Interactive Formal Verification (L21)

1 Replace, Reverse and Delete

▷ Define a function \texttt{replace}, such that \texttt{replace} \( x \ y \ zs \) yields \( zs \) with every occurrence of \( x \) replaced by \( y \).

\[
\texttt{replace} :: \ \langle'\texttt{a} \Rightarrow '\texttt{a} \Rightarrow '\texttt{a} \ 	exttt{list} \Rightarrow '\texttt{a} \ 	exttt{list}\rangle
\]

▷ Prove or disprove (by counterexample) the following theorems. You may have to prove some lemmas first.

- \texttt{theorem} "\texttt{rev} (\texttt{replace} \ x \ y \ zs) = \texttt{replace} \ x \ y \ (\texttt{rev} zs)"
- \texttt{theorem} "\texttt{replace} \ x \ y \ (\texttt{replace} \ u \ v \ zs) = \texttt{replace} \ u \ v \ (\texttt{replace} \ x \ y \ zs)"
- \texttt{theorem} "\texttt{replace} \ y \ z \ (\texttt{replace} \ x \ y \ zs) = \texttt{replace} \ x \ z \ zs"

▷ Define two functions for removing elements from a list: \texttt{del1} \( x \) \( xs \) deletes the first occurrence (from the left) of \( x \) in \( xs \), \texttt{delall} \( x \) \( xs \) all of them.

\[
\texttt{del1} :: \ \langle'\texttt{a} \Rightarrow '\texttt{a} \ 	exttt{list} \Rightarrow '\texttt{a} \ 	exttt{list}\rangle
\]
\[
\texttt{delall} :: \ \langle'\texttt{a} \Rightarrow '\texttt{a} \ 	exttt{list} \Rightarrow '\texttt{a} \ 	exttt{list}\rangle
\]

▷ Prove or disprove (by counterexample) the following theorems.

- \texttt{theorem} "\texttt{del1} \ x \ (\texttt{delall} \ x \ xs) = \texttt{delall} \ x \ xs"
- \texttt{theorem} "\texttt{delall} \ x \ (\texttt{delall} \ x \ xs) = \texttt{delall} \ x \ xs"
- \texttt{theorem} "\texttt{delall} \ x \ (\texttt{delall} \ x \ xs) = \texttt{delall} \ x \ xs"
- \texttt{theorem} "\texttt{delall} \ x \ (\texttt{delall} \ y \ zs) = \texttt{delall} \ y \ (\texttt{delall} \ x \ zs)"
- \texttt{theorem} "\texttt{delall} \ x \ (\texttt{delall} \ x \ zs) = \texttt{delall} \ y \ (\texttt{delall} \ x \ zs)"
- \texttt{theorem} "\texttt{delall} \ y \ (\texttt{replace} \ x \ y \ xs) = \texttt{delall} \ x \ xs"
- \texttt{theorem} "\texttt{delall} \ y \ (\texttt{replace} \ x \ y \ zs) = \texttt{delall} \ z \ (\texttt{replace} \ x \ y \ zs)"
- \texttt{theorem} "\texttt{replace} \ x \ y \ (\texttt{delall} \ z \ zs) = \texttt{delall} \ z \ (\texttt{replace} \ x \ y \ zs)"
- \texttt{theorem} "\texttt{rev} (\texttt{delall} \ x \ xs) = \texttt{delall} \ x \ (\texttt{rev} \ xs)"
- \texttt{theorem} "\texttt{rev} (\texttt{delall} \ x \ xs) = \texttt{delall} \ x \ (\texttt{rev} \ xs)"