initially $a_i = h_i = 1$
- At each step
$$a_i = \sum_{j->i} h_j$$
$$h_j = \sum_{j->i} a_i$$
- Normalize
$$\sum a_i = 1$$
$$\sum h_j = 1$$

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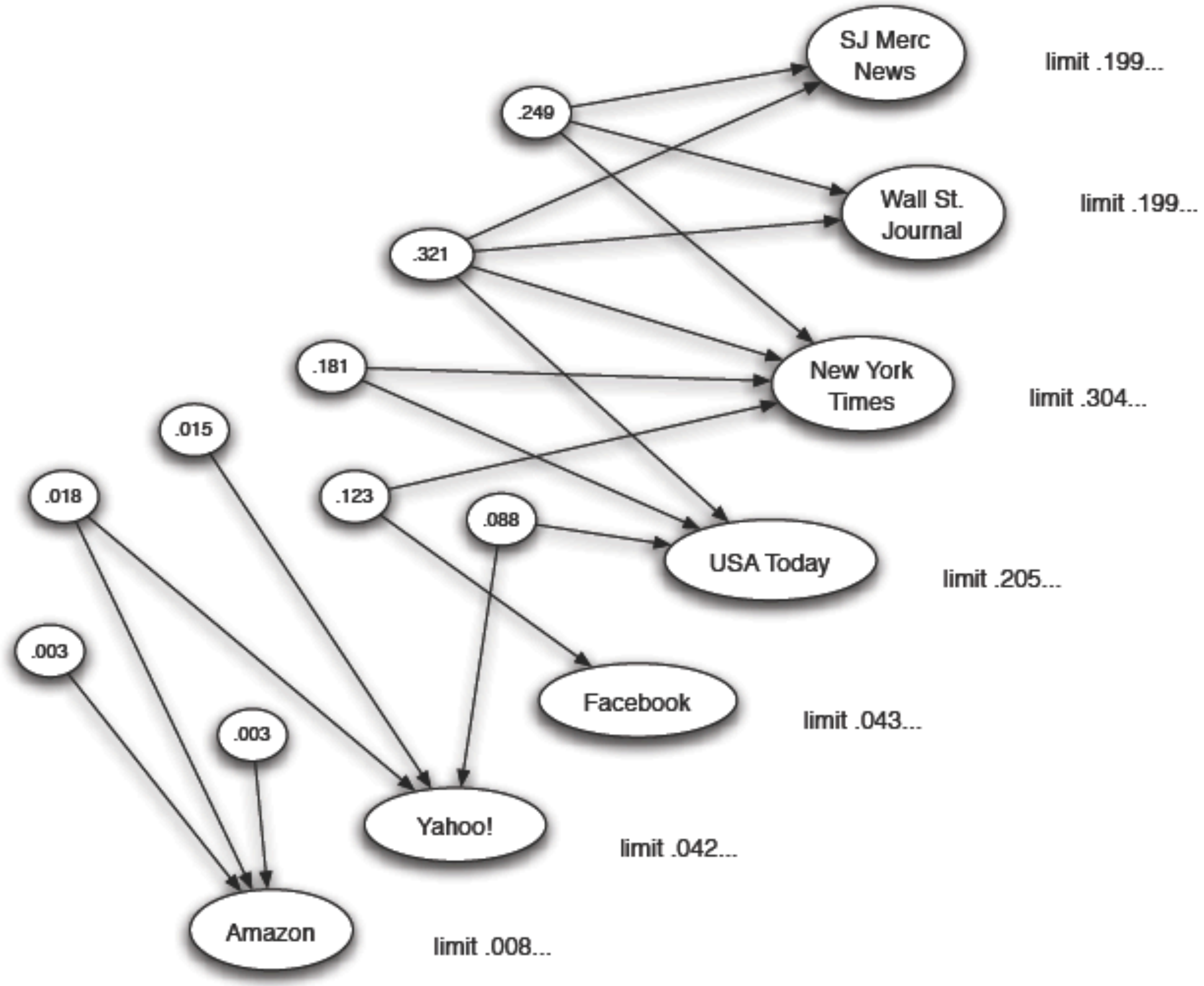
The process converges



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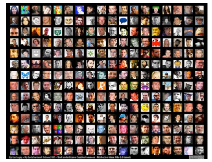

PageRank

- We have seen hubs and authorities
  - Hubs can “collect” links to important authorities who do not point to each others
  - There are other models: better for the web, where one prominent can endorse another.
- The **PageRank** model is based on transferrable importance.

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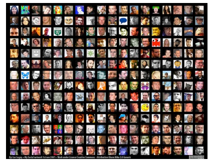
PageRank Concepts



- Pages pass endorsements on outgoing links as fractions which depend on out-degree
- Initial PageRank value of each node in a network of n nodes: 1/n.
- Choose a number of steps k.
- **[Basic] Update rule**: each page divides its pagerank equally over the outgoing links and passes an equal share to the pointed pages. Each page's new rank is the sum of received pageranks.

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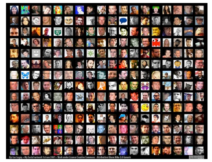
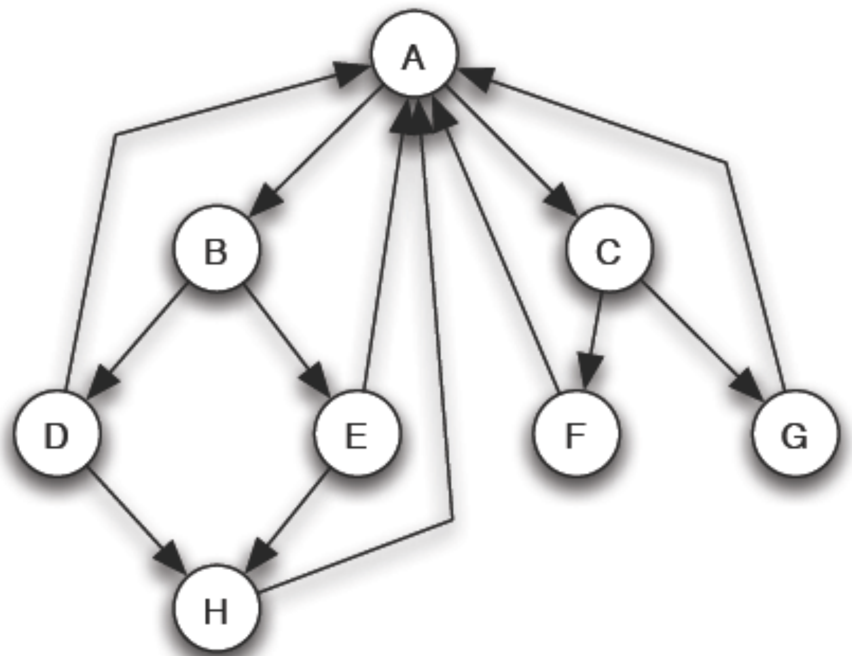


Example

- All pages start with PageRank= 1/8

Step	A	B	C	D	E	F	G	H
1	1/2	1/16	1/16	1/16	1/16	1/16	1/16	1/8
2	3/16	1/4	1/4	1/32	1/32	1/32	1/32	1/16

A becomes important and
B,C benefit too at step 2



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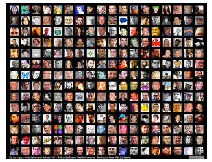
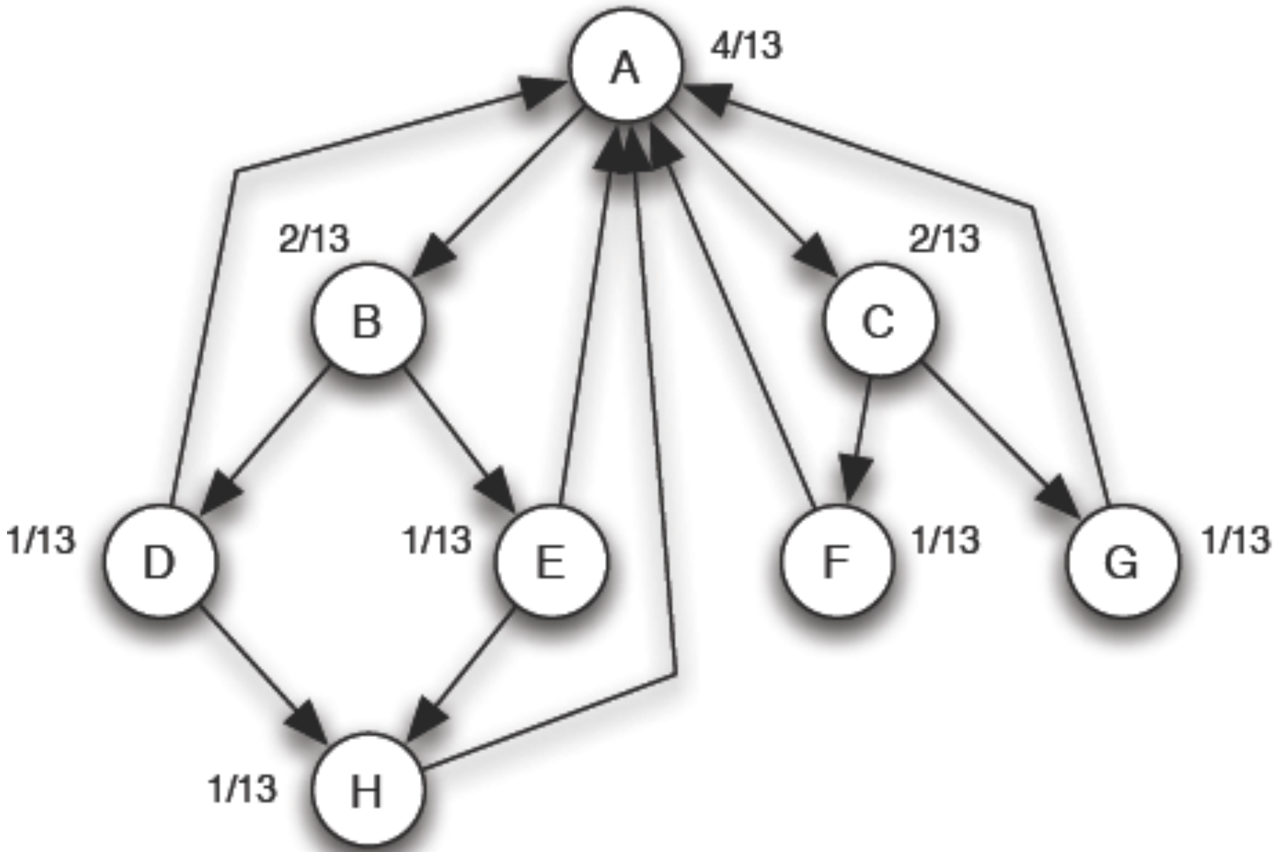


Convergence

- Except for some special cases, PageRank values of all nodes converge to limiting values when the number of steps goes to infinity.
- The convergence case is one where the PageRank of each page does not change anymore, i.e., they regenerate themselves.

Example of Equilibrium



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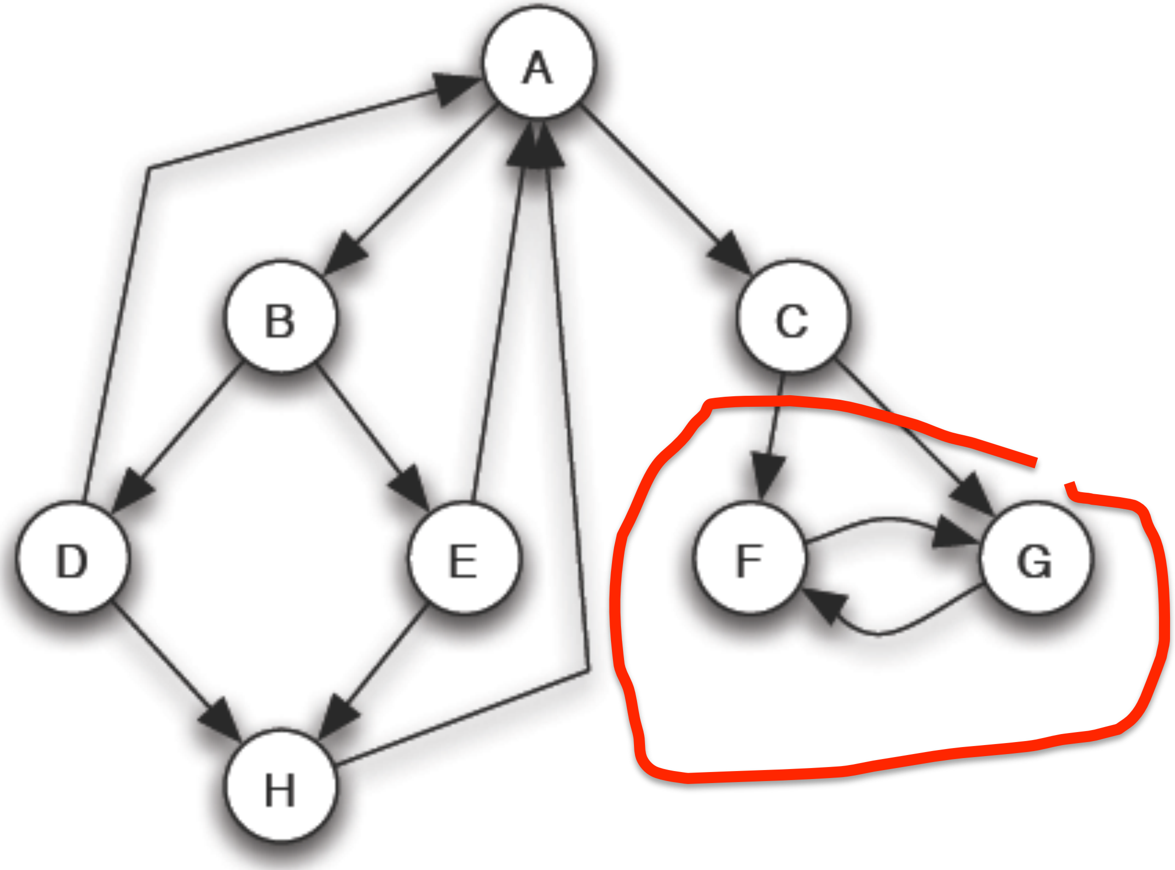
Problems with the basic PageRank
Dead ends

- F,G converge to ½ and all the other nodes to 0



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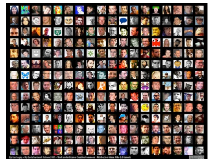
Solution: The REAL PageRank



- **[Scaled] Update Rule:**
  - Apply basic update rule. Then, scale down all values by scaling factor s [chosen between 0 and 1].
  - **[Total network PageRank value changes from 1 to s]**
  - Divide 1-s residual units of PageRank equally over all nodes: (1-s)/n each.

- It can be proven that values converge again.
- Scaling factor usually chosen between 0.8 and 0.9

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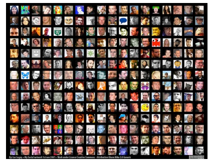
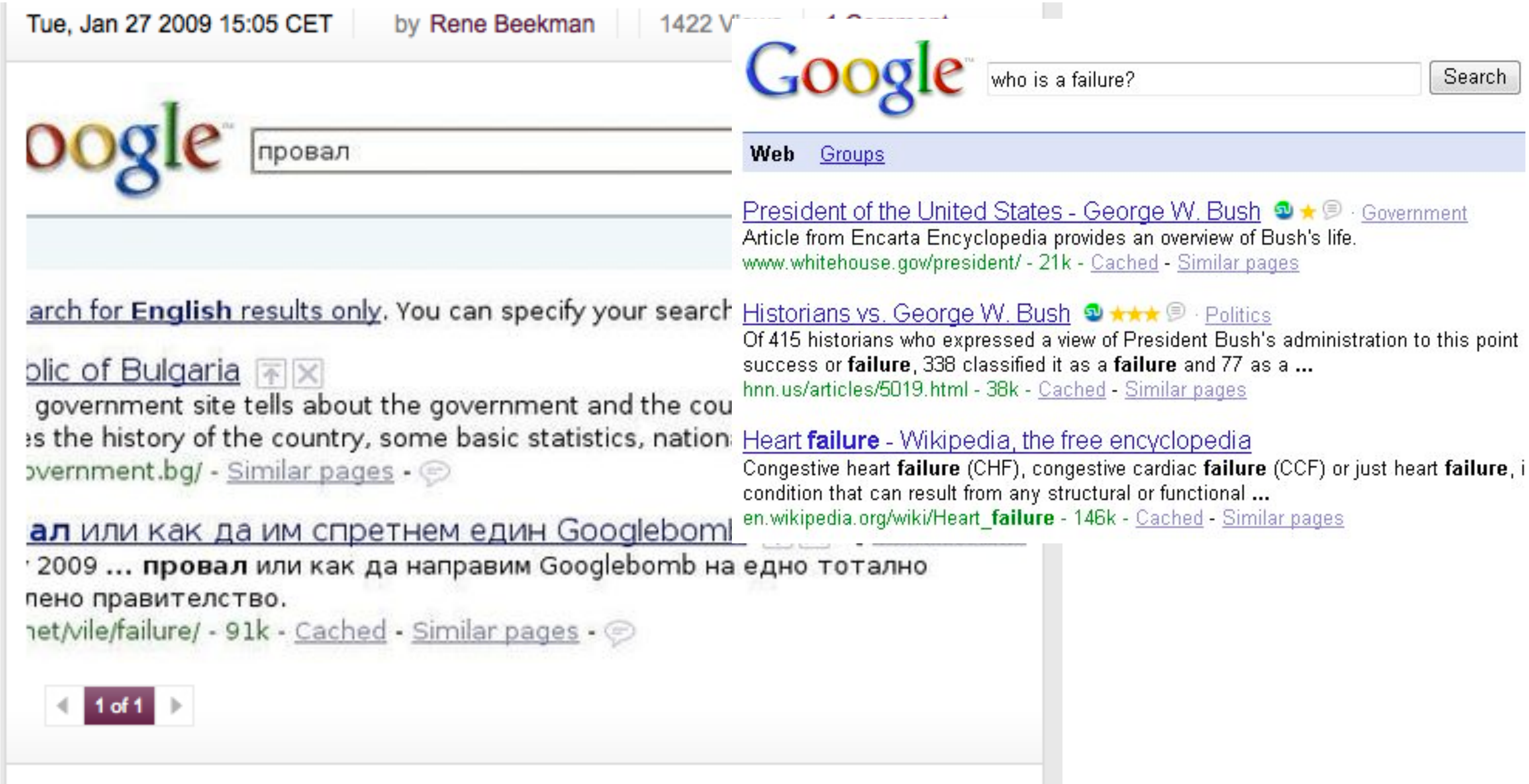
Search Ranking is very important to business



- A change in results in the search pages might mean loss of business
  - I.e., not appearing on first page.
- Ranking algorithms are kept very secret and changed continuously.

Examples of Google Bombs



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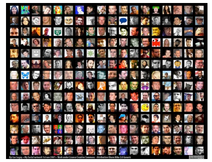
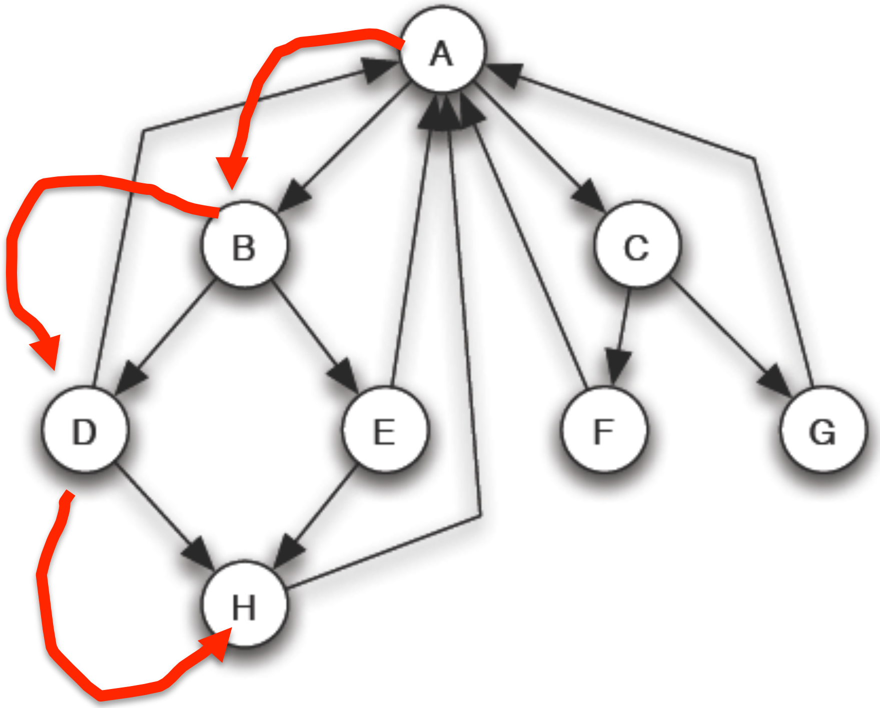
Random Walks



- Starting from a node, follow one outgoing link with random probability



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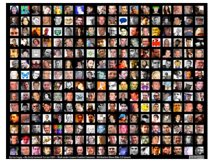
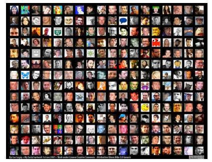
PageRank as Random Walk



- The probability of being at a page X after k steps of a random walk is precisely the PageRank of X after k applications of the Basic PageRank Update Rule
- Scaled Update Rule equivalent: follow a random outgoing link with probability s while with probability 1-s jump to a random node in the network.

References



- Chapter 13, 14 and 18

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