

Complexity Theory

Lent 2012

Suggested Exercises 2

1. Given a graph $G = (V, E)$, a set $U \subseteq V$ of vertices is called a *vertex cover* of G if, for each edge $(u, v) \in E$, either $u \in U$ or $v \in U$. That is, each edge has at least one end point in U . The decision problem **V-COVER** is defined as:

given a graph $G = (V, E)$, and an integer K , does G contain a vertex cover with K or *fewer* elements?

- (a) Show a polynomial time reduction from **IND** to **V-COVER**.
- (b) Use (a) to argue that **V-COVER** is **NP**-complete.

2. The problem of four dimensional matching, **4DM**, is defined analogously with **3DM**:

Given four sets, W, X, Y and Z , each with n elements, and a set of quadruples $M \subseteq W \times X \times Y \times Z$, is there a subset $M' \subseteq M$, such that each element of W, X, Y and Z appears in exactly one triple in M' .

Show that **4DM** is **NP**-complete.

3. Given a graph $G = (V, E)$, a *source vertex* $s \in V$ and a *target vertex* $t \in V$, a *Hamiltonian Path* from s to t in G is a path that begins at s , ends at t and visits every vertex in V exactly once. We define the decision problem **HamPath** as:

given a graph $G = (V, E)$ and vertices $s, t \in V$, does G contain a Hamiltonian path from s to t ?

- (a) Give a polynomial time reduction from the Hamiltonian cycle problem to **HamPath**.
- (b) Give a polynomial time reduction from **HamPath** to the problem of determining whether a graph has a Hamiltonian cycle.

Hint: consider adding a vertex to the graph.

4. We know from Cook-Levin theorem that every problem in **NP** is reducible to **SAT**. Sometimes it is easy to give an explicit reduction. In this exercise you are asked to give such explicit reductions for two graph problems: **3-Col** and **HAM**. That is,

- (a) describe how to obtain, for any graph $G = (V, E)$, a Boolean expression ϕ_G so that ϕ_G is satisfiable if, and only if, G is 3-colourable; and
- (b) describe how to obtain, for any graph $G = (V, E)$, a Boolean expression ϕ_G so that ϕ_G is satisfiable if, and only if, G contains a Hamiltonian cycle.