MPhil in Advanced Computer Science
Categorical logic

Leader:	Sam Staton
Timing:	Lent
Prerequisites:	'Category Theory for Computer Science', and some basic knowledge of logic and set theory
Structure:	16 Lectures

AIMS

In this module we will investigate models of constructive logics from the perspective of category theory. The course will be illustrated with examples from theoretical computer science.

SYLLABUS

1. **Simply-typed lambda calculus:** Intuitionistic logic. Propositions as objects in cartesian-closed categories (CCCs): the Curry-Howard correspondence. Examples of CCCs, including syntactic models. (4L)

2. **First-order logic.** 'Propositions as subobjects'. Finite limits, and Horn logic. Regular categories, and regular logic. Quantifiers as adjoints. Functors between categories. (5L)

3. **Higher-order logic.** Powerobjects and toposes. Properties of toposes. Toposes as set-theories. (3L)

4. **Dependent types.** Overview and examples of programming in Agda/Coq. (1L: a practical class.)

5. **Categorical type theory.** 'Propositions as arrows': dependent type theories in locally cartesian-closed categories. (3L)

OBJECTIVES

On completion of this module, students should:

- be familiar with various logical formalisms, including dependent type theory;
- be familiar with the basic principles of categorical logic for various fragments of logic.

COURSEWORK

I will hand out exercises, for each section of the course.

PRACTICAL WORK

One practical session will be timetabled for introducing a dependently-typed programming language, either Agda or Coq.

ASSESSMENT

The course will be assessed by a take home test, set and marked by the course lecturer. The test will account for 75% of the marks for this course. There will also be assessed practical exercises, accounting for the remaining 25% of the mark.

RECOMMENDED READING

This is a well-developed subject and there are many texts. Here are three recent ones.

Categorical logic. A. M. Pitts. In: *Handbook of Logic in Computer Science, Volume V: Algebraic and Logical Structures.* Edited by S. Abramsky, Dov M. Gabbay, and T. S. E. Maibaum. Oxford University Press, 2001. Available online.

Sketches of an elephant: a topos theory compendium. P. T. Johnstone. Oxford Logic Guides nos 43 and 44. Oxford University Press, 2002. (Portions of Chs A.1, A.2, D.1, D.4, are relevant.)

Practical foundations of mathematics. P. Taylor. Cambridge Studies in Advanced Mathematics, vol 59. Cambridge University Press, 1999. Partly available online.

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